Development of smooth and homogeneity of icosahedral grid using spring dynamics method

(Iga and Tomita 2014, J. Compt. Phys.)

Shin-ichi Iga

RIKEN/AICS, Japan

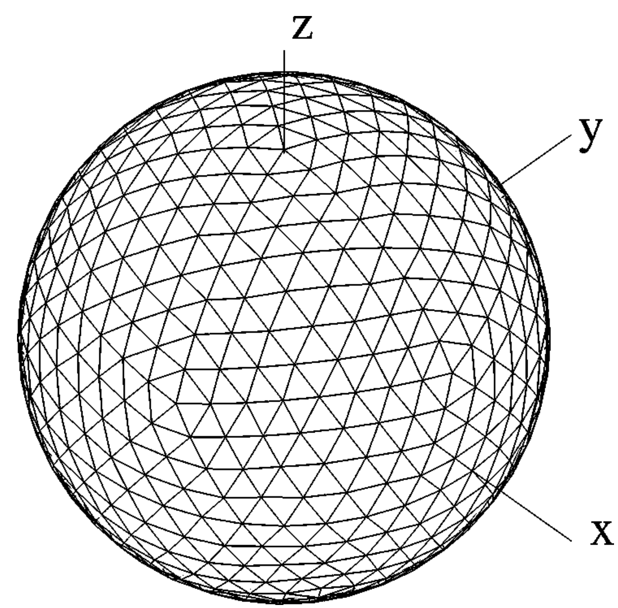
iga@riken.jp

K computer

Introduction

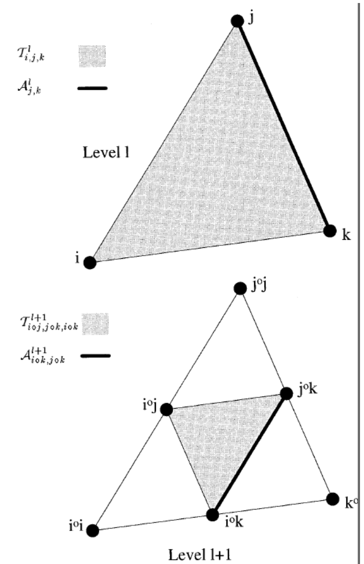
- Icosahedral grid is based on icosahedron

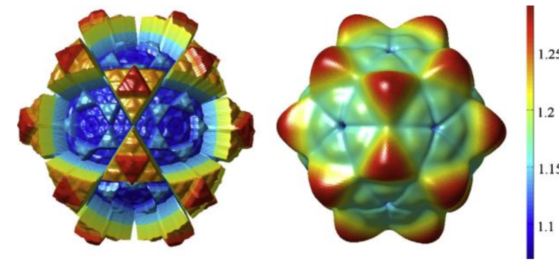
- Used in many AGCMs
  - High performance with high resolution.



There are many way to generate Icosahedral grid

- Recursive type
  - baumgardner-frederickson1985
  - Heikes and Randall1995a
  - Stune et al 1996
  - Xu et al 2006





Pudikiewis 2011

Left: very discontinuous!
Right: smoothed

- Non-recursive type
  - Williamson 1968
  - Sadourny et al. 1968
  - Steppeler et al. 2008

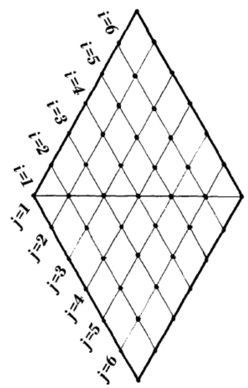
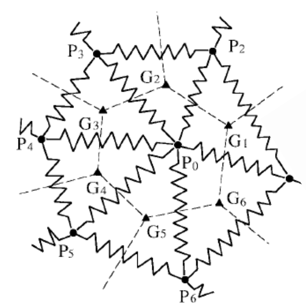


FIGURE 4.—Indexing of a rhombus cell, for $n=6$.

Sadourny et al.1968

- Spring dynamics method
  - Tomita et al. 2001, 2002

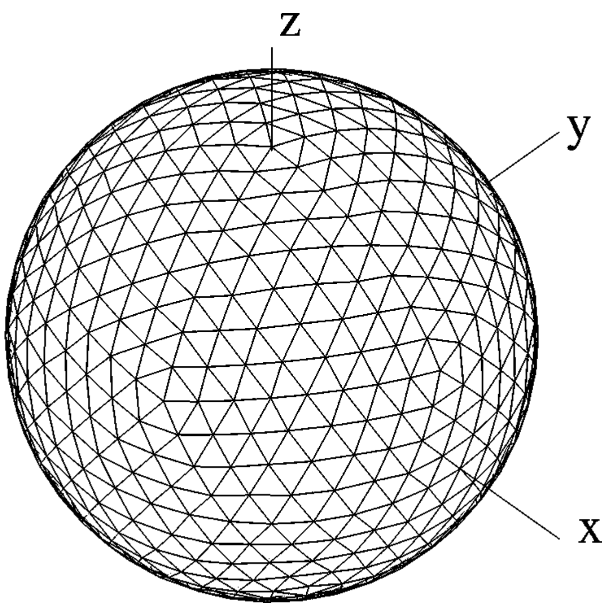
We focus on this



Tomita et al 2001, 2002

Our purpose is to generate high performance icosahedral grid !

1. **Large minimum-grid interval**
   – Because of CFL condition
2. **Small maximum-grid interval**
   – To decrease error
3. **Smooth (no-discontinuity)**
   – For stable calculation
4. **Locally isotropic**
   – **Each triangles are nearly regular**
5. **Applicable to any resolution**
   – **Even for high resolution**

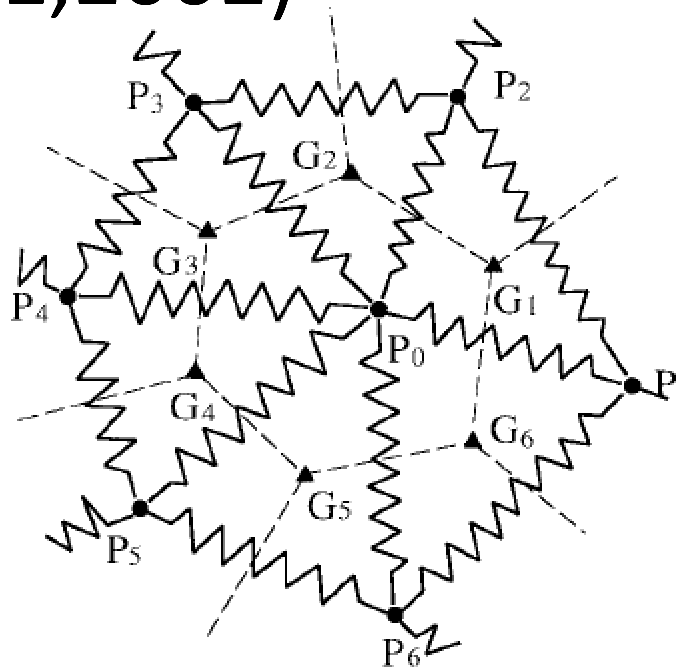


Spring dynamics method (SPR)
Tomita et al (2001,2002)

- Adjacent grid points are connected
- Initial location of grid points are arbitrarily given.

$$M\frac{d\mathbf{w}_0}{dt} = \sum_{i=1}^{n} k(d_i - \bar{d})\mathbf{e}_i - \alpha \mathbf{w}_0,$$

