Formulation and performance of the Variable-Cubic Atmospheric Model

John McGregor

CSIRO Marine and Atmospheric Research
Aspendale, Melbourne
Southern Hemisphere

PDEs on the Sphere
NCAR
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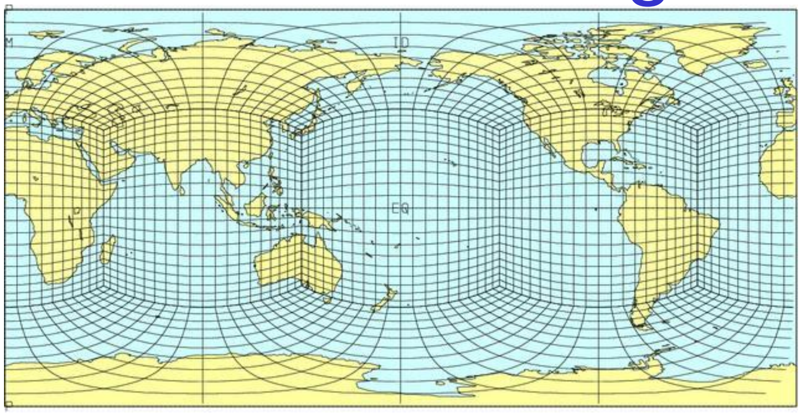
Outline

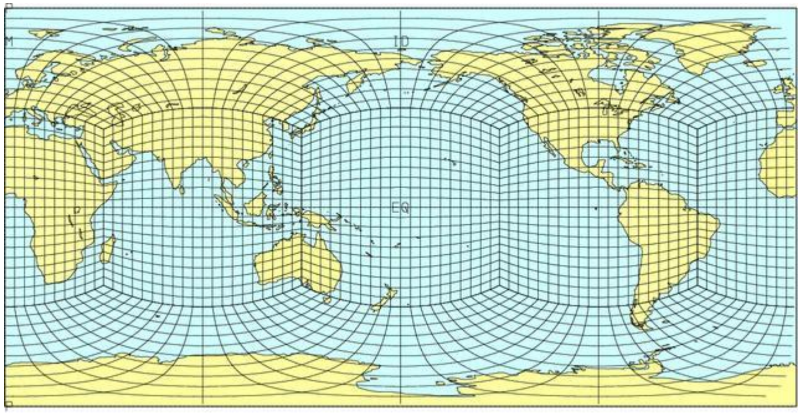
- CCAM description
  - conformal-cubic atmospheric model
- VCAM formulation
  - variable-cubic atmospheric model
- AMIP run (1979-1995)

