MASS CONSERVATION PROPERTIES OF CG/DG METHODS

on non-conforming dynamically adaptive meshes

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Objectives

Compare mass conservation properties of CG/DG methods using:

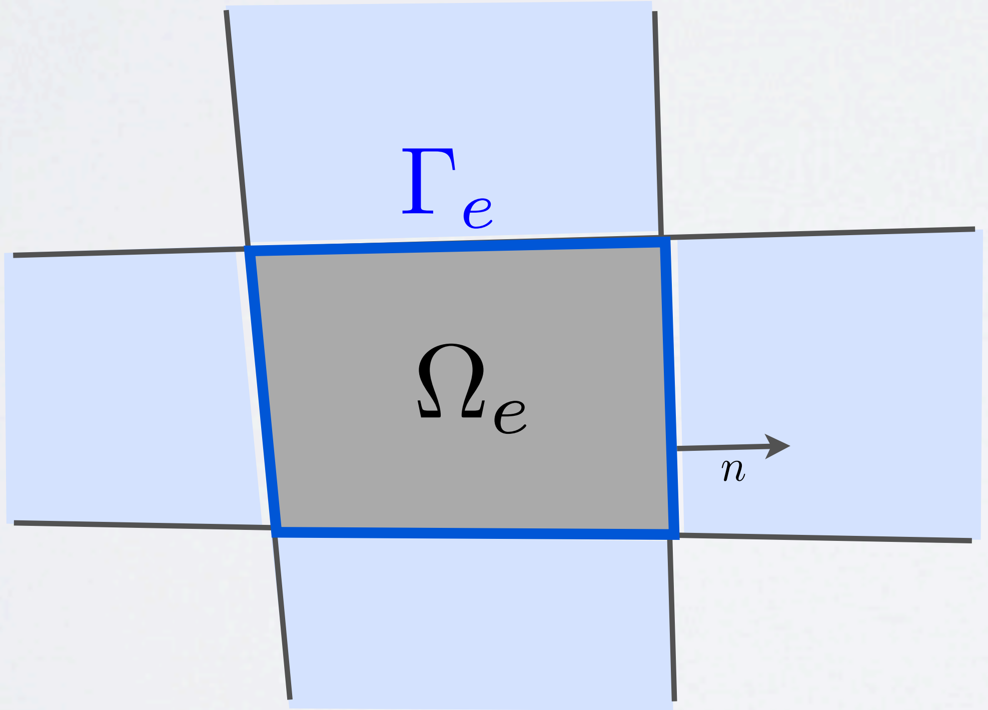
- static and dynamic non-conforming mesh refinement
- density current and rising thermal bubble test cases
- different polynomial order
- different resolution

Can CG perform as well as DG?

Unified CG DG method

$$\frac{\partial q}{\partial t} + \nabla \cdot F(q) = S(q)$$

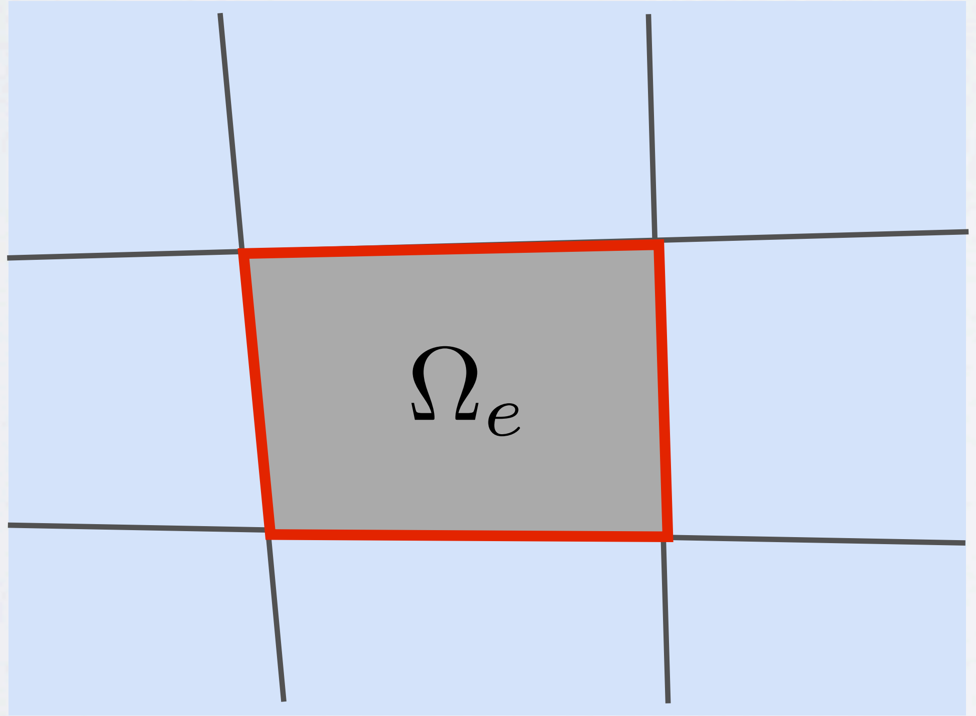
$$\int_{\Omega_e} \psi_i \frac{\partial q}{\partial t}\, d\Omega_e + \int_{\Gamma_e} n \cdot (\psi_i F(q))\, d\Gamma_e - \int_{\Omega_e} \nabla \psi_i \cdot F(q)\, d\Omega_e = \int_{\Omega_e} \psi_i S(q)\, d\Omega_e$$



Unified CG DG method

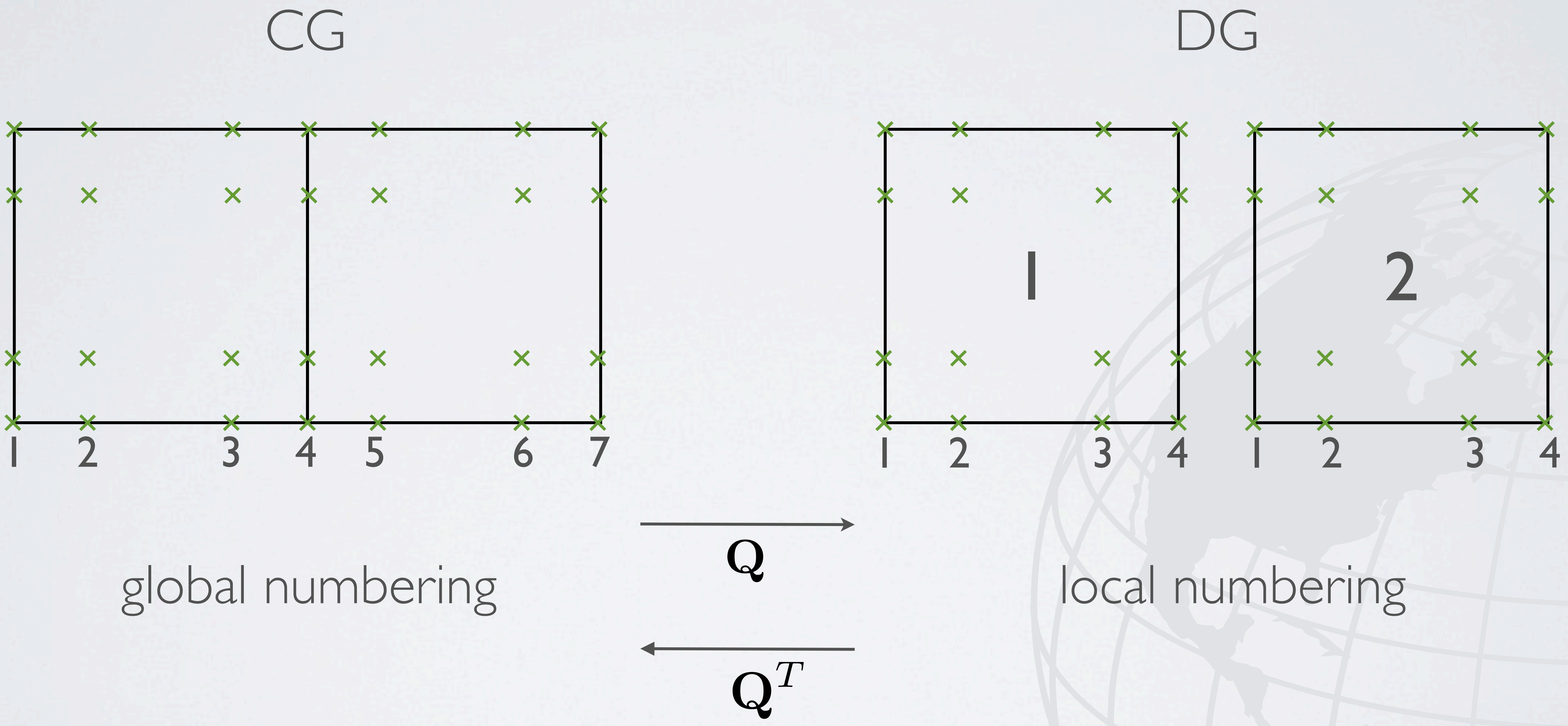
$$\frac{\partial q}{\partial t} + \nabla \cdot F(q) = S(q)$$