$$\frac{d\mathbf{r}_0}{dt} = \mathbf{w}_0,$$



Tomita et al. 2001,2002

Natural spring length: we can freely determine

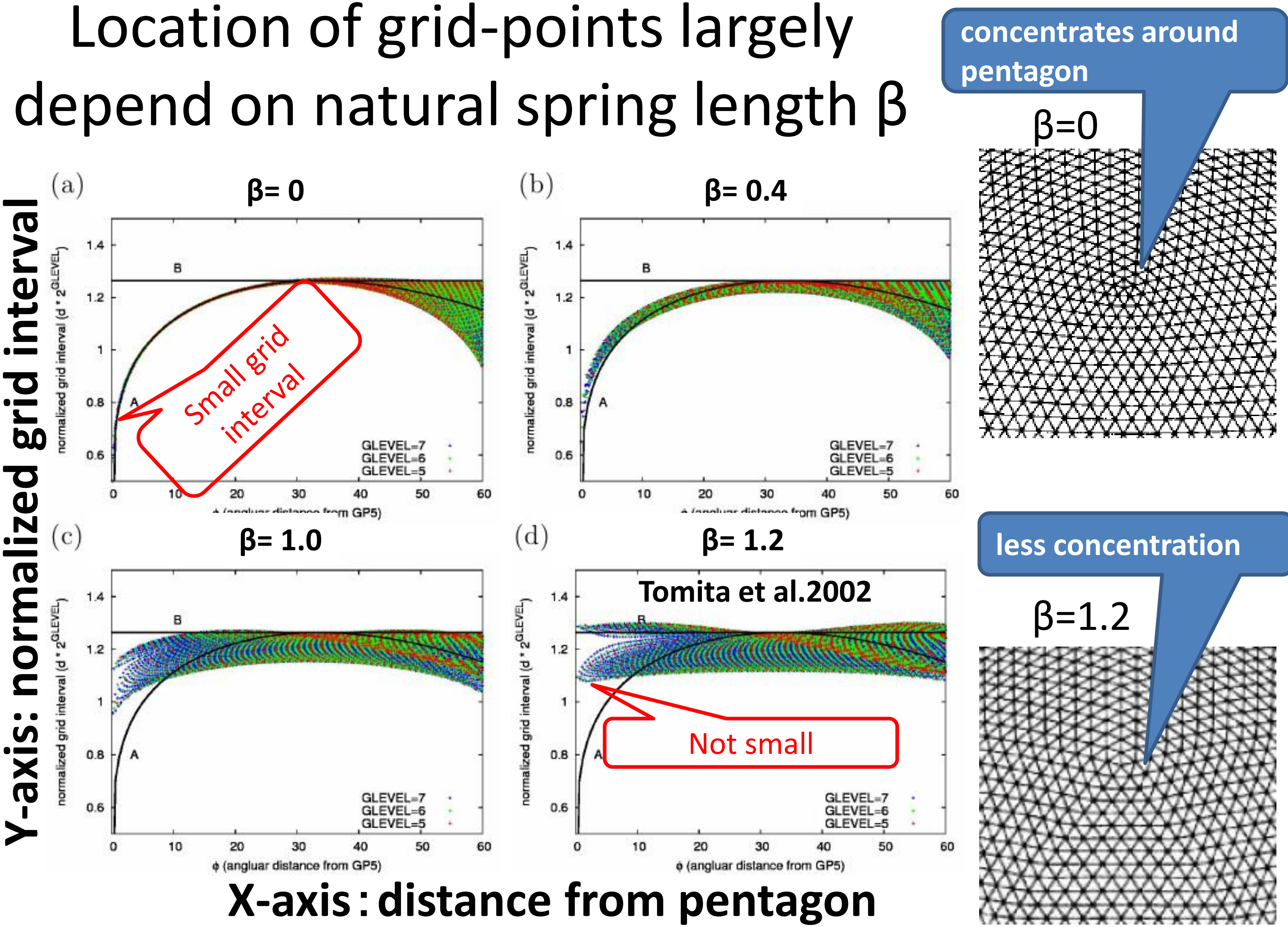
Following Tomita et al 2002, we introduce β

β : natural spring length normalized by averaged grid interval

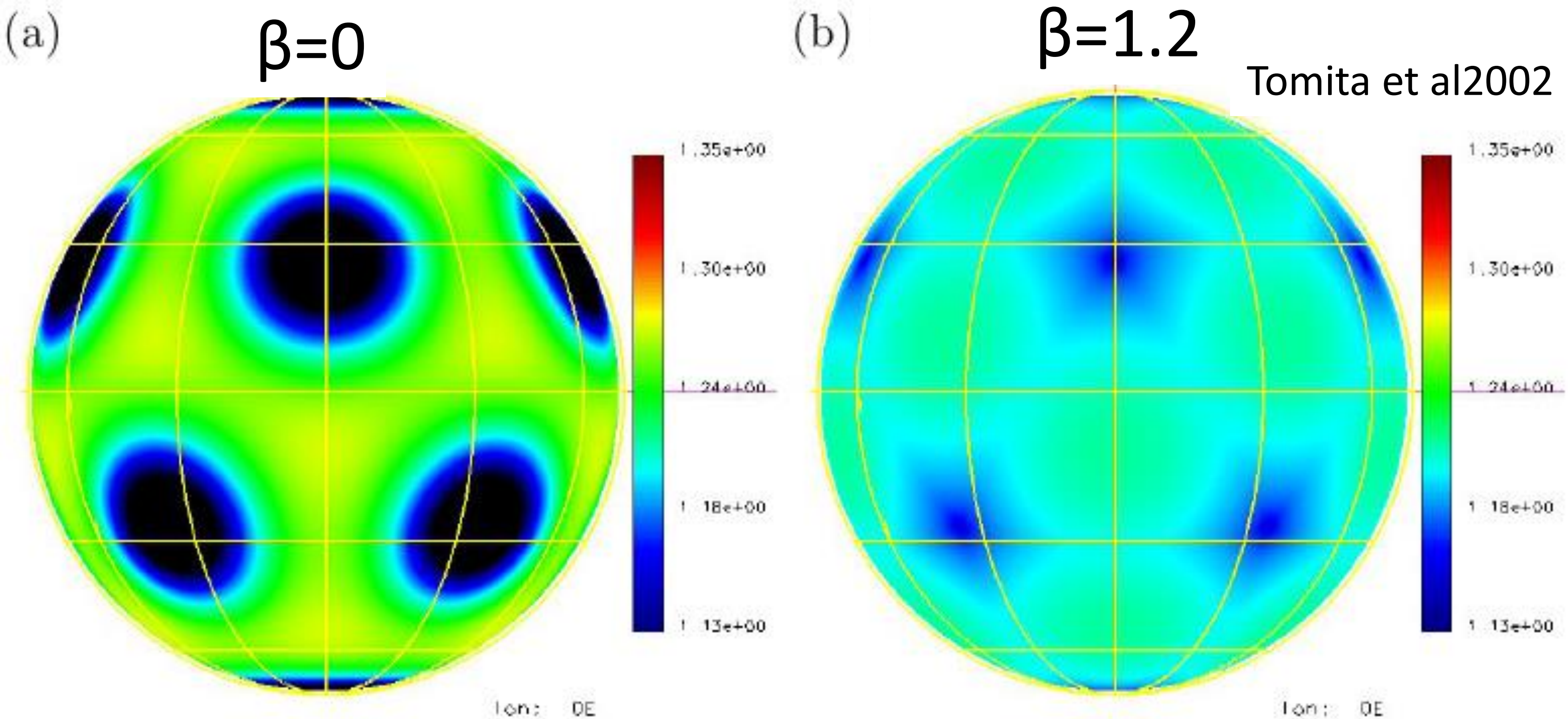
Repulsive when β>1, attractive when β<1,

zero natural spring length when β=0

Location of grid-points largely depend on natural spring length β



Angular mean resolution distribution



Much contrast

Less contrast

β = 1.2 is better for simulation

Spring dynamics grids are unstable for high resolution and large β!

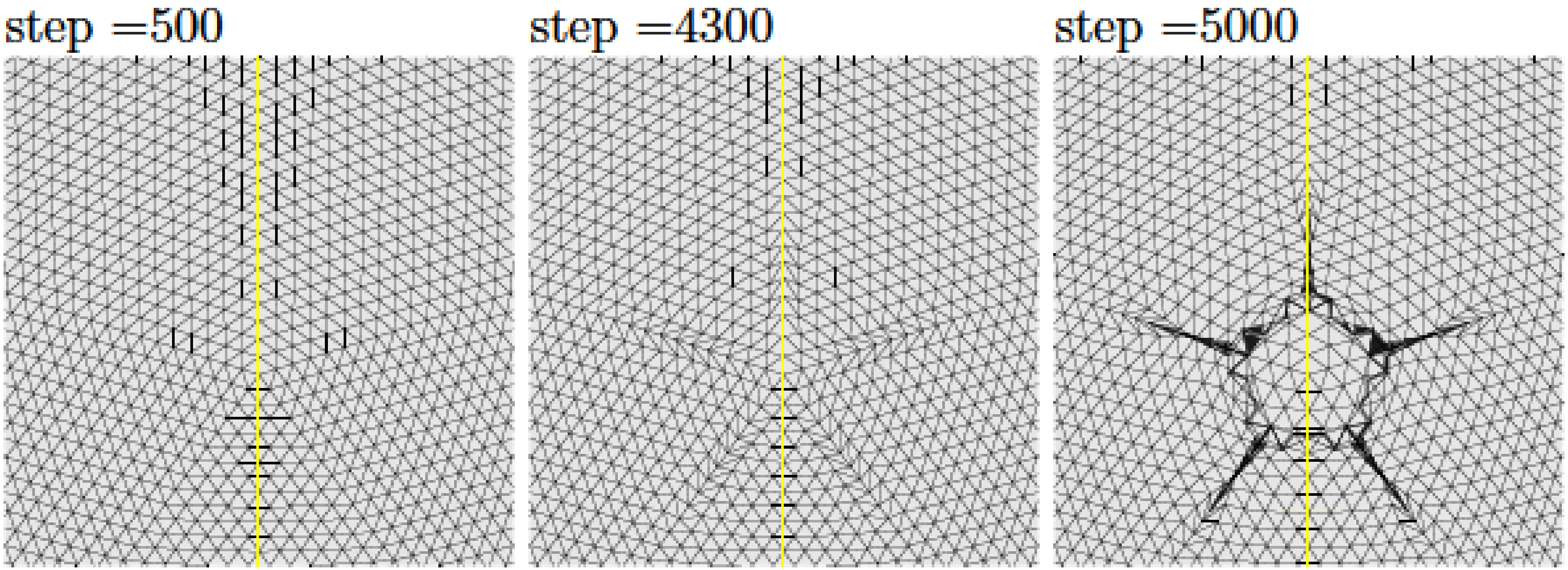


Figure 6: Collapse of the grid around GP5 for the case $\beta = 1.2$ and GLEVEL = 9.

To avoid collapse, β should be decreased when resolution is high

- Tomita et al 2002 used β = 1.2 but it works only when dx ≧ 28km.
- NICAM uses β = 1.15 but it works only when dx ≧ 3.5km.
- When dx ≦1.7km, β should be less than 1.15

However, homogeneity (d_{MIN} / d_{MAX}) is worse when β is small.

