A new dissipation mechanism for the spectral element dynamical core in the Community Atmosphere Model (CAM)

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Outline

The Spectral Element Method (SEM) and stabilization
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Variable resolution grids
Hyperviscosity for stabilization
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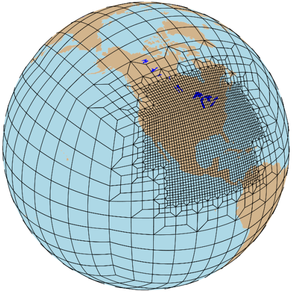
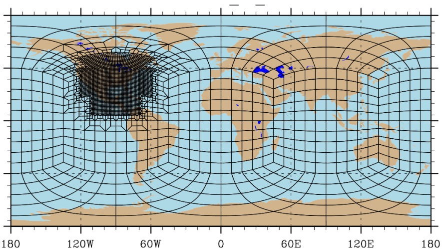
Convergence and performance
Topography smoothing





The Spectral Element Method and Stabilization

The Spectral Element Method (SEM)

- Continuous Galerkin finite element method with a diagonal mass matrix and Gauss-Lobatto quadrature leads to high scalability.
- Requires stabilization (hyperviscosity).

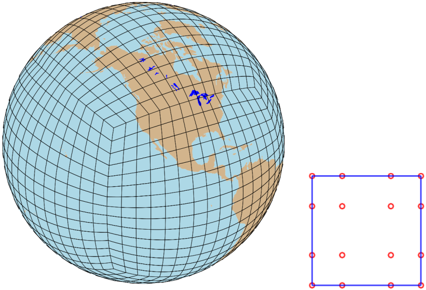
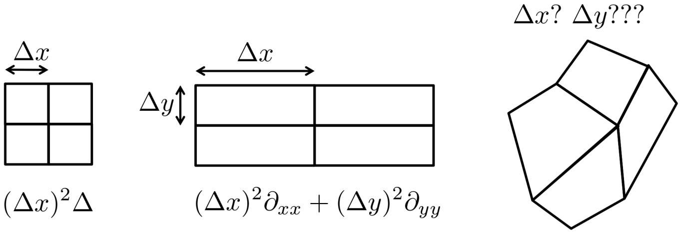


Figure: A uniform mesh

The Spectral Element Method and Stabilization

Both dynamics and tracers use vertical Lagrangian remap. For this research, only 2D discretizations are considered.
Stabilization is needed for both damping of $2\Delta x$ wave and enstrophy cascade. The hyperviscosity coefficient depends on spatial scales: $q_t = C(\Delta x)^4 \Delta^2 q$ or $q_t = C(\Delta x)^{3.2} \Delta^2 q$.



Hyperviscosity (HV) in SEM

Hyperviscosity:

$$\nu \Delta^2 q \quad \text{or} \quad \nu \Delta^2 \vec{u}$$

Coefficient ν scales like $(\Delta x)^p$ with $p = 4$ or $p = 3.2$.
Works well for uniform meshes.
In CAM-SE HV incorporates

$$\int_{sphere} \phi_i q_t = \int_{sphere} \phi_i \Delta q = -\int_{sphere} \nabla \phi_i \cdot \nabla q$$

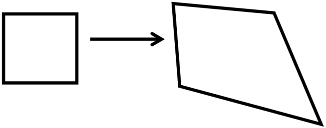
We focus on the local integral,

$$\int_{element} \nabla \phi_i \cdot \nabla q.$$

Elements in Physical and Reference Spaces

Transform:

$$\xi, \eta \in [-1,1] \times [-1,1] \qquad x(\xi,\eta), y(\xi,\eta)$$



$$\begin{aligned}\int_{element} \nabla_{xy}\phi_i \cdot \nabla_{xy} q &= \int_{[-1,1]^2} J D^{-T} \nabla_{\xi\eta}\phi_i \cdot D^{-T} \nabla_{\xi\eta} q \\ &= \int_{[-1,1]^2} J \nabla_{\xi\eta}\phi_i \cdot D^{-1} D^{-T} \nabla_{\xi\eta} q\end{aligned}$$

$$D = \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{pmatrix}$$

Local Dimensions from Metric Tensors

$$D^T D = E \Lambda E = E \begin{pmatrix} \left(\frac{\Delta x}{2}\right)^2 & 0 \\ 0 & \left(\frac{\Delta y}{2}\right)^2 \end{pmatrix} E^T$$