Alternative cubic grids

Original
Sadourny (1972)
C20 grid

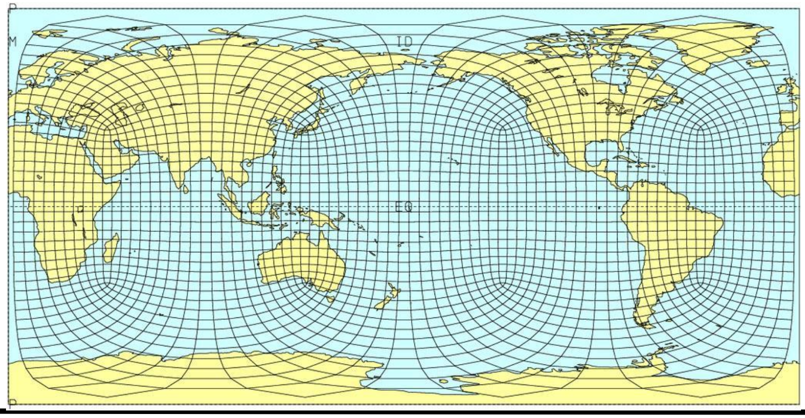


Equi-angular
C20 grid



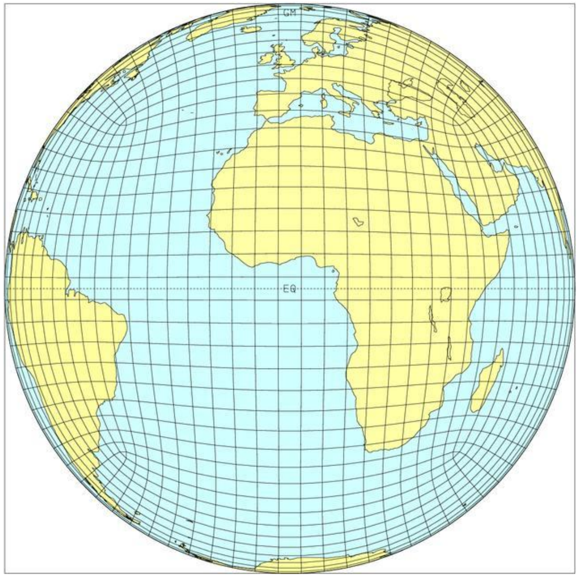
Used by VCAM

Conformal-cubic
C20 grid

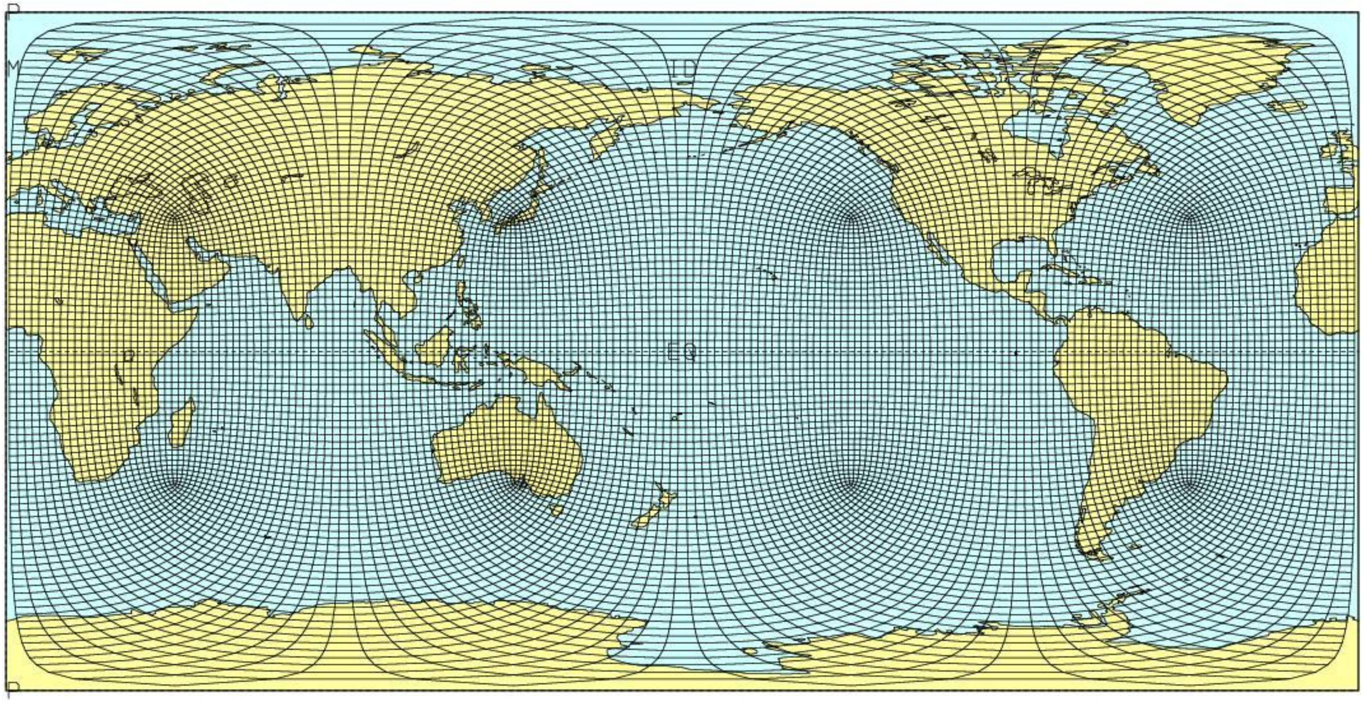


Used by CCAM

The conformal-cubic atmospheric model



- CCAM is formulated on the conformal-cubic grid
- Orthogonal
- Isotropic
- Semi-Lagrangian
- Reversible staggering