γ_1 GLEVEL	β=0				β=1.05	β=1.1	β=1.15	β=1.2		
5	0.533	0.611	0.722	0.784	0.800	0.816	0.829	**0.840**	0.825	0.837
6	0.464	0.560	0.694	0.768	0.787	0.806	0.820	**0.835**	0.824	0.837
7	0.404	0.516	0.669	0.755	0.776	0.796	0.814	**0.833**	0.824	0.837
8	0.343	0.478	0.650	0.744	0.767	0.788	0.809	**0.830**	0.824	0.837
9	0.298	0.446	0.633	0.735	0.758	0.783	**0.806**	x	0.824	0.837
10	0.260	0.419	0.620	0.726	0.752	0.778	**0.804**	x	0.824	0.837
11	0.226	0.395	0.609	0.720	0.748	0.775	**0.801**	x	0.824	0.837
12	0.197	0.376	0.600	0.715	0.744	**0.773**	x	x	0.824	0.837
13	0.171	0.359	0.593	0.712	0.742	**0.769**	x	x	0.824	0.837
14	0.149	0.345	0.586	0.709	**0.739**	x	x	x	0.825	0.837

To generate dx= 400m grid, β should be 1.05, and d_{MIN} / d_{MAX} = 0.739 …..→ less homogeneous

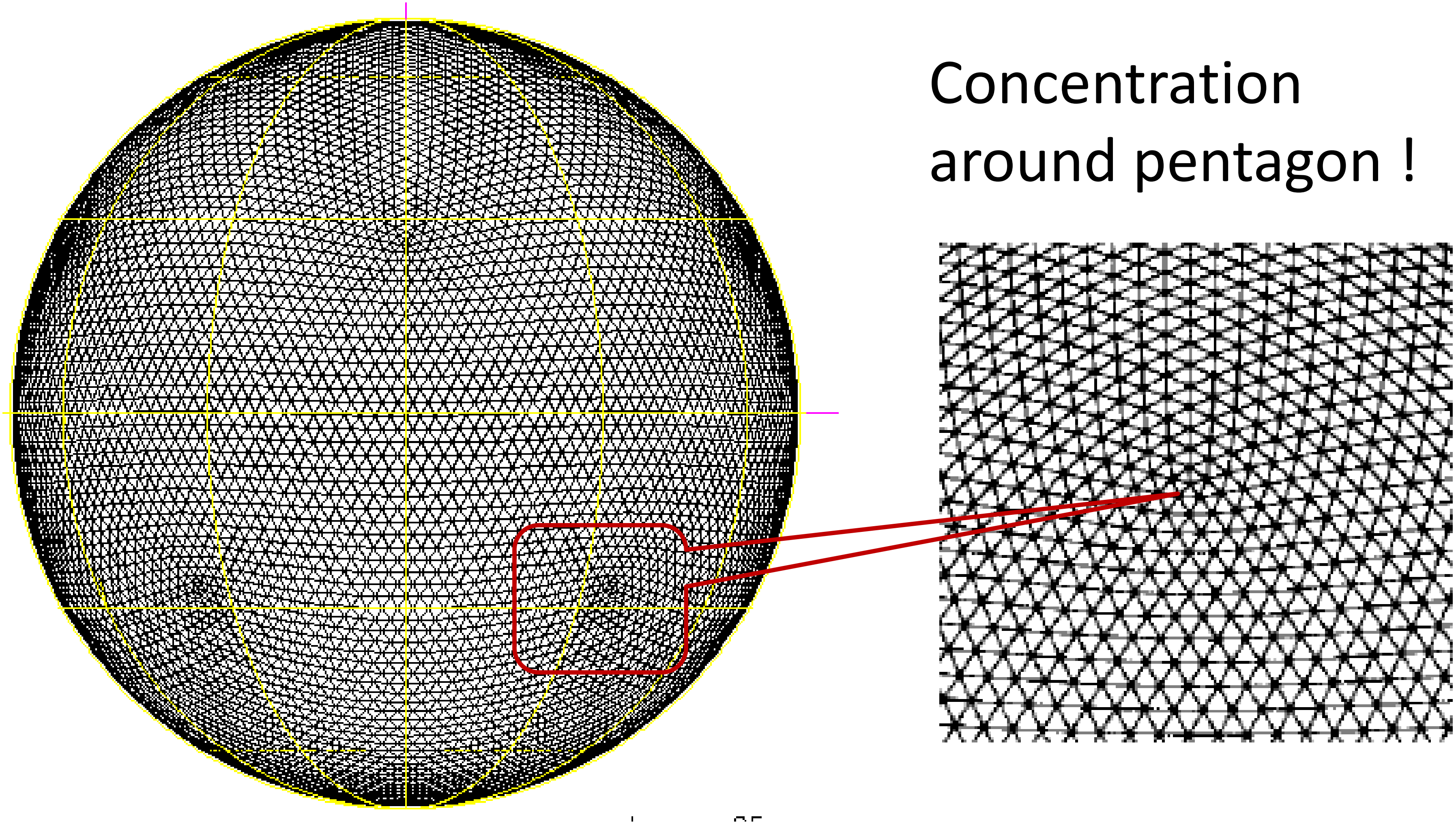
Newly proposed method resolves the problem!

Generation Method

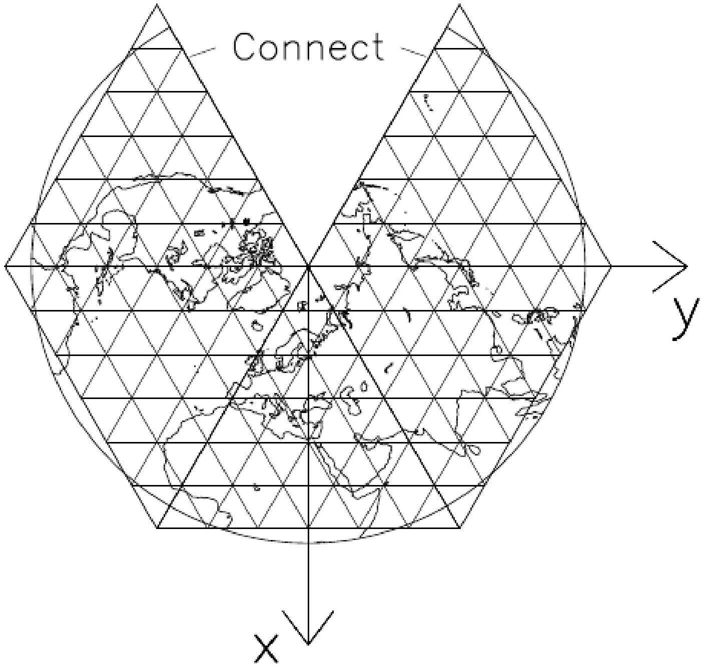
1. Generate Spring dynamic grid with $\beta = 0$
2. Applies transformation by smooth analytic function around pentagon

Spring dynamic grid with β = 0

Concentration around pentagon !

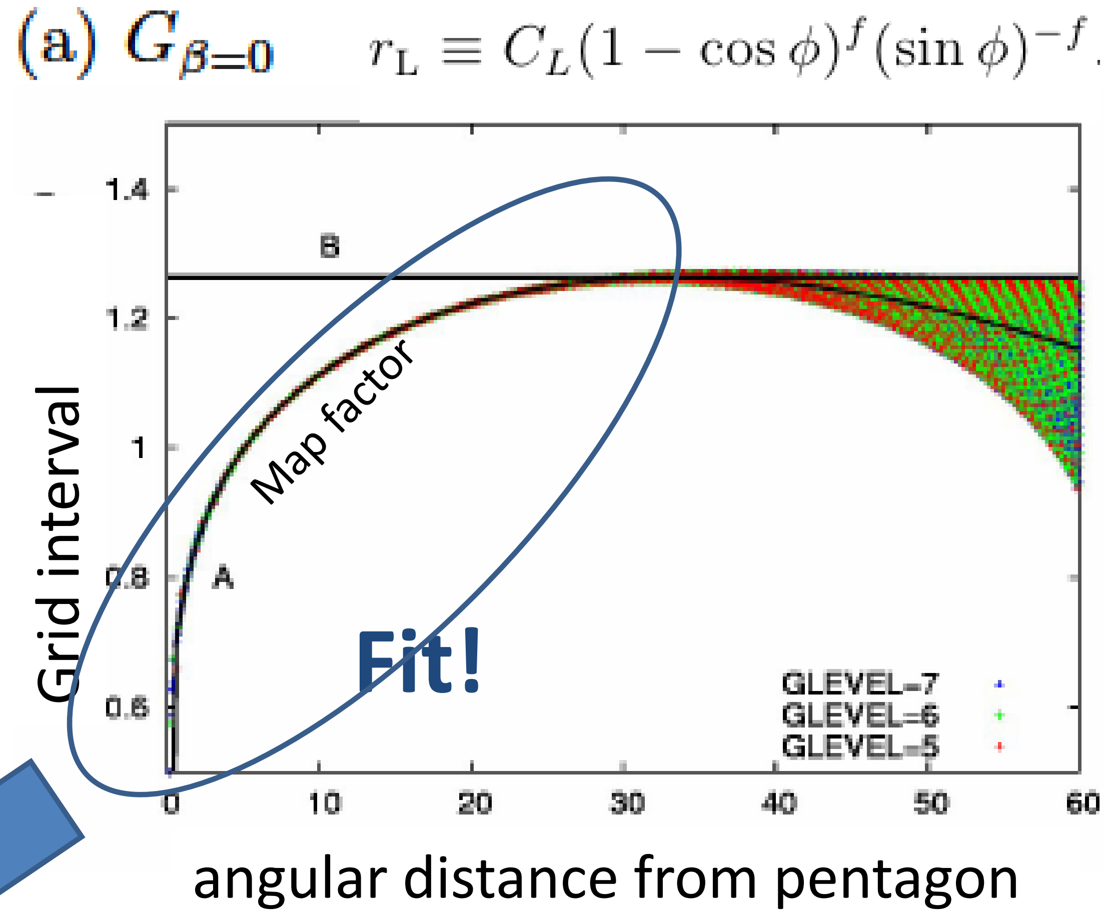


Distribution of grid interval fits with map factor of Lambert Comformal Conic Projection (LCCP) with map angle of 300°