Δx, Δy are interpreted as dimensions of an element.

Tensor HV:
Instead of $\nabla_{\xi\eta}\phi_i \cdot D^{-1} D^{-T} \nabla_{\xi\eta} q$, take

$$\nabla_{\xi\eta}\phi_i \cdot D^{-1} \mathbf{V} D^{-T} \nabla_{\xi\eta} q$$

$$\mathbf{V} = DE \begin{pmatrix} \left(\frac{\Delta x}{2}\right)^{p-2} & 0 \\ 0 & \left(\frac{\Delta y}{2}\right)^{p-2} \end{pmatrix} E^T D^T$$

Transition from the traditional HV: Instead of $q_t = C(\Delta x)^p \Delta^2$ we take

$$q_t = C(\nabla \cdot \mathbf{V}\nabla)^2 q \quad \text{or} \quad q_t = C(\nabla \cdot \mathbf{V}^2 \nabla)(\nabla \cdot \nabla q).$$

Shallow Water Tests

Standard tests for dycores in Williamson et al. (JCP 1992)

Test Case # 2: Global steady state nonlinear zonal geostrophic flow

Convergence rates are expected to be as in theory. 4th order tensor HV is used.

Test Case # 5: Zonal flow over a mountain

An analytic solution does not exists. Errors are obtained from a hi-res solution. Theoretical convergence rates are not expected, vorticity field is examined for oscillations. Tensor HV of order 3.2.

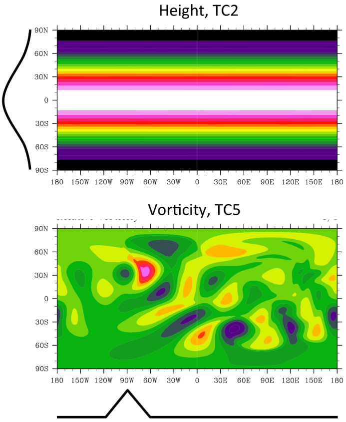


Figure: A uniform mesh

Meshes for SW Tests

Presence of a refinement should not affect global errors. In the refined region, local scales are expected to be resolved.

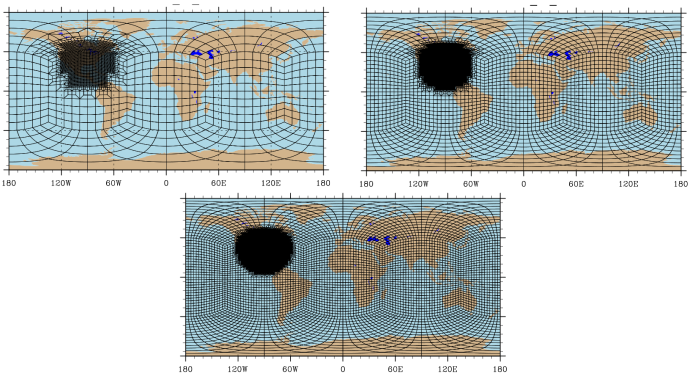
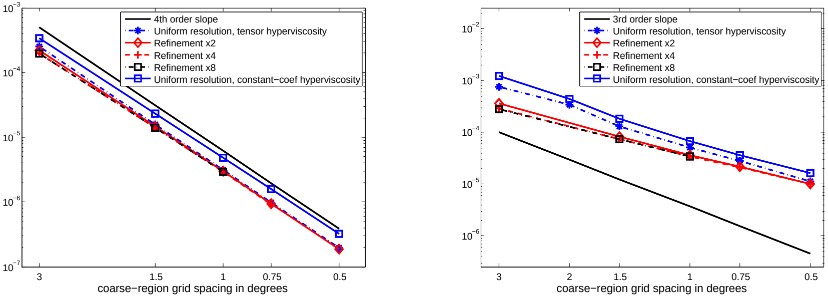