$$\int_{\Omega_e} \psi_i \frac{\partial q}{\partial t}\, \mathrm{d}\Omega_e + \int_{\Gamma_e} n \cdot (\psi_i F(q))\, \mathrm{d}\Gamma_e - \int_{\Omega_e} \nabla \psi_i \cdot F(q)\, \mathrm{d}\Omega_e = \int_{\Omega_e} \psi_i S(q)\, \mathrm{d}\Omega_e$$

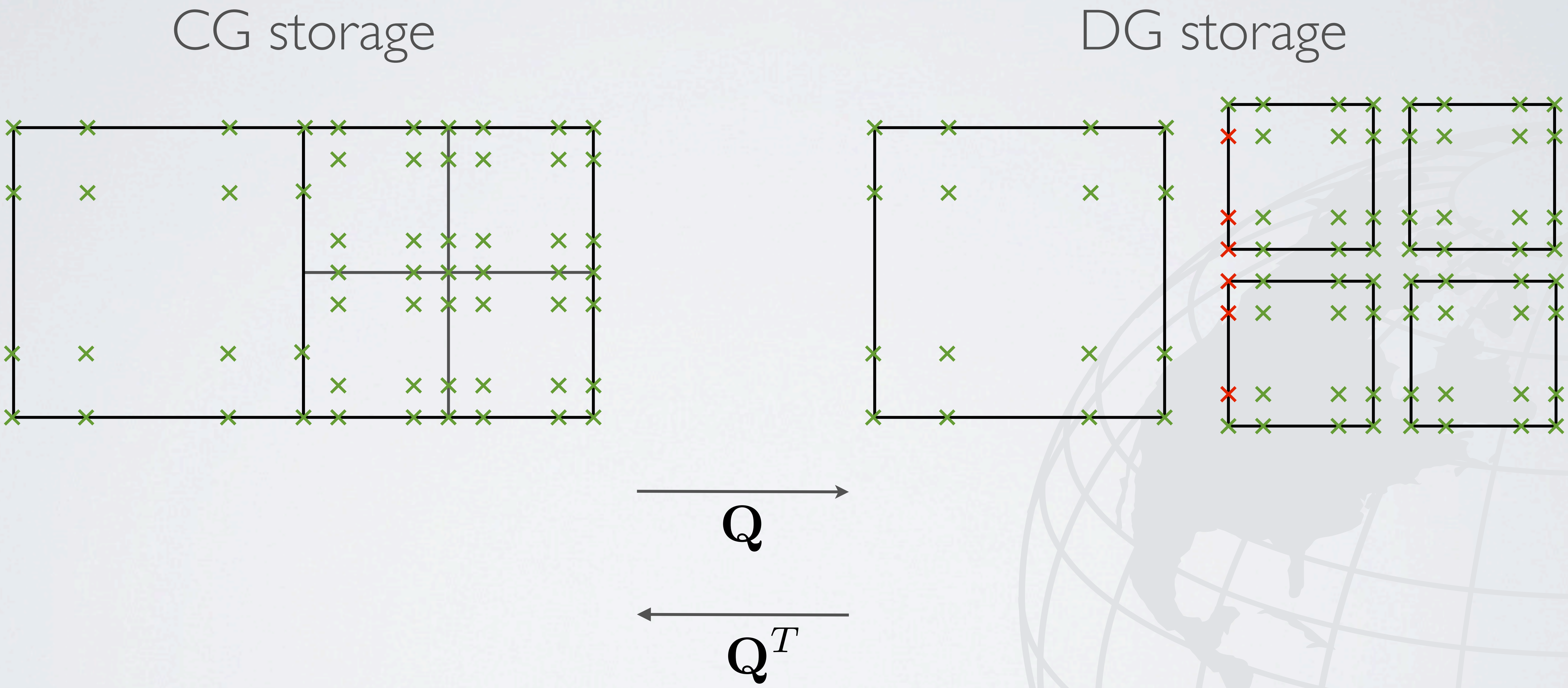


Direct Stiffness Summation

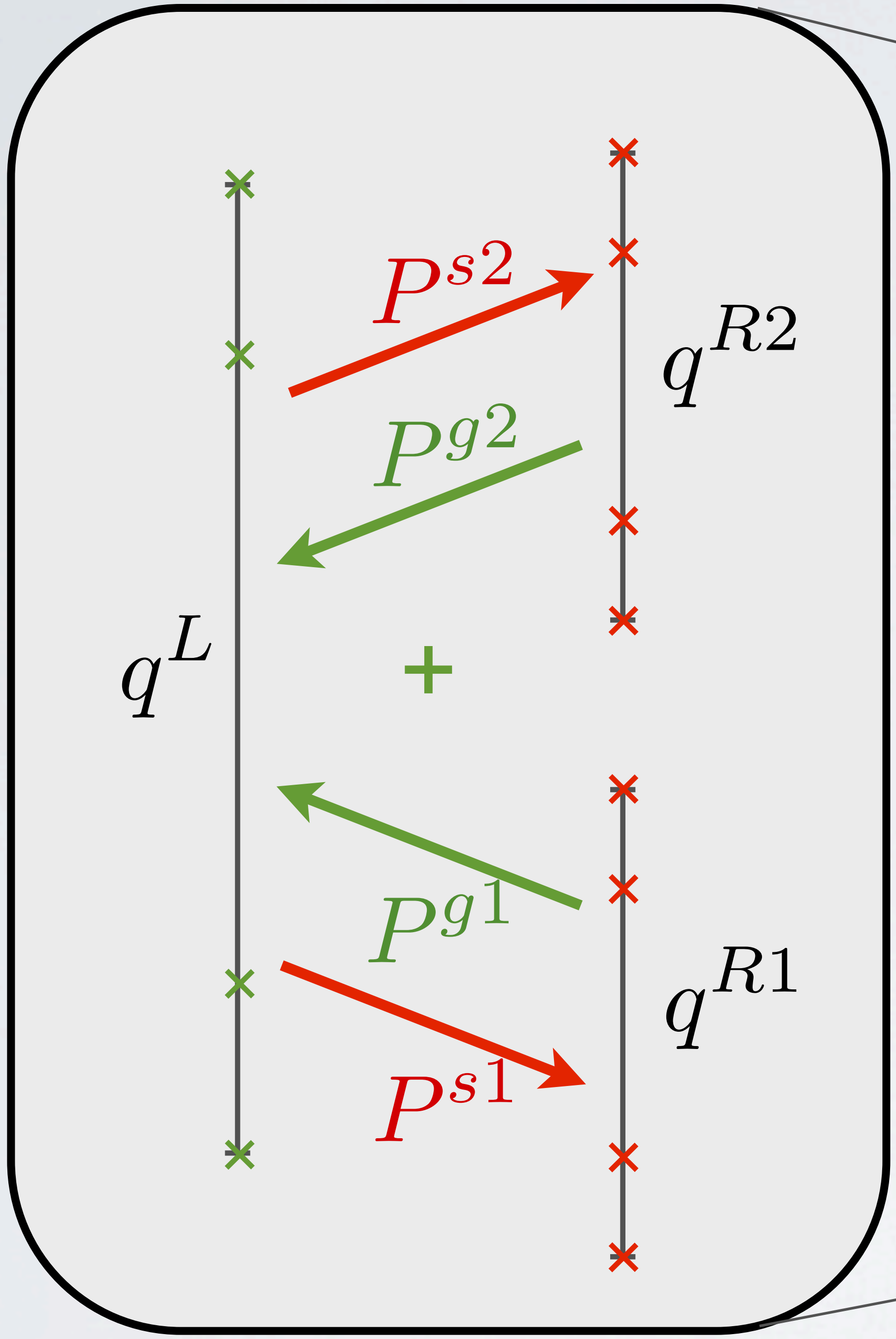
Data structures - storage



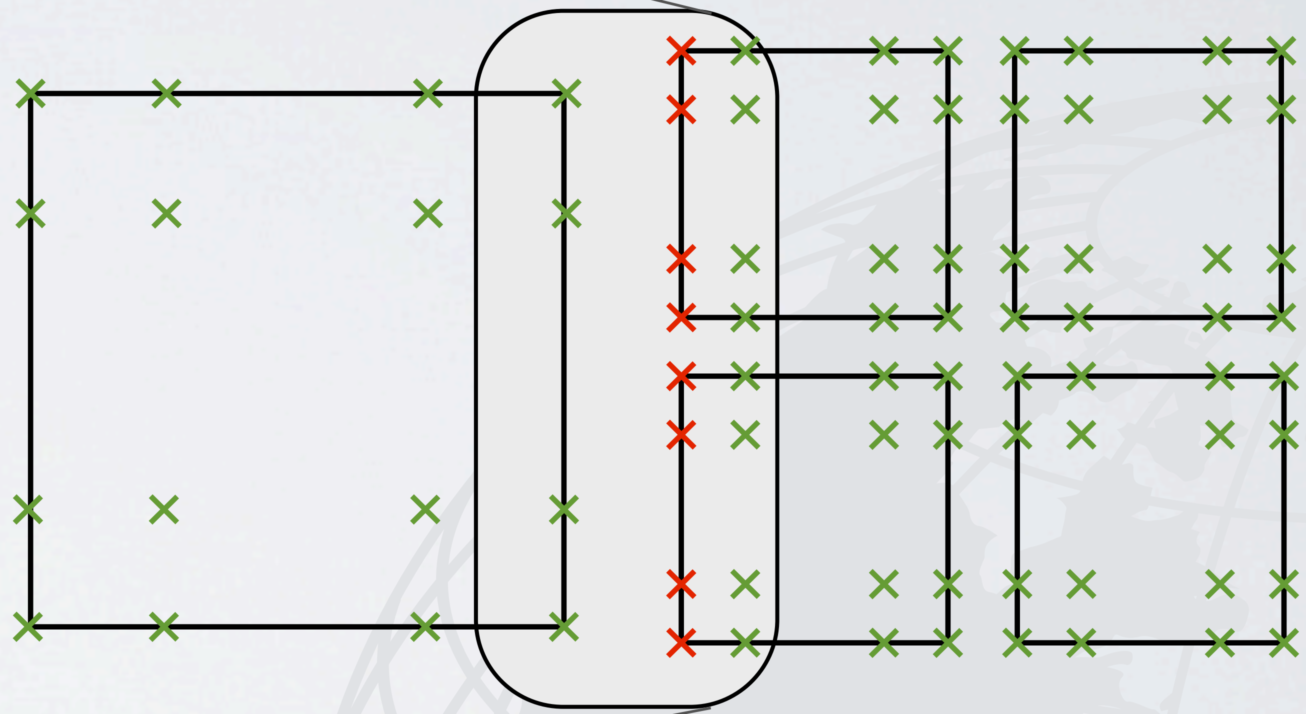
DSS on non-conforming elements



DG flux on non-conforming elements



$$\int_{-1}^{1} \left(q^{Lk}(z^{(k)}) - q^{L}(s \cdot z^{(k)} + o^{(k)})\right) \psi_i(z^{(k)})\ dz^{(k)} = 0$$