Map factor

$$r_{\mathrm{L}} \equiv C_L (1 - \cos\phi)^f (\sin\phi)^{-f}.$$



Reason is shown later

Transformation by analytic function

ϕ : Angular distance from pentagon

ϕ is transformed by

$$\phi_{\mathrm{modif}} = r_{\mathrm{modif}}^{-1}(r_{\mathrm{L}}(\phi_{\beta=0}))$$

where

$$r_{\mathrm{modif}}(\phi) \equiv A r_{\mathrm{A}}(\phi) + (1 - A) r_{\mathrm{L}}(\phi)$$

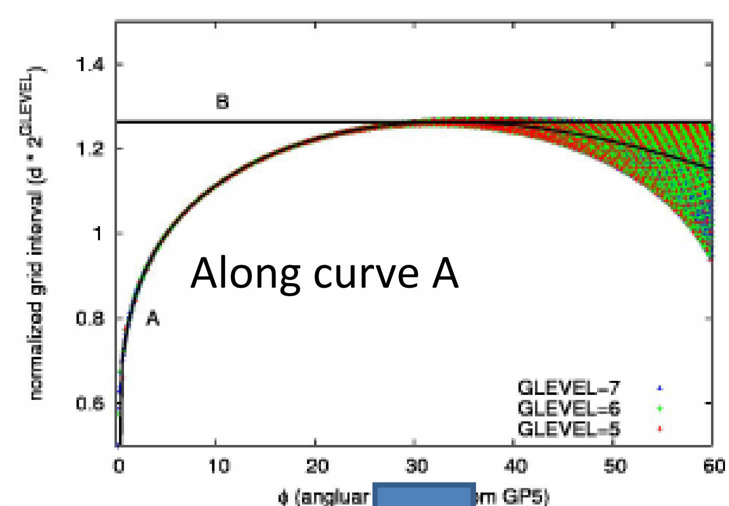
$$A \equiv 0.5\left[1 - \tanh\left(\frac{\phi - \phi_{c2}}{\Delta}\right)\right]$$

$$r_{\mathrm{A}}(\phi) \equiv (1 - \cos\phi_{c1})^{f} (\sin\phi_{c1})^{-(f+1)} \sin\phi.$$

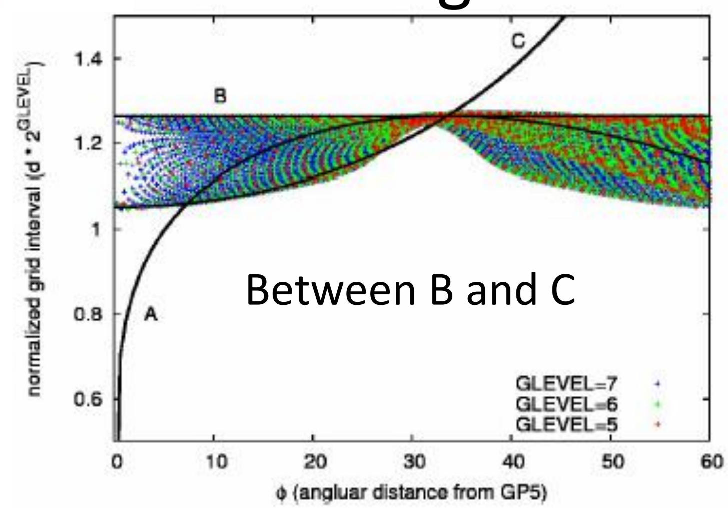
$$r_{\mathrm{L}} \equiv C_L (1 - \cos\phi)^{f} (\sin\phi)^{-f}.$$

- Then proposed grid is generated!