Figure: Meshes with 3^o, 1.5^o, 0.75^o resolutions almost everywhere

A sequence of uniform 3^o, 1.5^o, 0.75^o, etc. simulations is compared to the sequence of meshes from above.

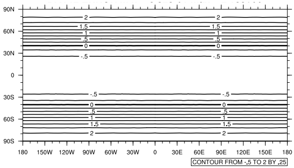
Global Errors, TC2 and TC5



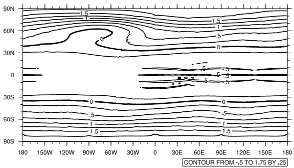
Test Case 2: All convergence rates are of 4th order. Global errors are not affected by refinements.
Test Case 5: Global errors for refined meshes demonstrate the same behavior as for uniform meshes, plus, errors are sligtly improved due to the location of the mountain.

Performance of TC2, Contour Plots for Errors

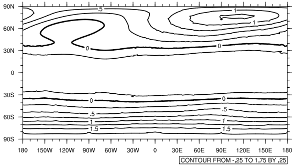
Uniform mesh



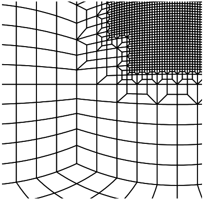
A highly distorted mesh with refinement



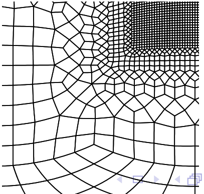
A less distorted mesh with refinement



A highly distorted grid, 6-valence nodes are common.

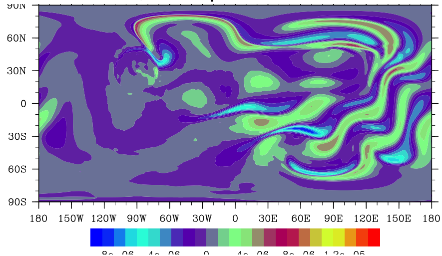


A low-connectivity mesh with very few (and avoidable) 6-valence nodes. See SQuadGen, http://climate.ucdavis.edu/squadgen.php .

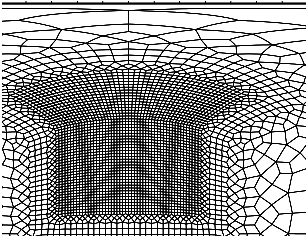


Performance of TC5

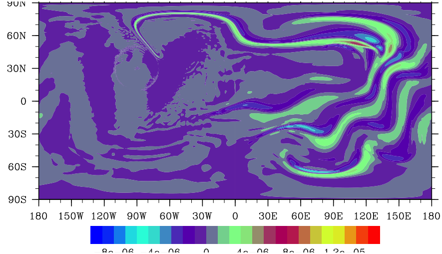
Uniform mesh, error plot



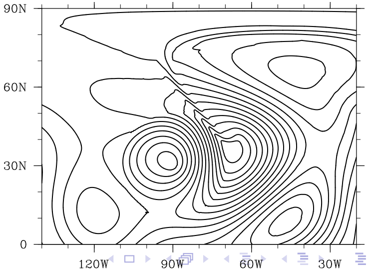
Grid



A mesh with refinement, error plot



Vorticity field (smooth!)



Topography Smoothing

$$h_{new} = h_{old} + \nu(\nabla \cdot \mathbf{V}\nabla)(h_{old})$$