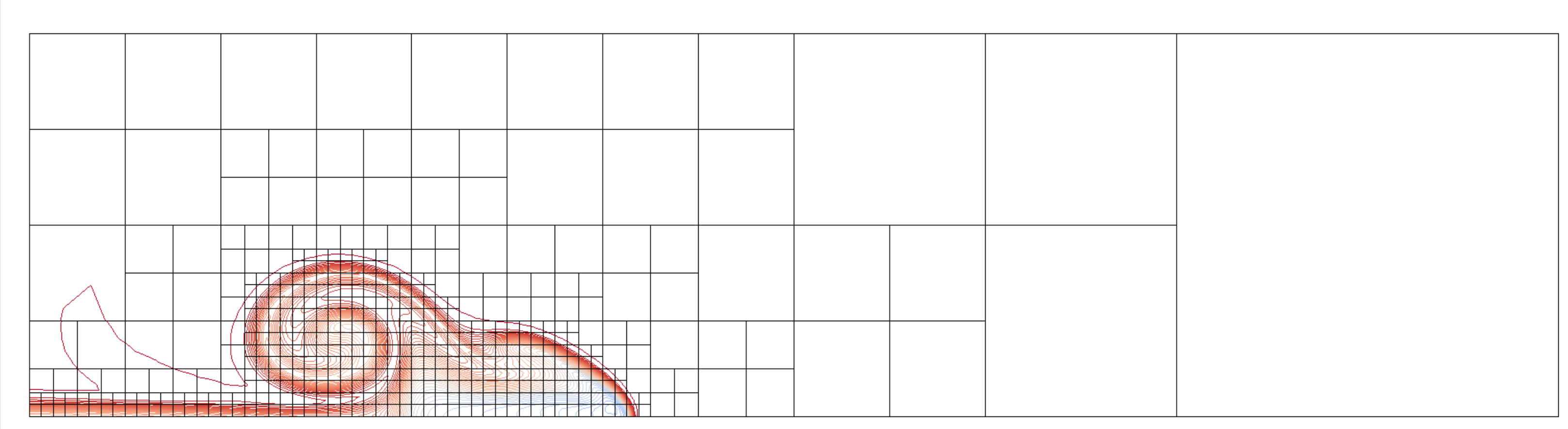


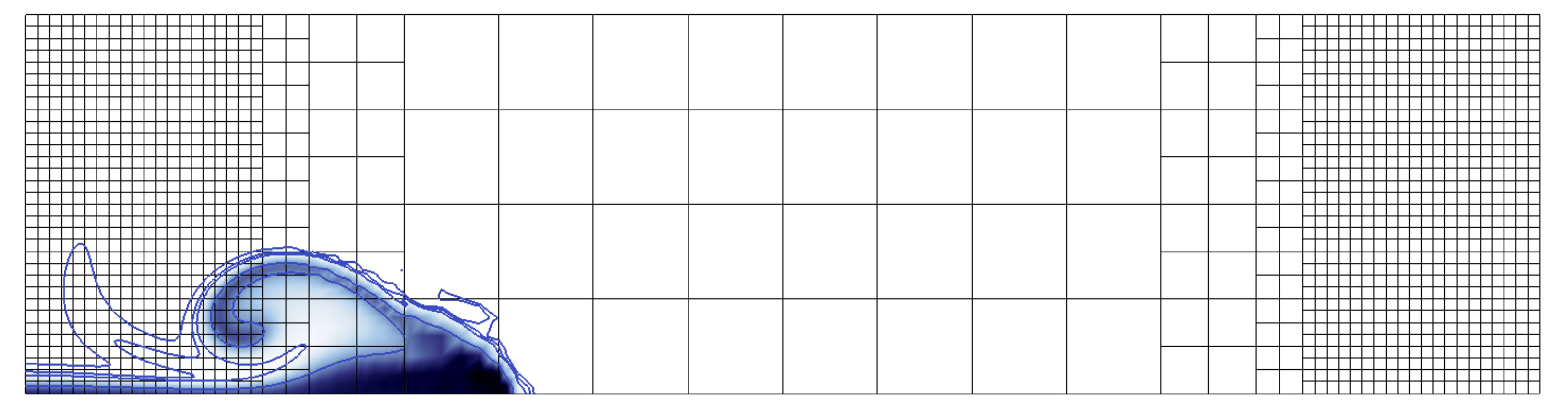
$$\int_{-1}^{1} \left(q^{R}(\xi) - \tilde{q}^{R}(\xi)\right) \psi_i(\xi) d\xi = 0$$

Kopriva, D.A. (1996), *A Conservative Staggered-Grid Chebyshev Multidomain Method for Compressible Flows*, J. Comp. Phys. **128**, 475-488