(a) β = 0



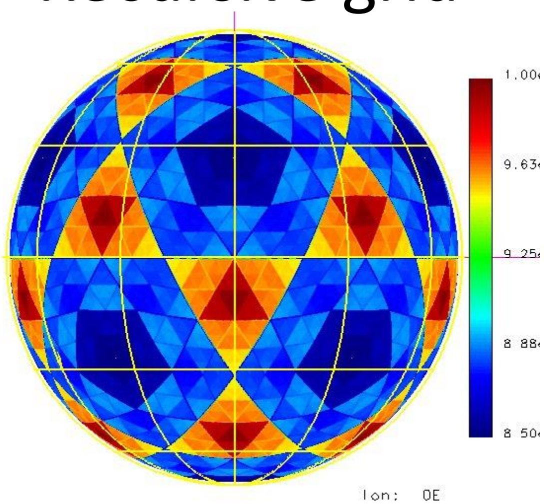
modified grid



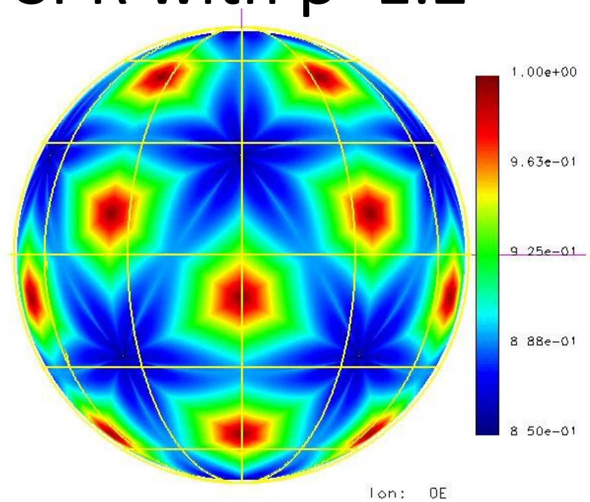
Comparison

Regularity

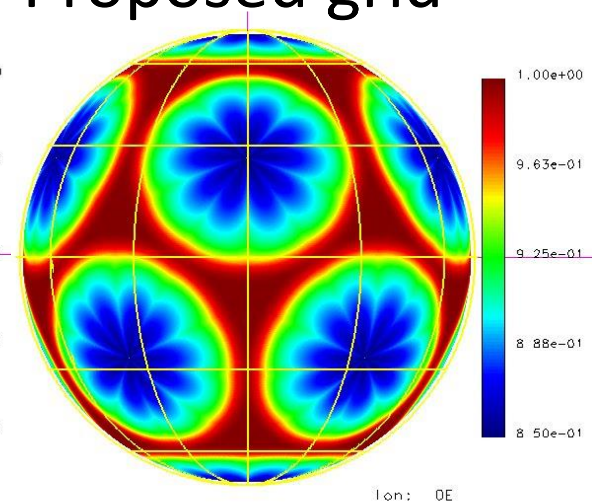
Recursive grid



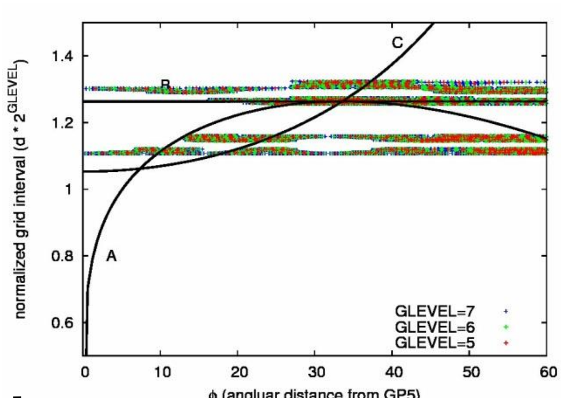
SPR with β=1.2

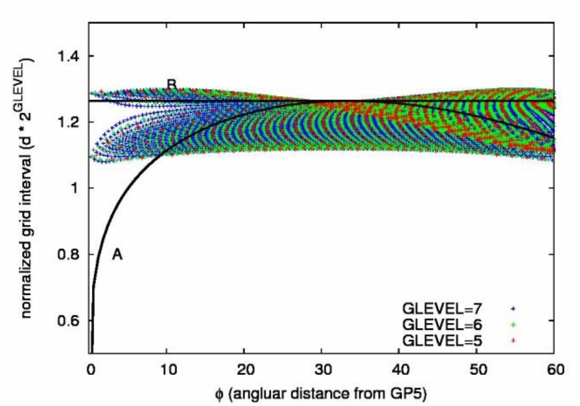


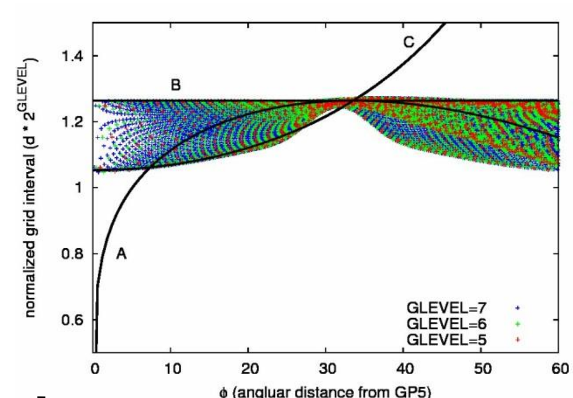
Proposed grid



Grid interval



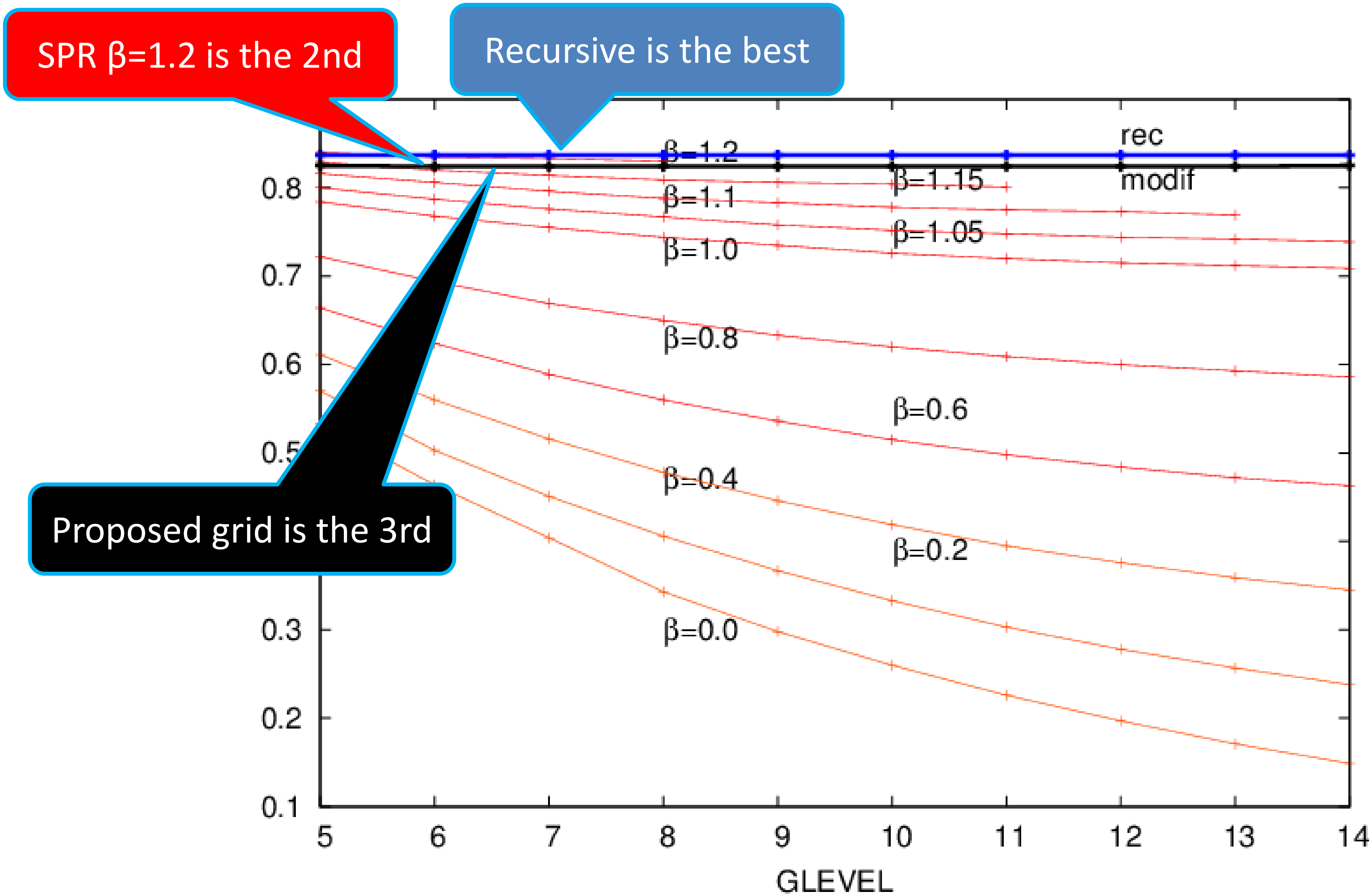




Exceed Line B

Does not exceed Line B

Homogeneity defined as d_{MIN} / d_{MAX}

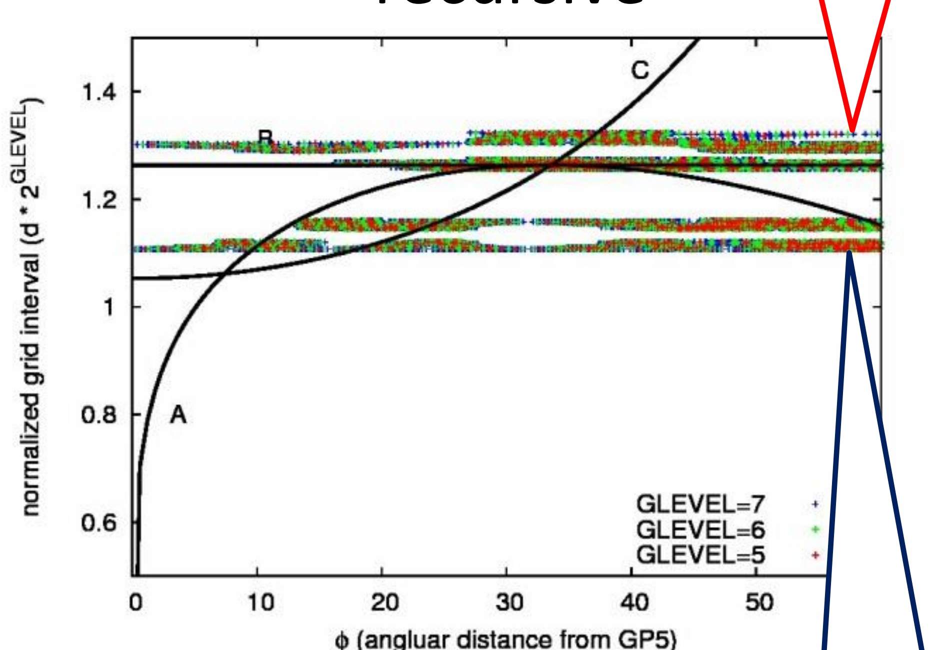


Which is better ?