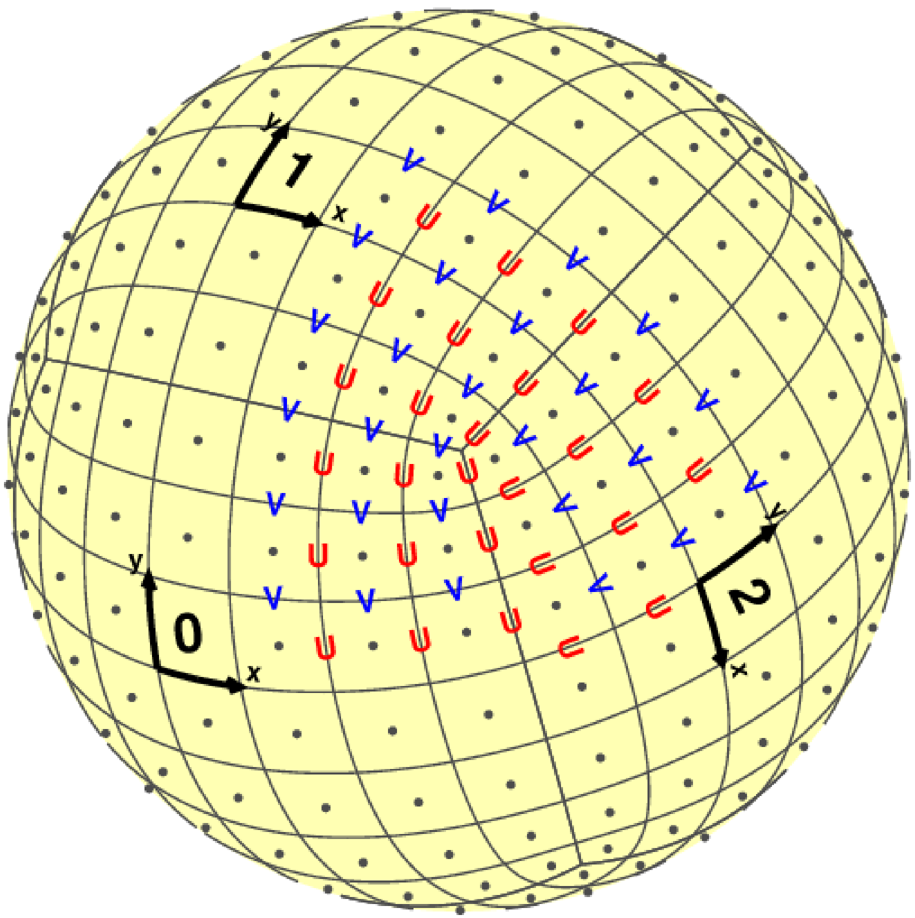


Example of quasi-uniform C48 grid with resolution about 200 km

A major issue for dynamical cores is how to stagger the winds to accurately balance the pressure gradients, whilst also accurately handling the Coriolis terms, i.e. obtain good geostrophic adjustment.

The approach in CCAM and VCAM is based on “reversible staggering” of velocity components.

Location of variables in grid cells



All variables are located at the centres of quadrilateral grid cells.

In CCAM, during semi-implicit/gravity-wave calculations, u and v are transformed reversibly to the indicated C-grid locations.

Produces same excellent dispersion properties as spectral method (see McGregor, MWR, 2006), but avoids any problems of Gibbs' phenomena.

2-grid waves preserved. Gives relatively lively winds, and good wind spectra.

Reversible staggering

pivot

| X | X * | X |

m-1 m-1/2 m m+1/2 m+1 m+3/2 m+2

m+3/4

Where U is the unstaggered velocity component and u is the staggered value, define (Vandermonde formula)

$$\frac{u_{m-\frac{1}{2}} + 10u_{m+\frac{1}{2}} + 5u_{m+\frac{3}{2}}}{16} = \frac{5U_m + 10U_{m+1} + U_{m+2}}{16}$$

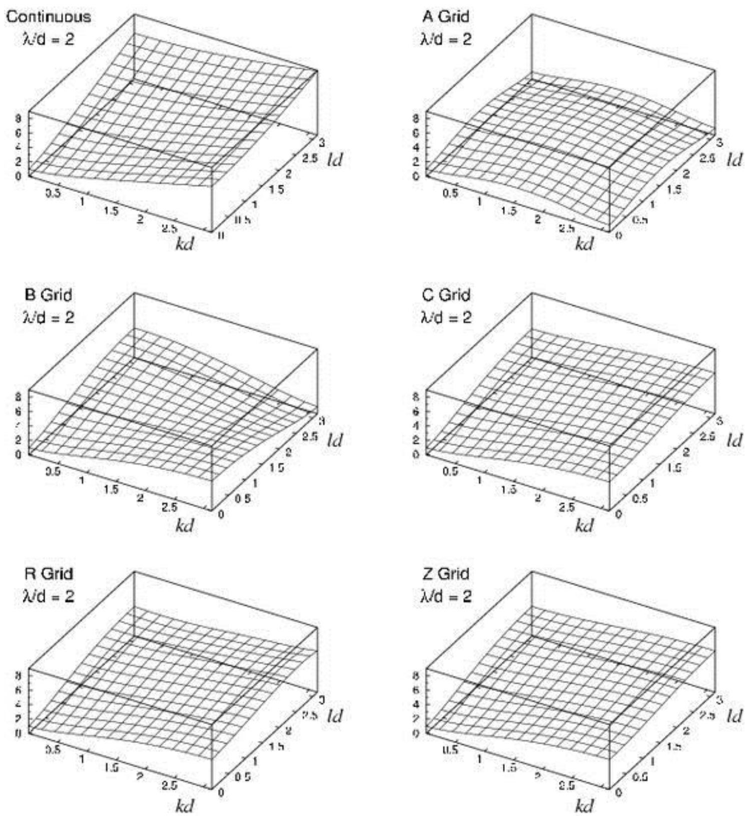
- accurate at the pivot points for up to 4^{th} order polynomials
- solved iteratively, or by cyclic tridiagonal solver
- excellent dispersion properties for gravity waves, as shown for the linearized shallow-water equations