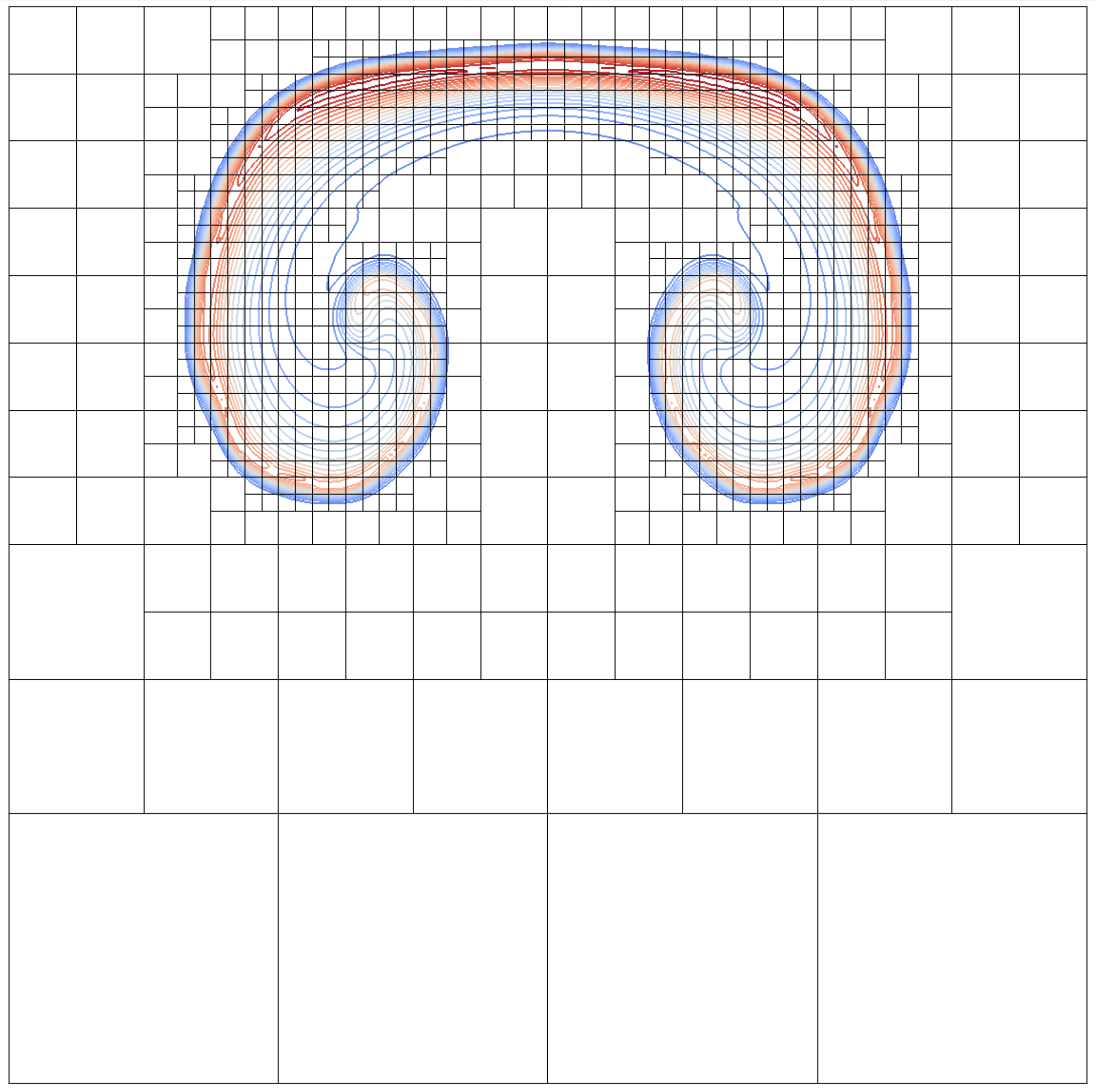
Test cases

Density current

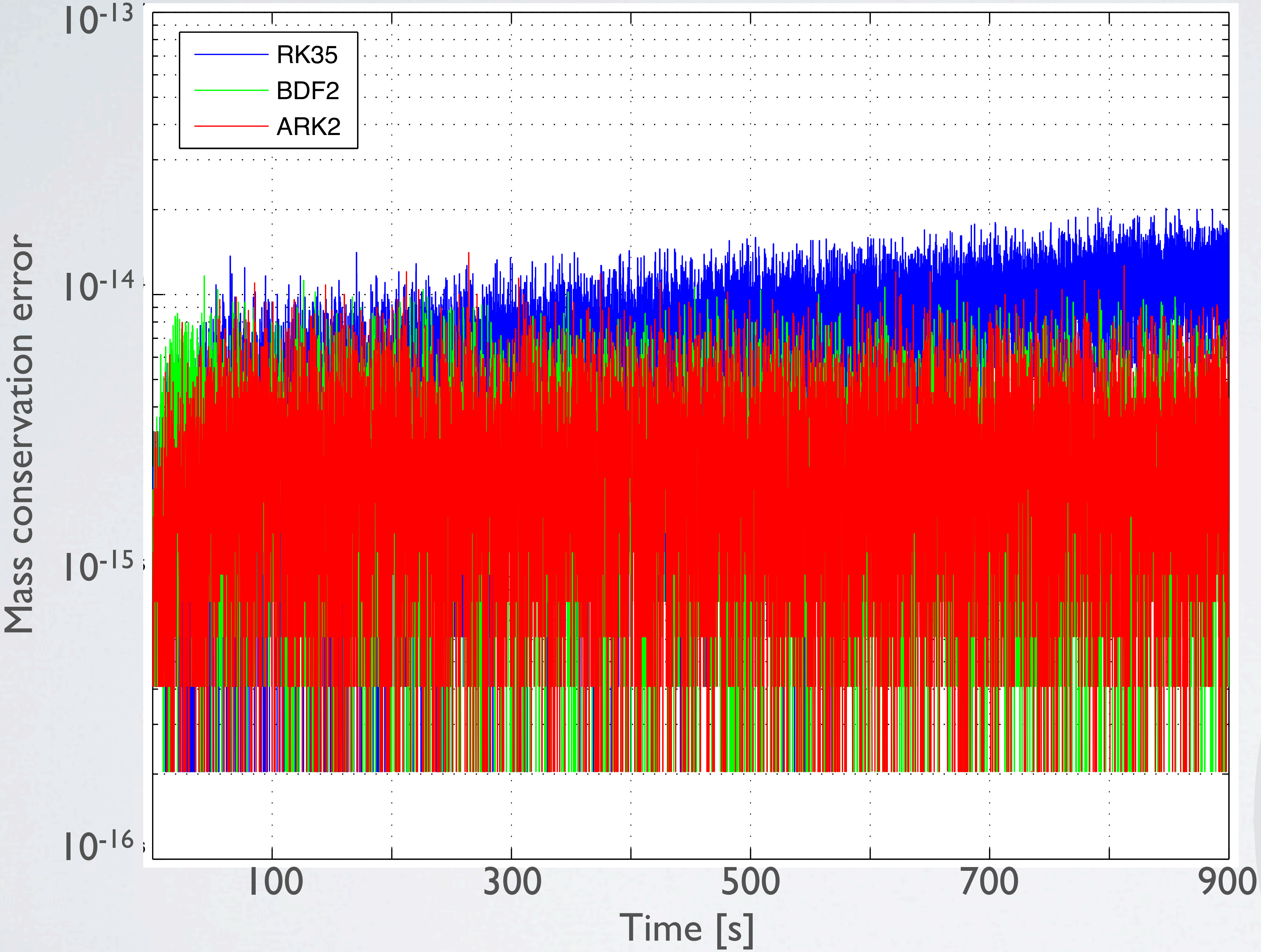




Rising thermal bubble



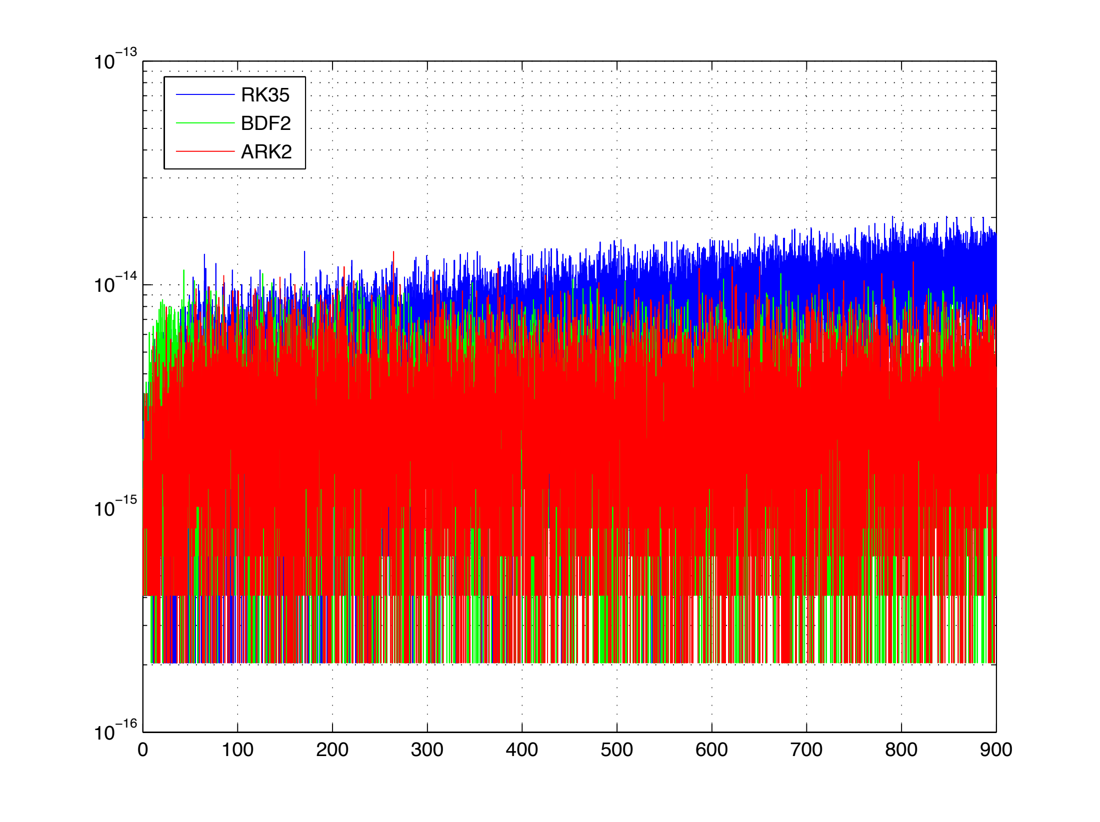
Initial tests - no AMR DG