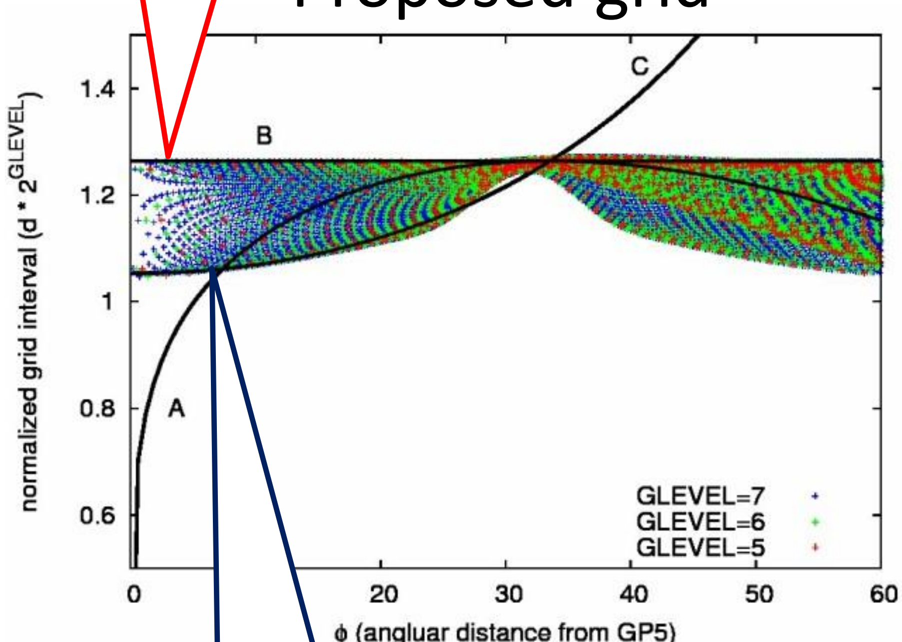
Larger maximum

smaller maximum

recursive



Proposed grid



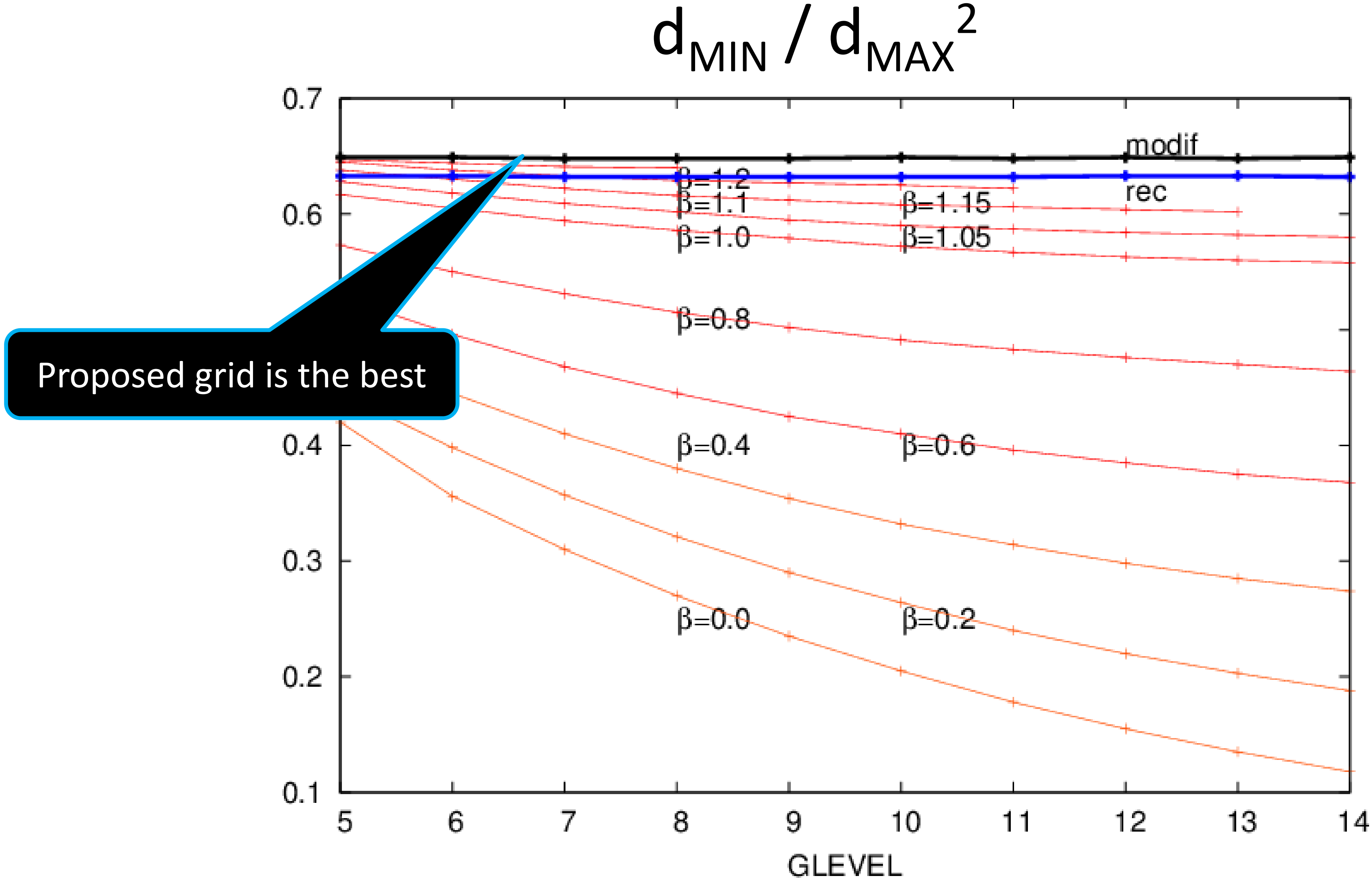
Larger minimum

Smaller minimum

Weighted homogeneity (d_{MIN} / d_{MAX}^2)

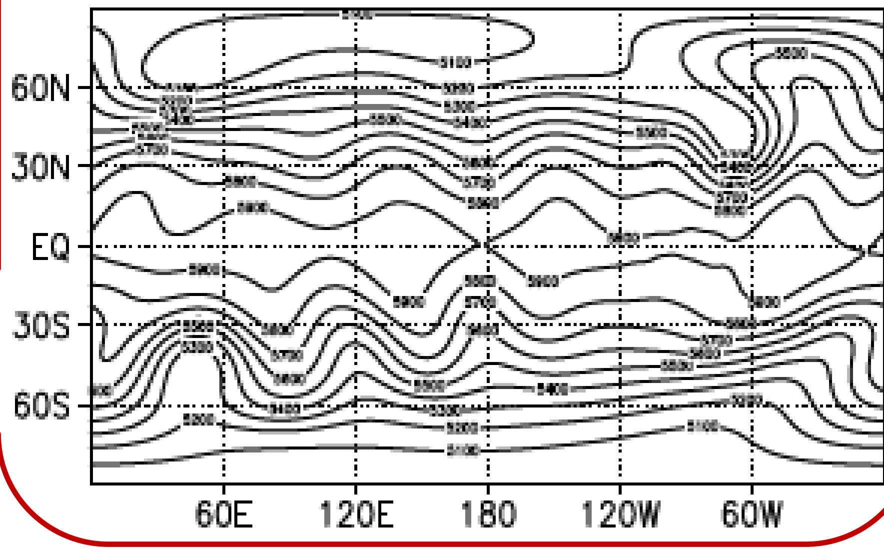
- This indicates cost-efficiency of simulation because
  - Maximum error may be proportional to d_{MAX}
  - Therefore, required grid points for some limited error is proportional to d_{MAX}^2
  - Time step needed by CFL constraint is proportional to d_{MIN}
  - In total, d_{MIN} / d_{MAX}^2 means cost-efficiency of calculation

Weighted homogeneity (cost-efficiency)



Williamson's test case 5

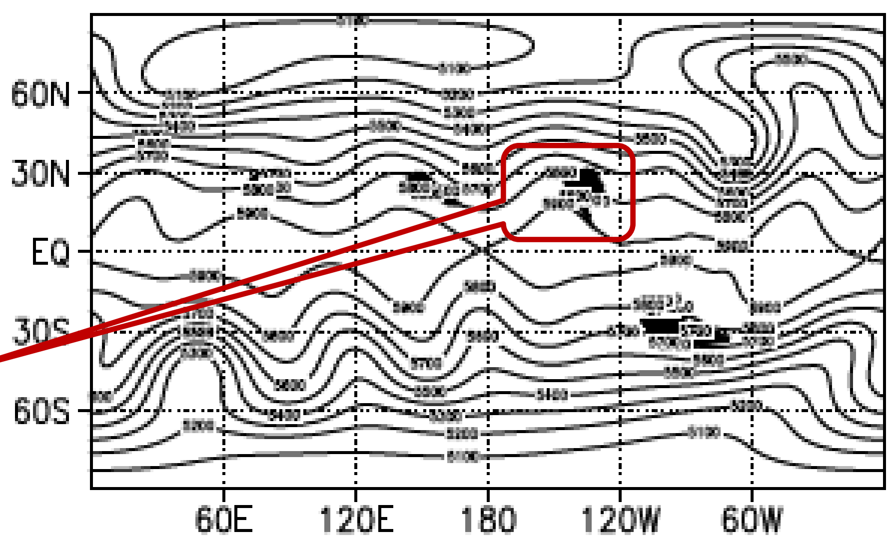
Proposed grid



More stable!

15 days later
Same viscosity is used

Spring dynamics grid β=1.2 (Tomita et al2002)