$$\frac{\partial u}{\partial t} - fv + g\frac{\partial h}{\partial x} = 0,$$

$$\frac{\partial v}{\partial t} + fu + g\frac{\partial h}{\partial y} = 0,$$

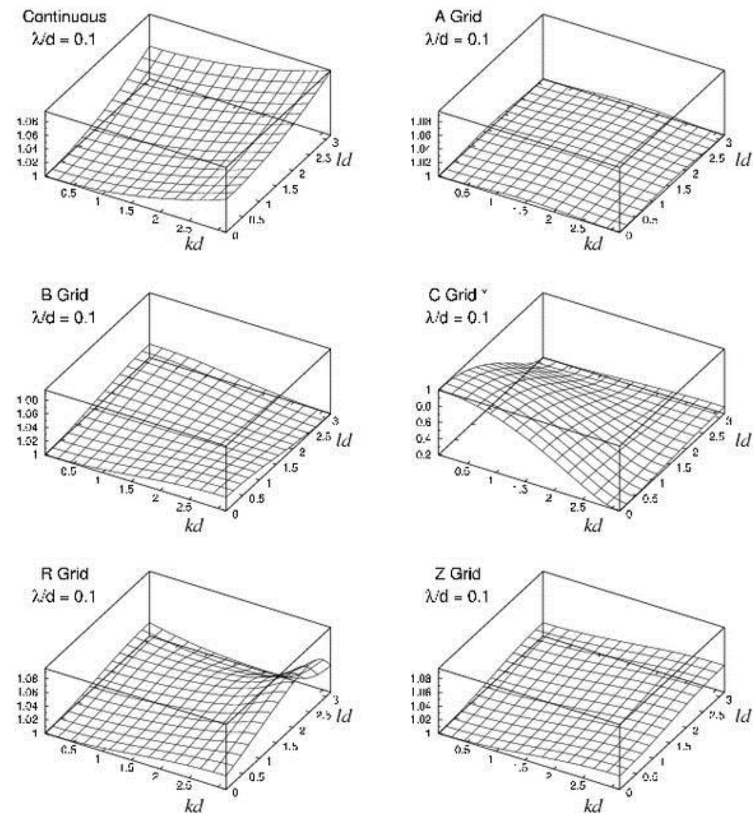
$$\frac{\partial h}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0.$$

Dispersion behaviour for linearized shallow-water equations



Typical atmosphere case

- large radius deformation



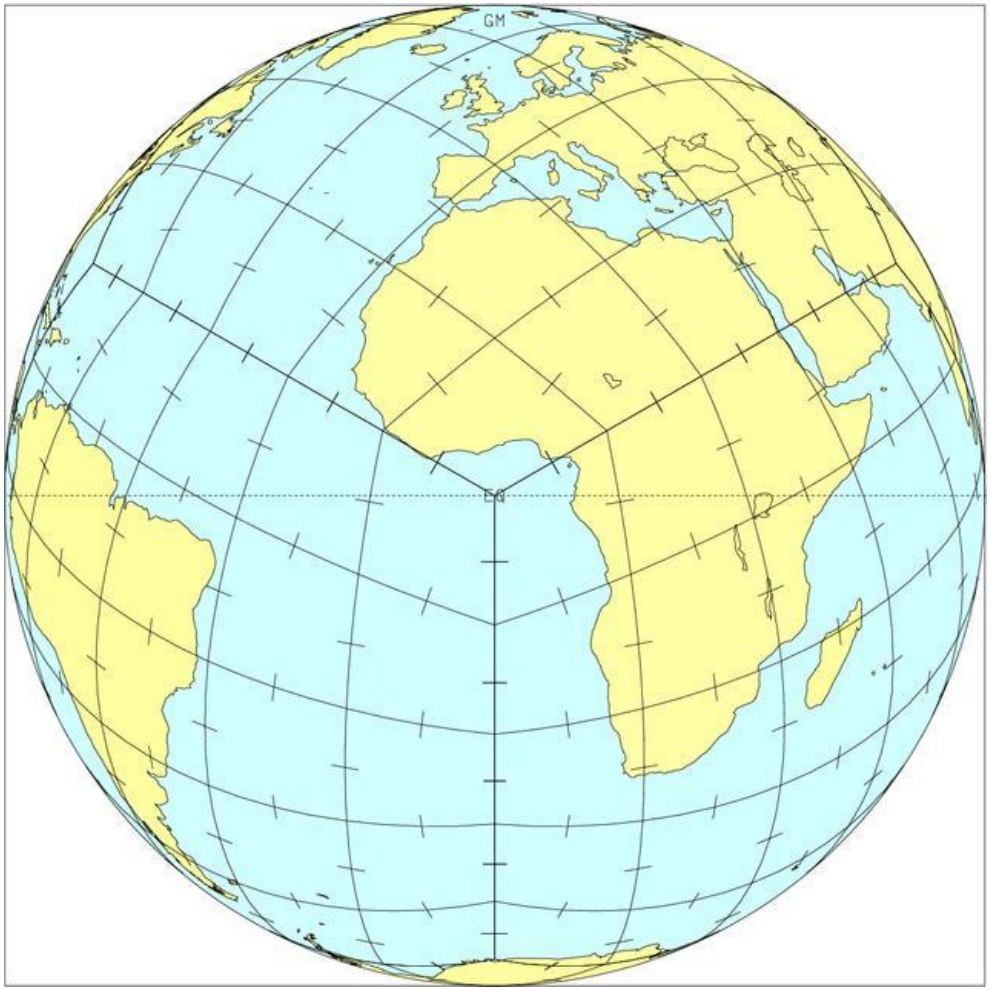
Typical ocean case

- small radius deformation

N.B. the asymmetry of the R grid response disappears by alternating the reversing direction each time step, giving the same response as Z (vorticity/divergence) grid

Gnomonic grid (for VCAM) showing orientation of the contravariant wind components

Illustrates the suitability of the gnomonic grid for reversible interpolation – thanks to smooth changes of orientation of contravariant components



New dynamical core for VCAM
- Variable Cubic Atmospheric Model

- uses equi-angular gnomonic-cubic grid
  - provides highly uniform resolution
  - less issues for resolution-dependent parameterizations
- reversible staggering transforms the contravariant winds to the edge positions needed for calculating divergence and gravity-wave terms
- forward-backward (F-B) solver for gravity waves (split explicit)
  - no need for Helmholtz solver
  - linearizing assumptions avoided in gravity-wave terms
  - avoids semi-Lagrangian off-centring
- flux-conserving form of equations
  - preferable for trace gas studies
- finite volume advection with TVD can preserve sharp gradients

Flux form of equations

Denote staggered velocities by u^* and v^* (reversibly obtained from u and v).