$$m(t) = \sum_e \int_{\Omega_e} \rho(t)\, \mathrm{d}\Omega_e$$

$$M_1 = \frac{|m(t_1) - m(0)|}{m(0)}$$

Initial tests - no AMR DG

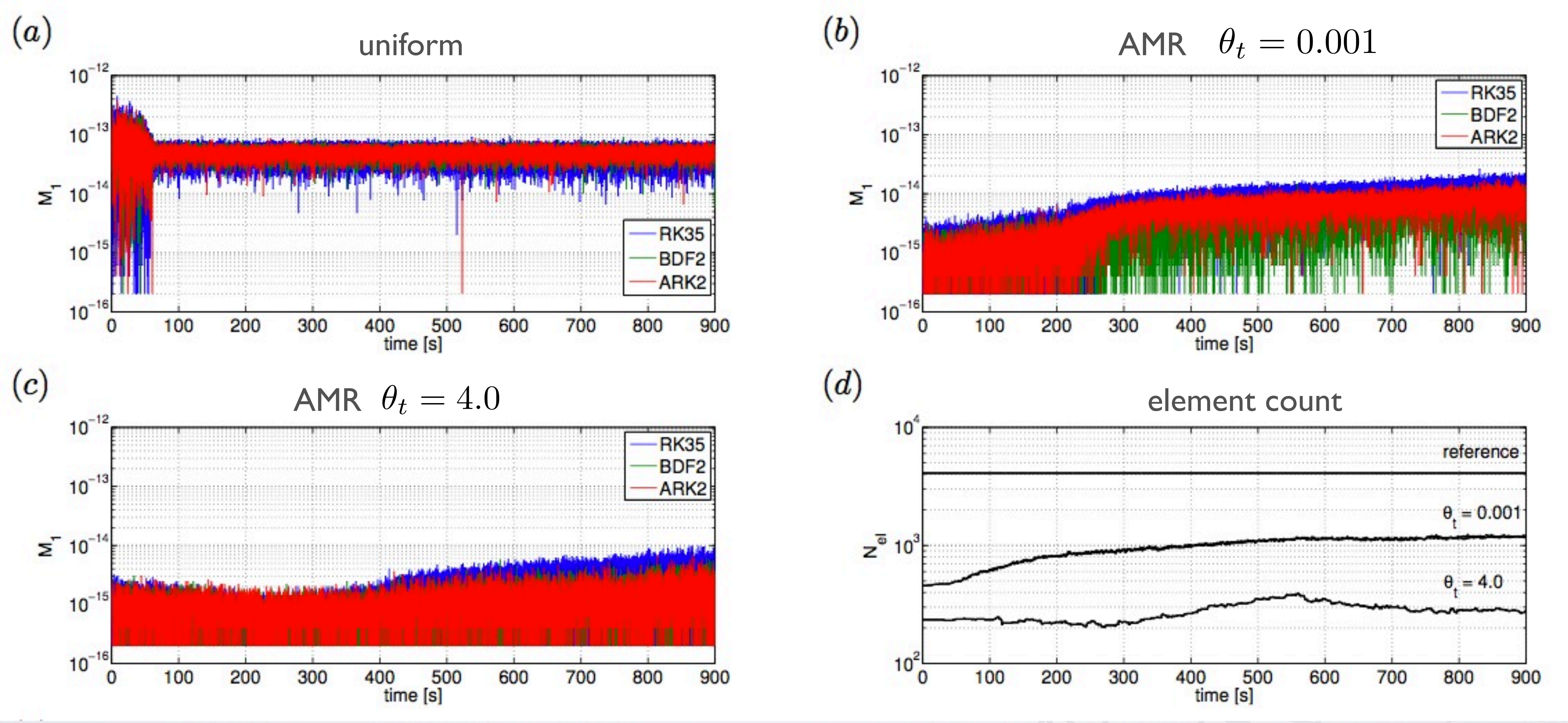


RK35 coefficients are slightly off !!!