Noise appeared

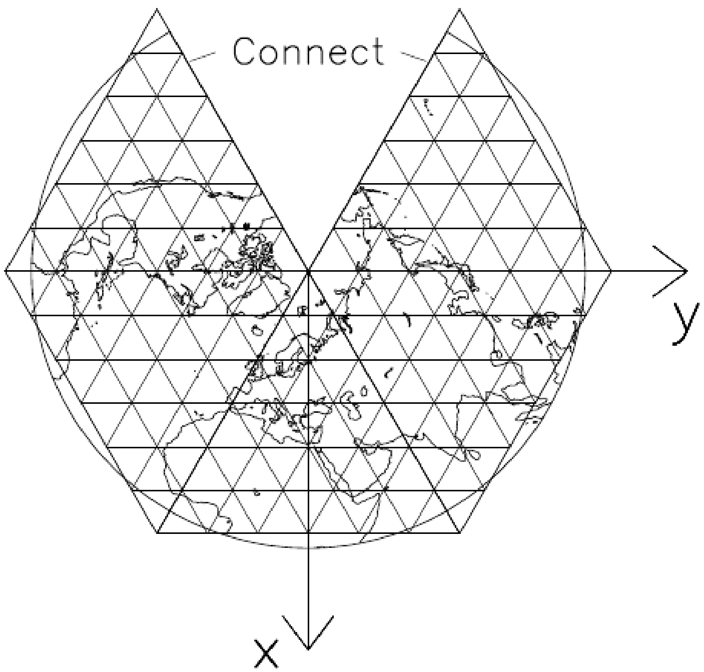
Summary

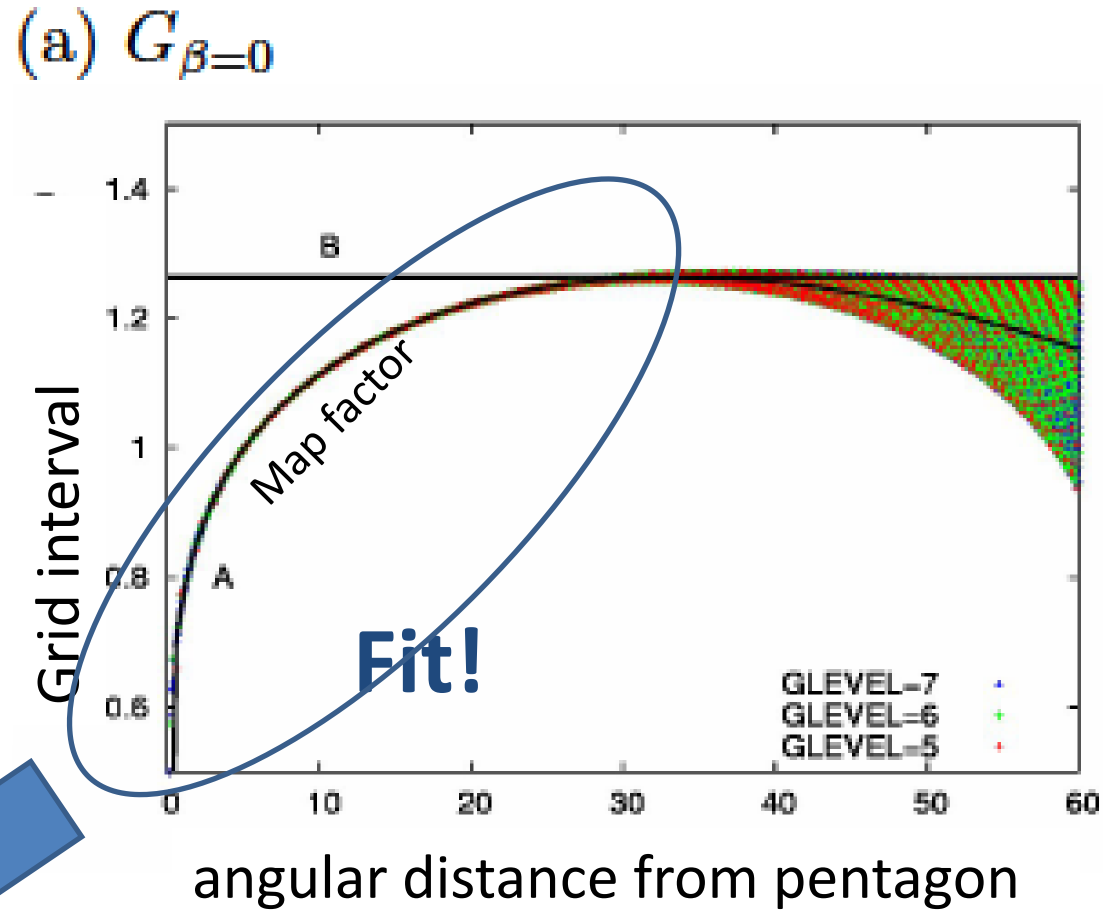
Compared with proposed grid, the other grids are

	Regularity of triangles	smoothness	homogeneity	Weighted homogeneity	stability	accuracy
Spring grid (β＝1．2)	worse	equal	better	worse	worse	worse
Recursive grid	worse	worse	better	worse	worse	worse
Non-recursive grid	Not examined	worse	worse	worse	Not examined	Not examined

Proposed grid is better than the other grid in many properties.

Distribution of grid interval fits with map factor of Lambert Comformal Conic Projection (LCCP) with map angle of 300°





Reason is shown later

Map factor $r_{\mathrm{L}} \equiv C_L(1-\cos\phi)^f(\sin\phi)^{-f}$.

reason

- In the case β=0, when all triangles are regular, potential energy is minimum. (proven)
- Since potential energy tend to be minimum, triangles might become regular. (speculation)

Imagine Lambert map filled with regular triangle grids → Lambert grid

- It might be resemble to Lambert grid (speculation)
- So, resolution distribution of spring grid with β=0 is similar to map factor of Lambert map.

In the case β=0, when all triangles are regular, potential energy is minimum.

From Heron's fomula, total area of whole triangles is

$$S = \sum_{j=1}^{N_T} \sqrt{s_j(s_j - \tilde{d}_{1j})(s_j - \tilde{d}_{2j})(s_j - \tilde{d}_{3j})}, \quad (3)$$

It is rewritten as $S = \sum_{h=1}^{N_S} \frac{\sqrt{3}}{6} \alpha'_h d_h^2,$ With $0 < \alpha_j \leqq 1$

It is rewritten as

$$S = \sum_{j=1}^{N_T} \frac{\sqrt{3}}{12} (\tilde{d}_{1j}^2 + \tilde{d}_{2j}^2 + \tilde{d}_{3j}^2) \alpha_j, \quad (4)$$

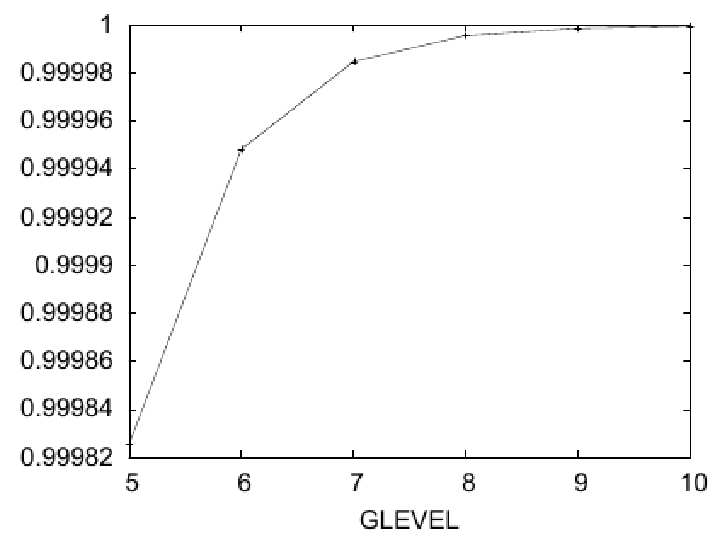
Where $\alpha'_h \equiv \alpha_i + \alpha_j,$ $0 < \alpha'_h \leqq 1$

On the other hand, potential energy is $PE = \sum_{h=1}^{N_S} \frac{1}{2} k d_h^2.$

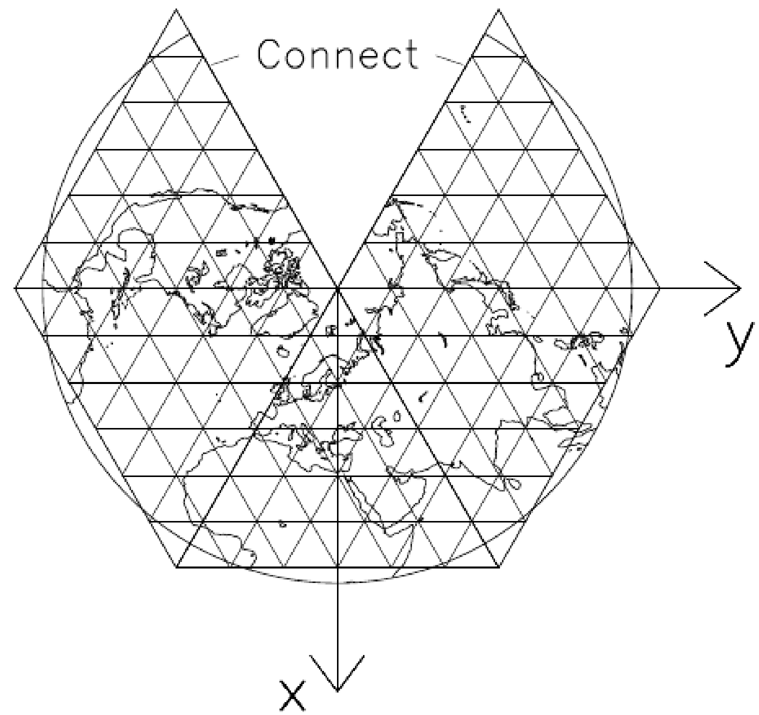
If all triangle are regular, α'_h is unity (maximum) because potential energy is minimum.

Therefore, if potential energy approaches to minimum, we speculate that each triangles of spring dynamics grid with β=0 approaches to regular.

In reality, it is true !
right figure is α'_H
It approach to 1 when resolution increases



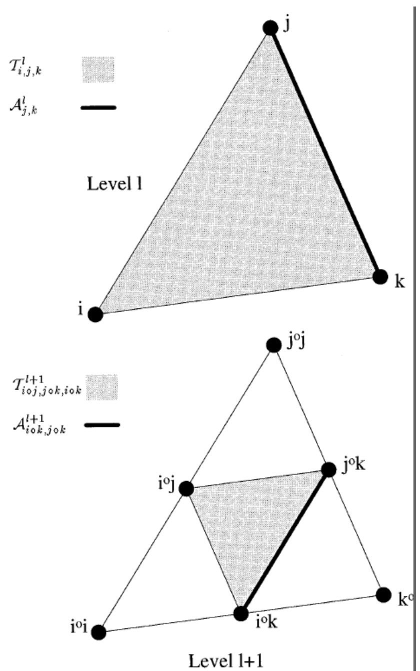
- Lambert conformal conic projection map is filled with regular triangular mesh.
- Since it is conformal, corresponding sphere is also filled with regular triangular mesh which has singular points at the north pole.
- It can be presumed to be resemble to spring dynamics grids with β=0 which is also composed of regular triangles.
- If the presumption is true, grid interval of spring dynamics grid with β=0 is proportional to the map factor.