Continuity

$$\frac{\partial p_s}{\partial t} + m^2 \left\{ \frac{\partial (p_s u^*/m)}{\partial x} + \frac{\partial (p_s v^*/m)}{\partial y} \right\} + p_s \frac{\partial \dot{\sigma}}{\partial \sigma} = 0,$$

$\llcorner$..........................$A\ \Delta t'$........................$\lrcorner$

Temperature and velocity equations

$$\frac{\partial p_s T}{\partial t} + m^2 \left\{ \frac{\partial p_s u^* T/m}{\partial x} + \frac{\partial p_s u^* T/m}{\partial y} \right\} + p_s \frac{\partial \dot{\sigma} T}{\partial \sigma} - \frac{R_d T \omega}{c_p \sigma} = \tilde{N}_T,$$

$\llcorner$......................$C\ adv \Delta t$....................$\lrcorner$ $\llcorner A\ \Delta t' \lrcorner$

$$\frac{\partial p_s u}{\partial t} + m^2 \left\{ \frac{\partial p_s u^* u/m}{\partial x} + \frac{\partial p_s v^* u/m}{\partial y} \right\} + p_s \frac{\partial \dot{\sigma} u}{\partial \sigma} + m p_s \frac{\partial \phi_v}{\partial x} + m R_d T_v \frac{\partial p_s}{\partial x} = f p_s v^T$$

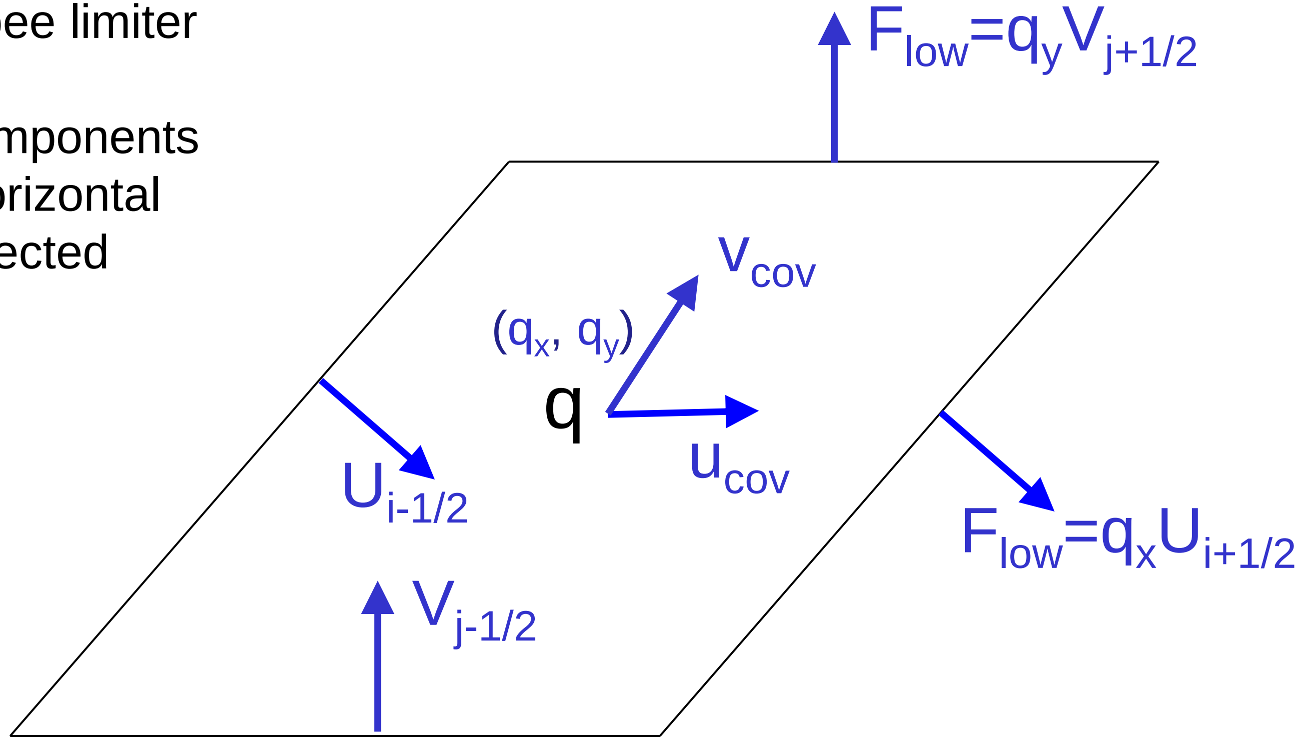
$\llcorner$.....................$C\ adv \Delta t$......................$\lrcorner$ $\llcorner$..................$B\ \Delta t'$..................$\lrcorner$

with a similar equation for v.