S.J. Ruuth, Global optimization of explicit strong-stability-preserving Runge Kutta methods, *Math. Comp. 75 (253)* (2006) 183-207

stage	α			β
1	1	0	0	0.377268915331368
2	0	1	0	0.377268915331368
3	0.355909775063327	0.644090224936674	0	0.242995220537396
4	0.367933791638137	0.632066208361863	0	0.238458932846290
5	0	0.762406163401431	0.237593836598569	0.237593836598569

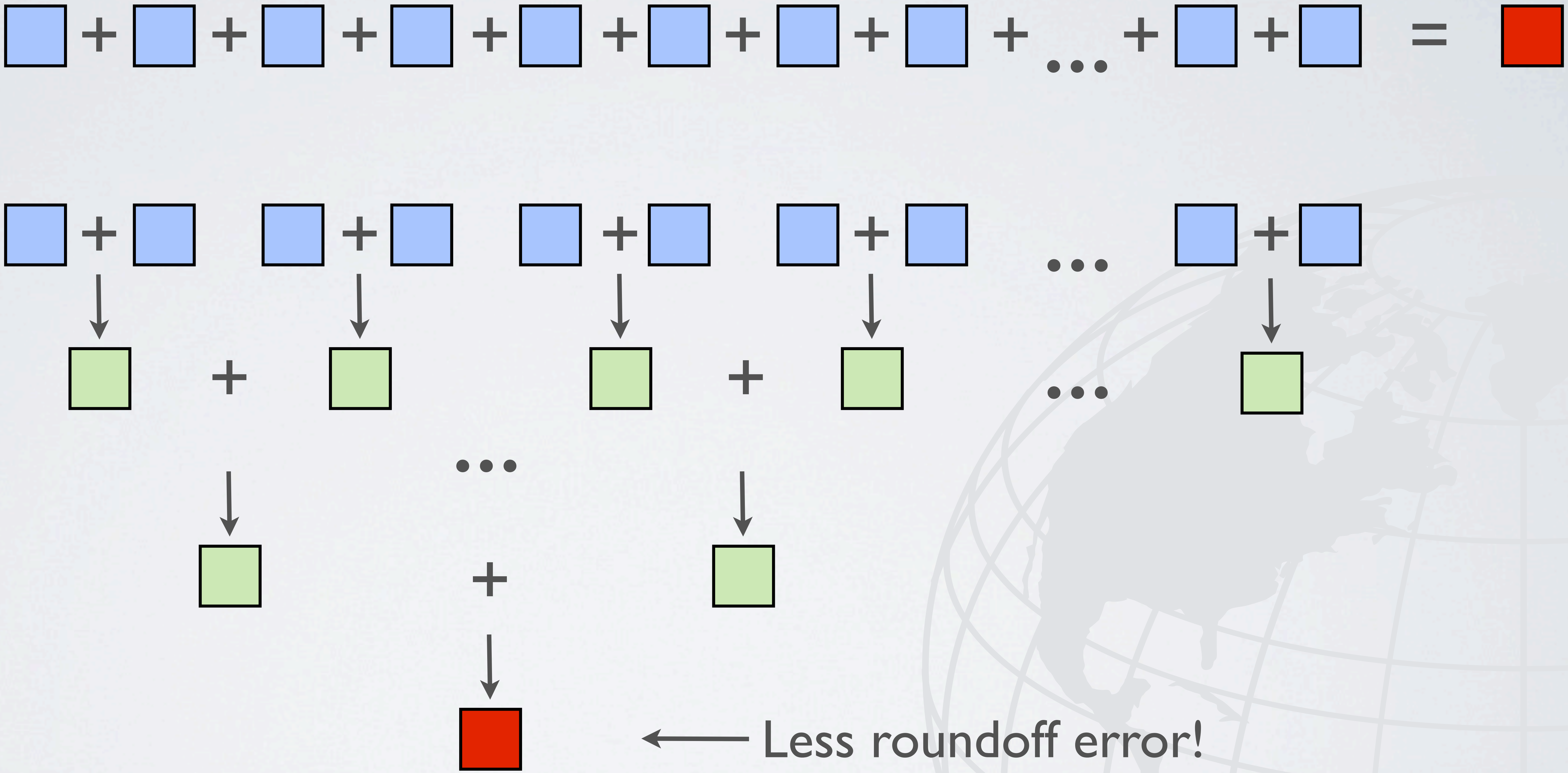
Initial tests - AMR DG



Summing algorithm

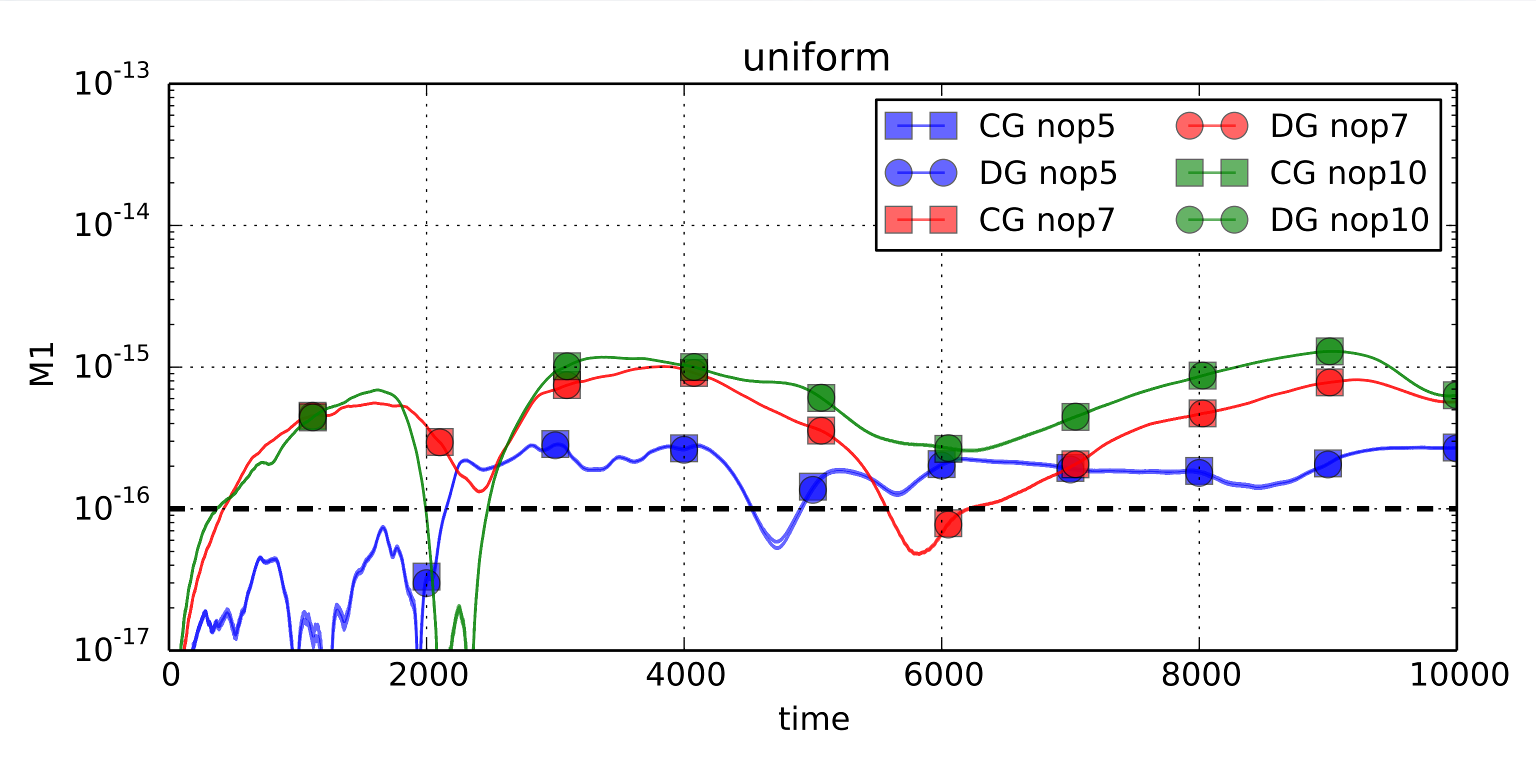
Serial

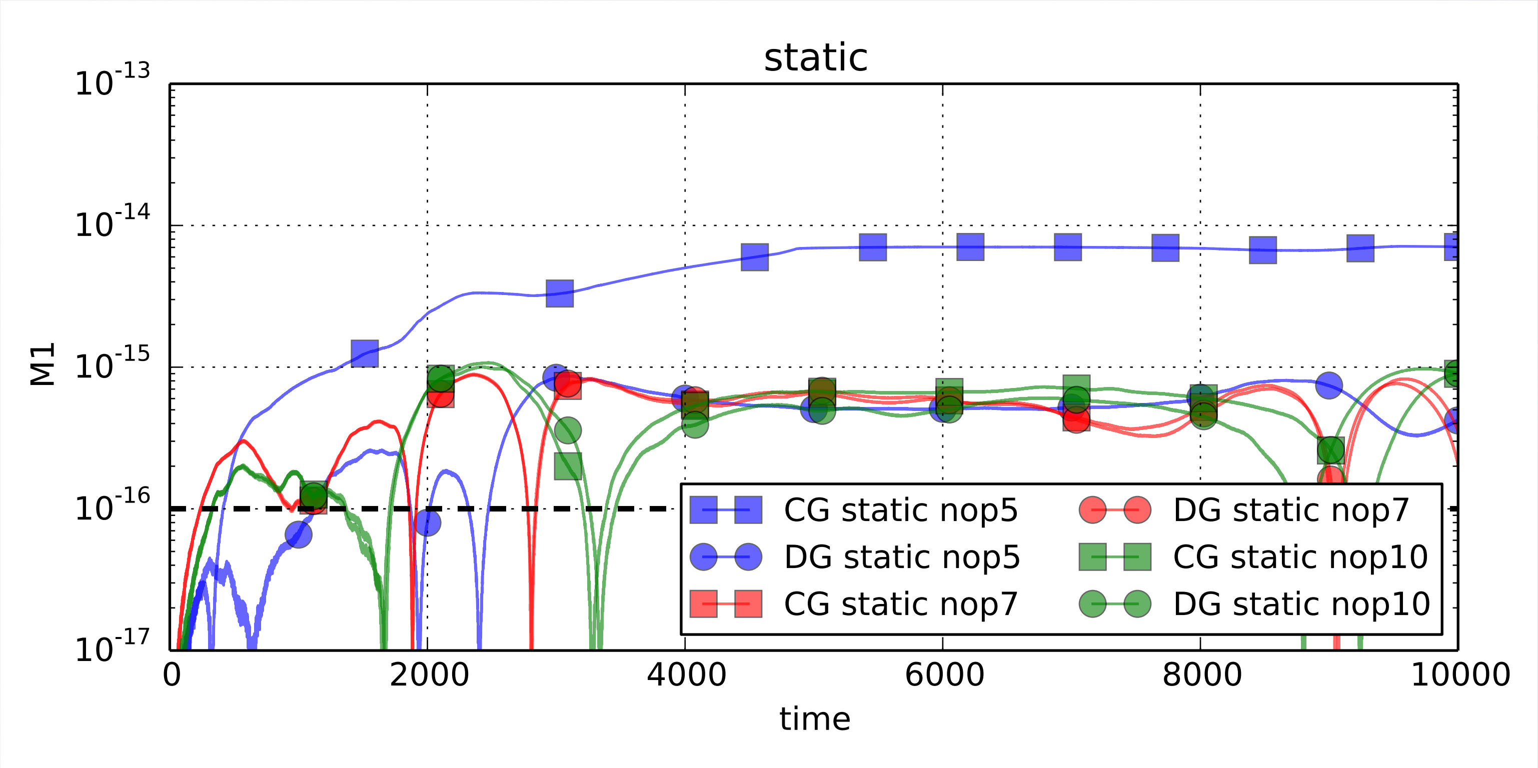
Pairwise sum

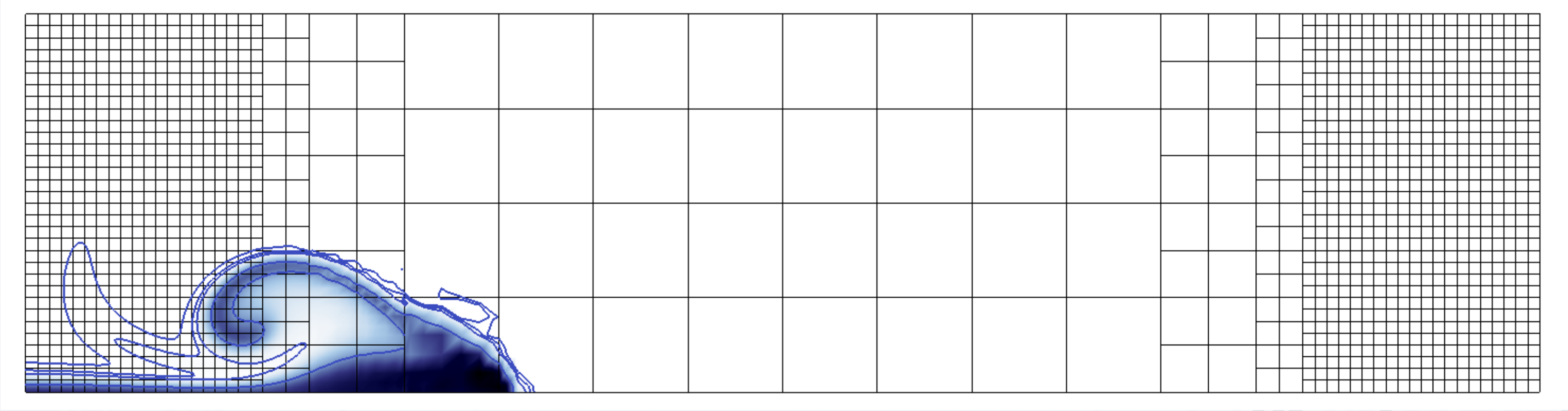


N. J. Higham, The accuracy of floating point summation, *SIAM Journal on Scientific Computing 14 (4)* (1993) 783–799.

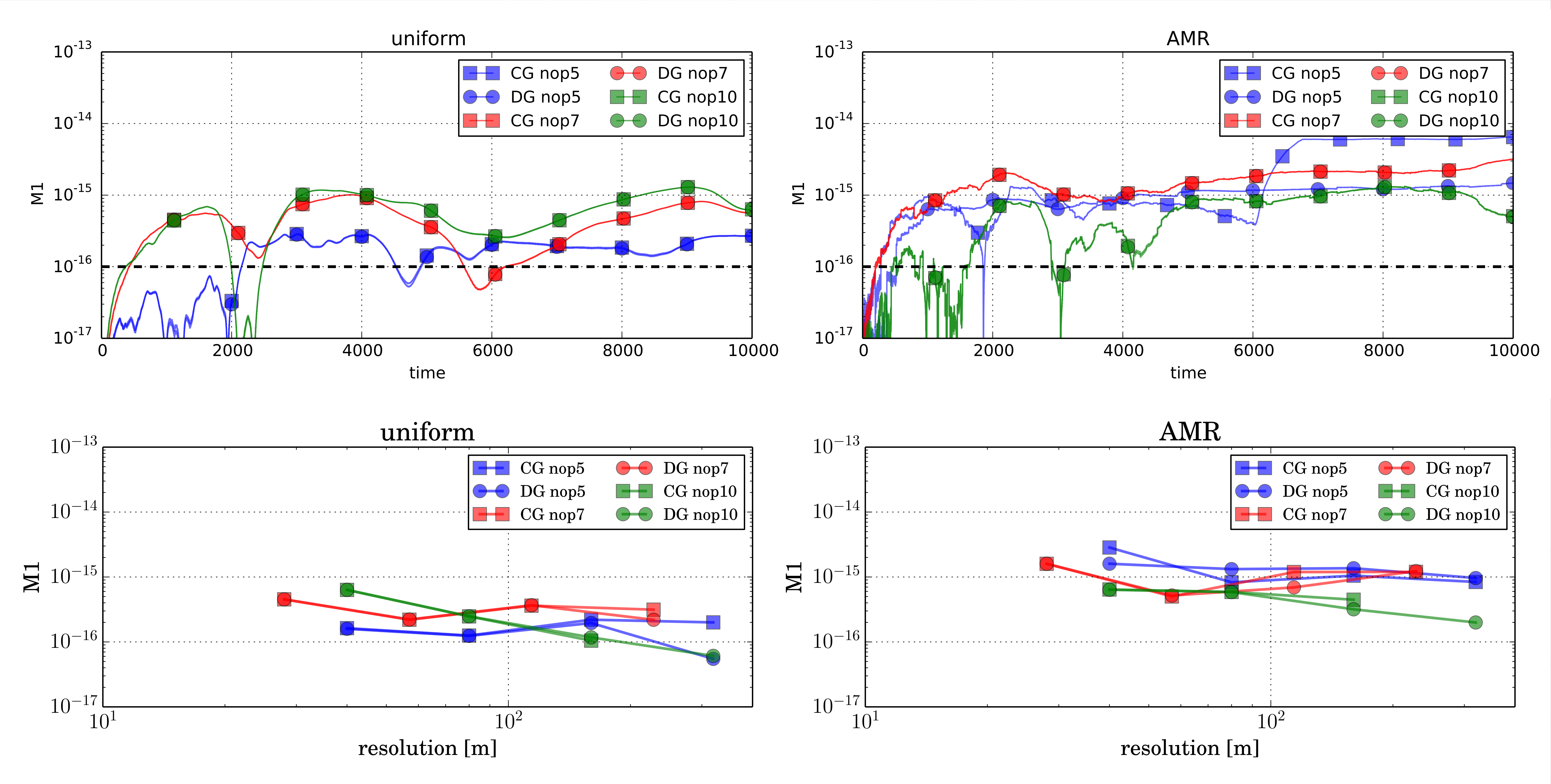