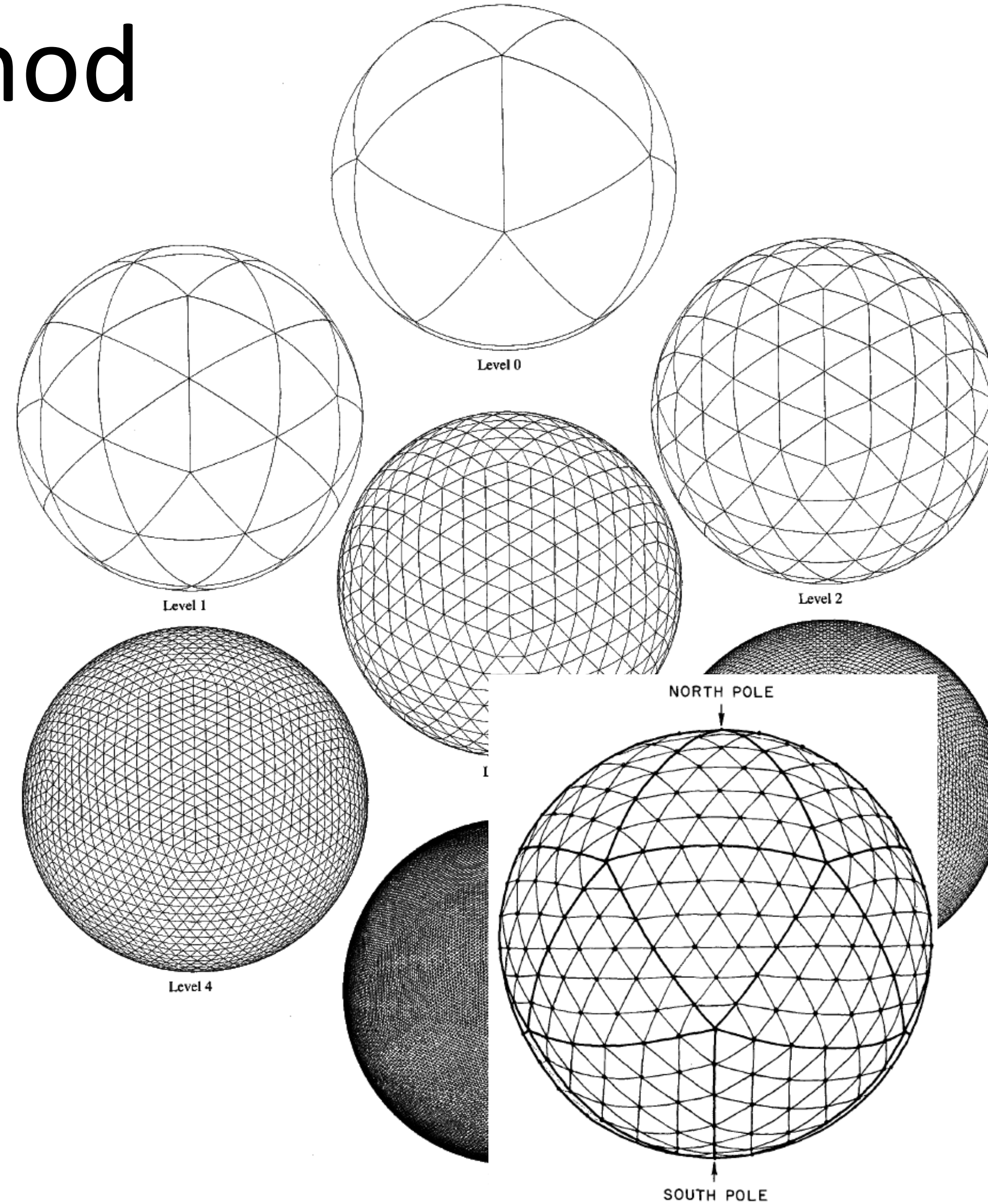


Map factor $r_{\mathrm{L}} \equiv C_L(1-\cos\phi)^f(\sin\phi)^{-f}$.

Recursive method

- ジオデシック面上の大三角形の中点を結び、ジオデシック面で小三角形を作る。これを再帰的に繰り返す。



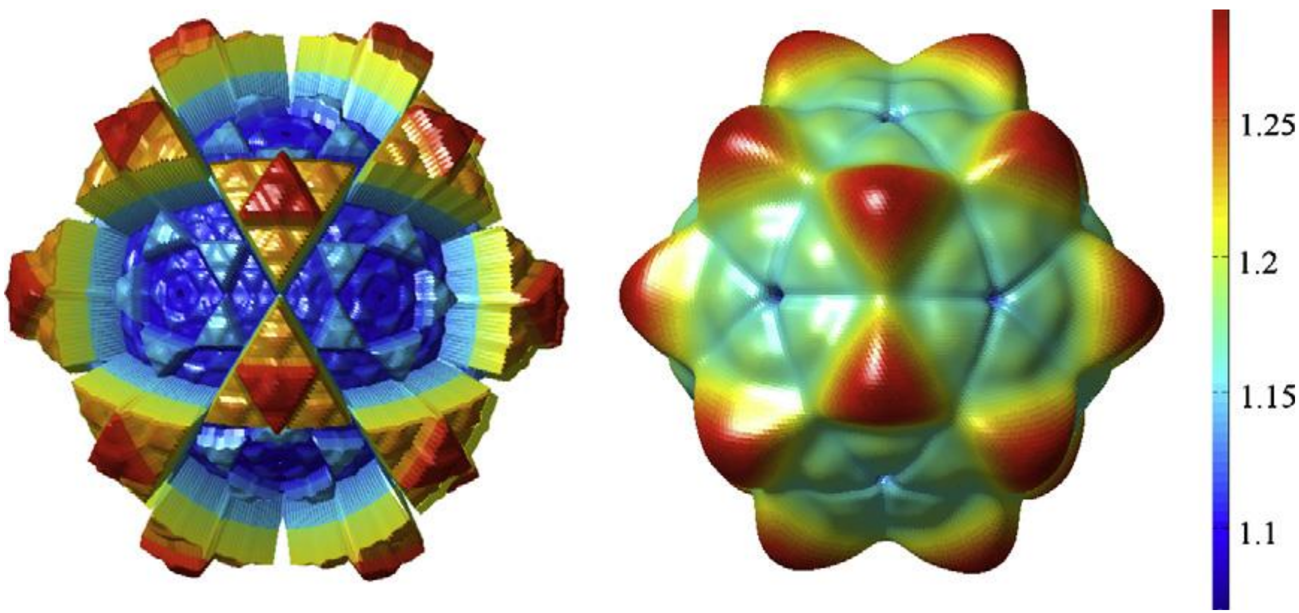


Stuhne et al. 1996

FIG. 2. The computational mesh structures for refinement levels $l = 0$ through $l = 6$ of the basic icoshedron.

Recursive grid

- Original recursive grid is very discontinuous.
- In Xu et al.2006, it is smoothed by Laplacian



Color indicates grid interval

Original recursive Smoothed by Laplacian

Pudikiewis 2011

Non-recursive method

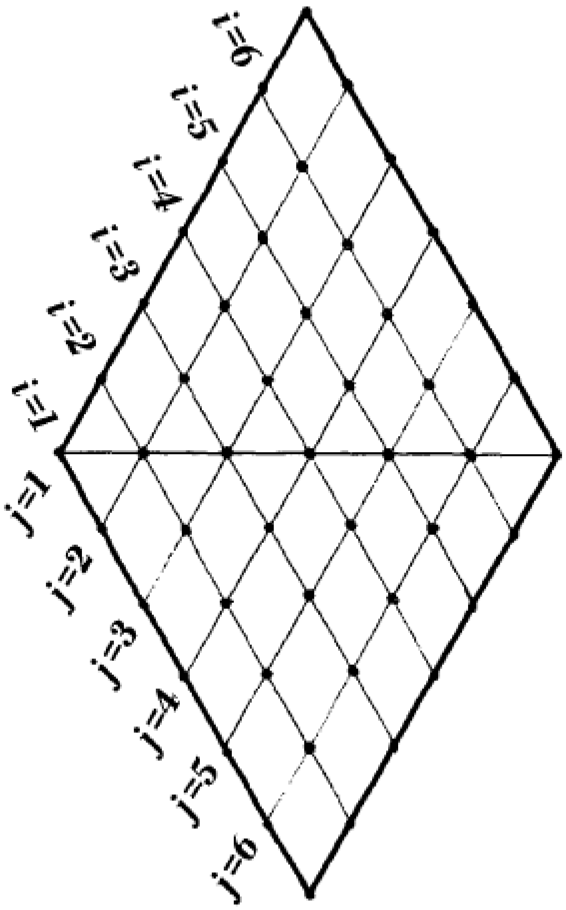


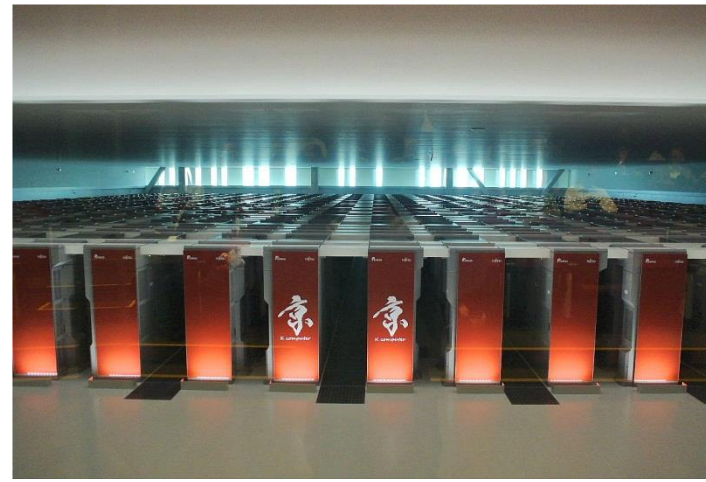
FIGURE 4.—Indexing of a rhombus cell, for $n=6$.

Sadourny et al.1968

- 正二十面体の大三角形(ひし形)を等間隔の三角形(ひし形)に分割し、球面に投影。
- 辺の個所に不連続は残る。
- 最大・最小格子間隔比は大きい。

本来の動機

- 京コンピュータを使用した、計算科学研究機構のグランドチャレンジプロジェクト（2013年）
  - 水平400m解像度での全地球球気象シミュレーションデモラン。（格子点数54万点）



しかし、チームが用いている既存の格子作成方法では不具合が生じるので、対処したい！