Advection

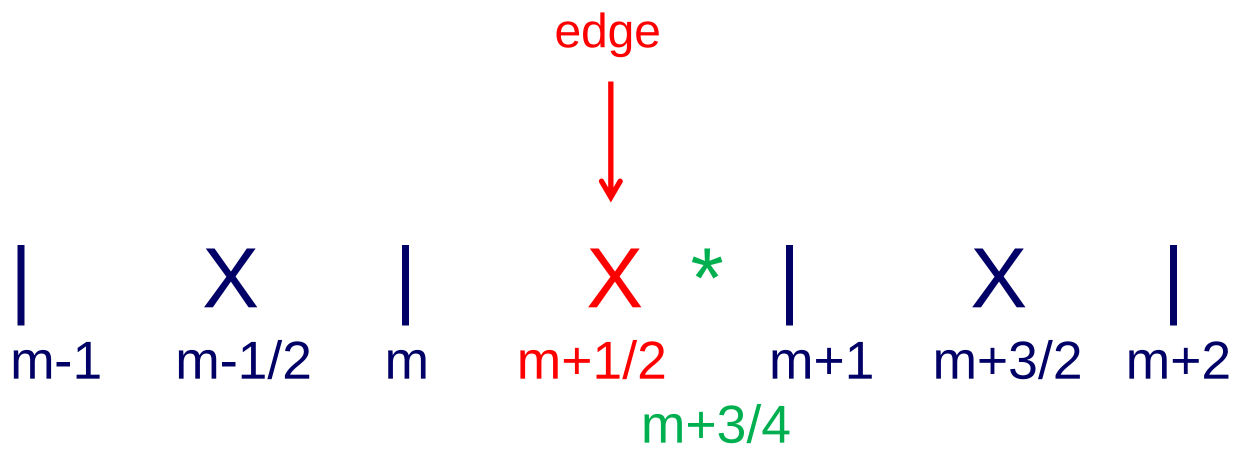
Low-order and high-order fluxes combined using Superbee limiter

Cartesian components (U,V,W) of horizontal wind are advected



Transverse components are included in both low/high order fluxes calculated at the edges of the grid cells. Follows LeVeque, but different calculation of transverse terms both horizontally and vertically.

Improved treatment of pivot points at panel edges



Usual pivot velocity (in terms of staggered u) is

$$u_{m+3/4} = (u_{m+1/2}+u_{m+3/2})/2$$

In terms of unstaggered U, it is

$$U_{m+3/4} = (2U_m+6U_{m+1})/8$$

But adjacent to panel edge it is better to use

$$U_{m+3/4} = (-U_{m-1} + 3U_m + 7U_{m+1} - U_{m+2})/8$$

which is derived by using an estimate for $U_{m+1/2}$ provided by averaging 1-sided left and right extrapolations of U.

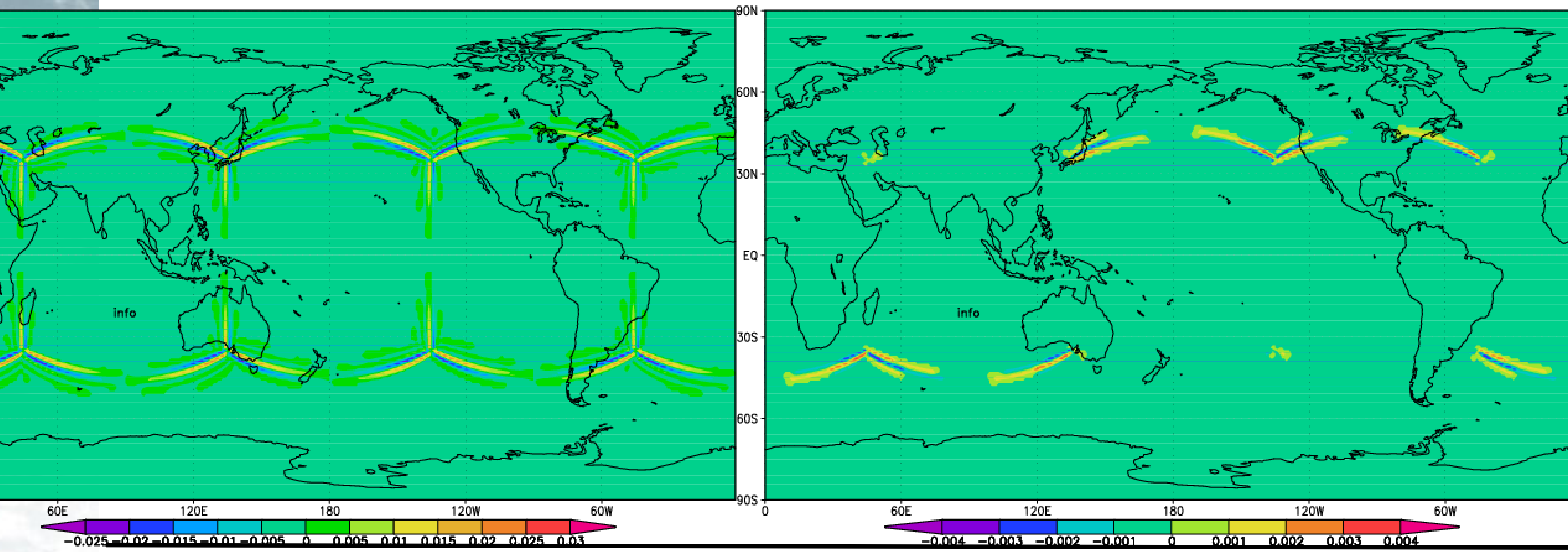
These extrapolations will be very accurate for velocities such as solid body rotation