Results - density current



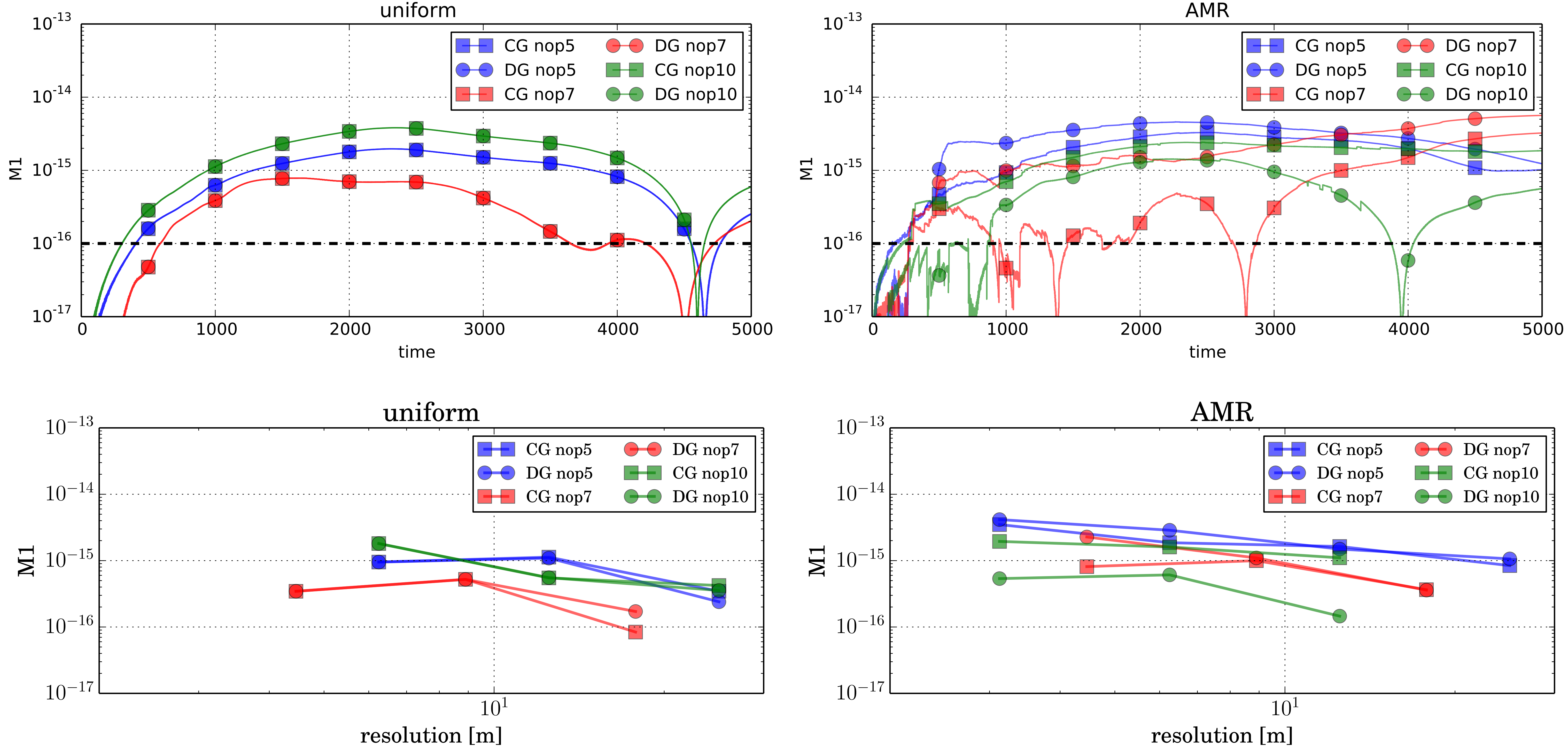




Results - density current



Results - rising thermal bubble



Conclusions

- AMR affects mass conservation only slightly
- CG can conserve as well as DG in AMR simulations
- CG is less robust for AMR simulations
- devil is in the details!

To do

- Investigate the filter effect