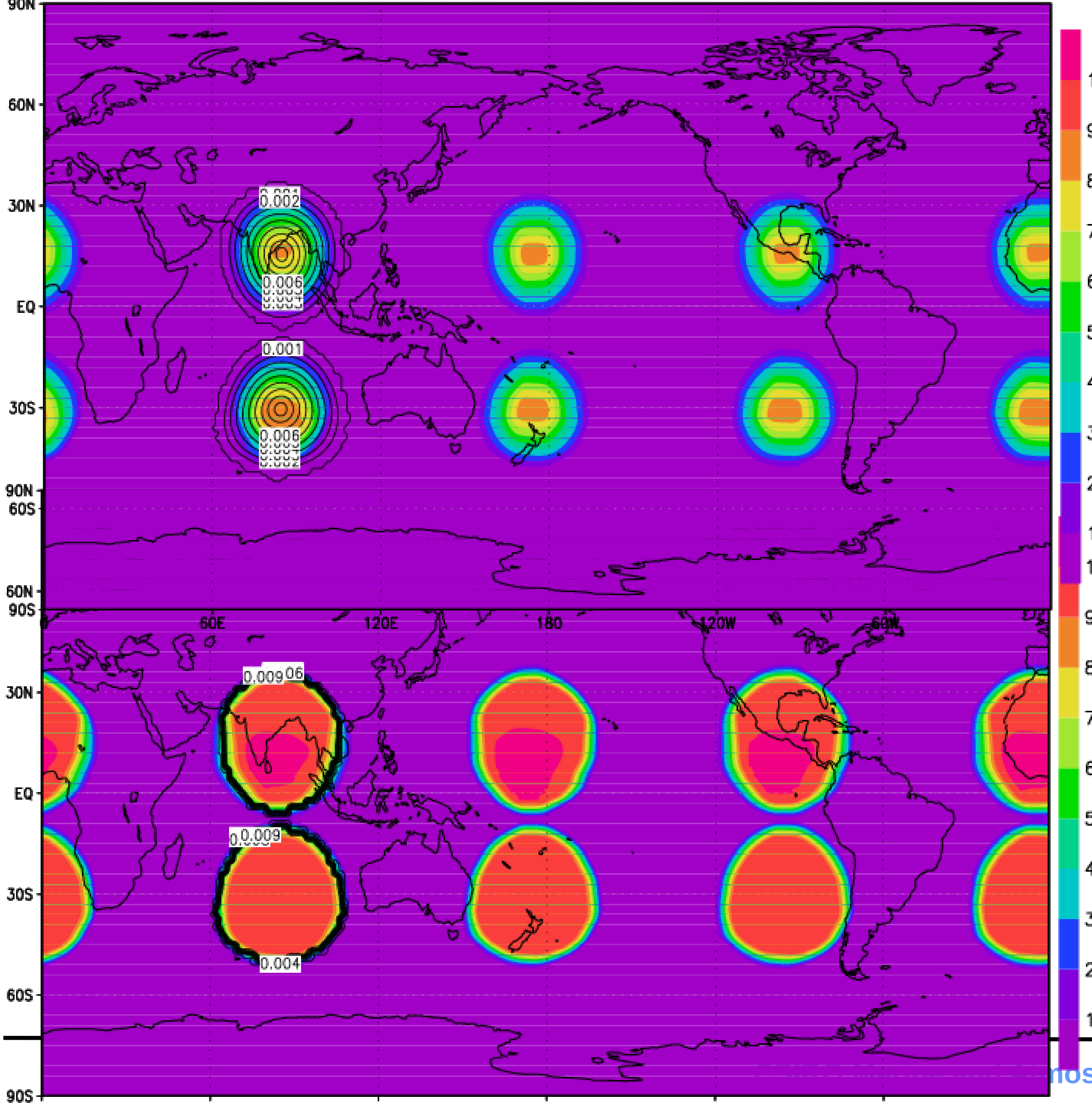
N.B. Special treatment not needed if using Jim Purser's uniform-cubic grid

That will also avoid ANY grid imprinting.

Reduction of “grid imprinting”

- Spurious vertical velocities reduced by factor of 8 by improved calculation of pivot velocities near panel edges (for use with reversible interpolation)
- As seen from vertical velocities of solid-body rotation (after staggering)





Eastwards solid body rotation in 900 time steps Using superbee limiter

Solution procedure

- Start τ loop

Nx(Δt/N) forward-backward loop
Stagger (u, v) τ+n(Δt/N)
Average ps to (psu, psv) τ+n(Δt/N)
Calc (div, sdot, omega) τ+n(Δt/N)
Calc (ps, T) τ+(n+1)(Δt/N)
Calc phi and staggered pressure gradient terms, {then unstagger these
Including Coriolis terms, calc unstaggered (u, v) τ+(n+1)(Δt/N)}
End Nx(Δt/N) loop

Perform TVD advection (of T, qg, Cartesian_wind_components) using average ps*u, ps*v, sdot from the N substeps
Calculate physics contributions

- End τ loop

F-B can be performed in staggered space

- Can avoid reversible staggering during the F-B steps to allow whole F-B to be done in staggered space
- At the start of F-B, stagger u, v, $(fu)^{\tau}$, $(fv)^{\tau}$
- During F-B, use $(fu)^{\tau}$, $(fv)^{\tau}$ in momentum eqns
- At end of F-B, unstagger the net pressure-gradient terms, and apply these over the full Δt (implicit Coriolis) by solving the simple simultaneous equations:

$$U^{\tau+1} = U^{\tau} + \Delta t * pgx_{uns} + \Delta t * f(V^{\tau+1} + V^{\tau})/2$$

$$V^{\tau+1} = V^{\tau} + \Delta t * pgy_{uns} - \Delta t * f(U^{\tau+1} + U^{\tau})/2$$

- Behaviour is very similar to more-expensive unstaggering-staggering with Coriolis calculated In unstaggered space

Treatment of pressure gradient terms using first principles

- Calculate dphi/dx on pressure surfaces at u-staggered positions (similarly to description by Kurihara 1968)
- Requires vertical interpolation on adjacent columns and use of standard lapse rate below ground, but NO MPI involved
- Easy for split-explicit models, but not practicable for semi-explicit models, which usually employ an eigenvector decomposition in the vertical
- Before introducing this method had noise near orography, needing hybrid vertical coordinates
- Now no noise and don’t need hybrid coordinates
- Maybe can use unfiltered orography
- N.B. presently hydrostatic

Noise

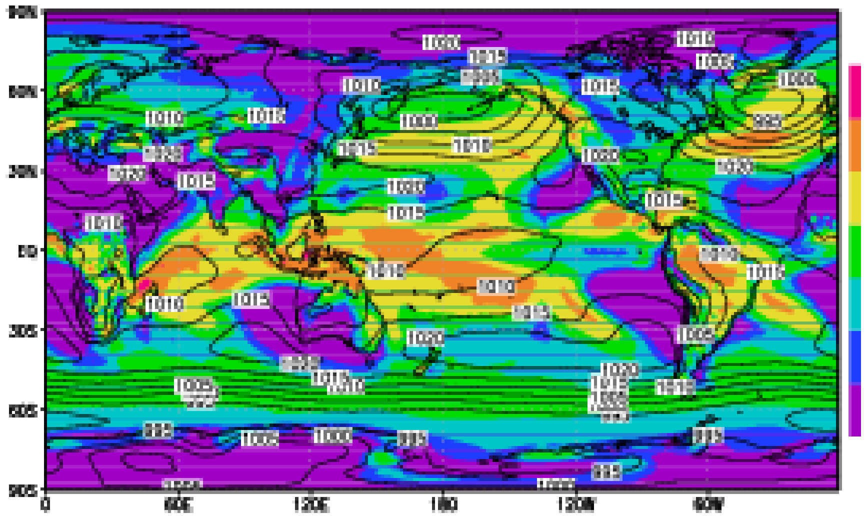
- See minor noise in PMSL in tropics only
  - seems unrelated to split-explicit
  - removed by using weak divergence damping
  - may be due to computational modes as in WRF
- No other noise issues, thanks to
  - use of reversible staggering (N.B. significant noise is seen if simple interpolation of velocity components is used in Coriolis terms)
  - “First principles” pressure gradient calculation
  - application of convective heating distributed over the forward-backward time steps

Message passing required during F-B steps

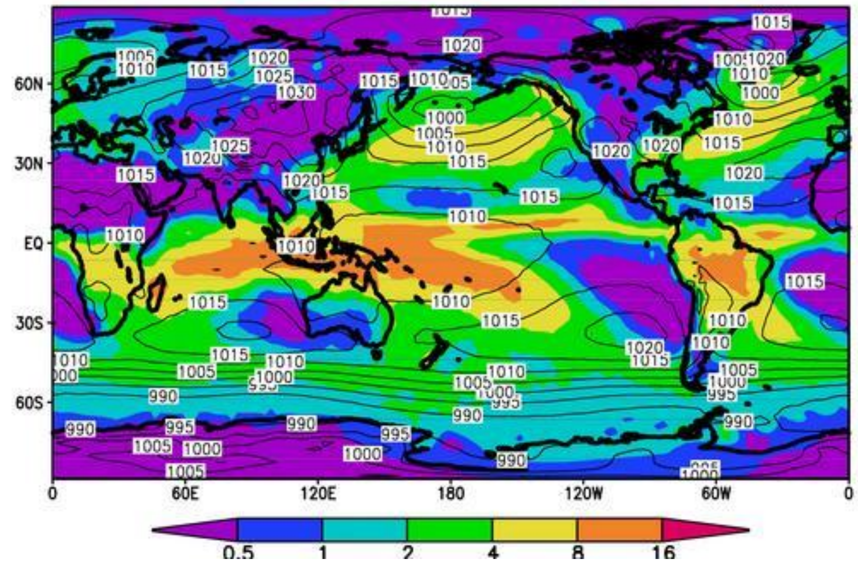
- Typically 10 F-B steps per advective step
- Need to pass T, ps (for calculating pressure gradients)
- Need to pass staggered u and v (for calculating divergence)
- No MPI needed for reversible wind staggering

Rainfall AMIP run 1979 - 1995

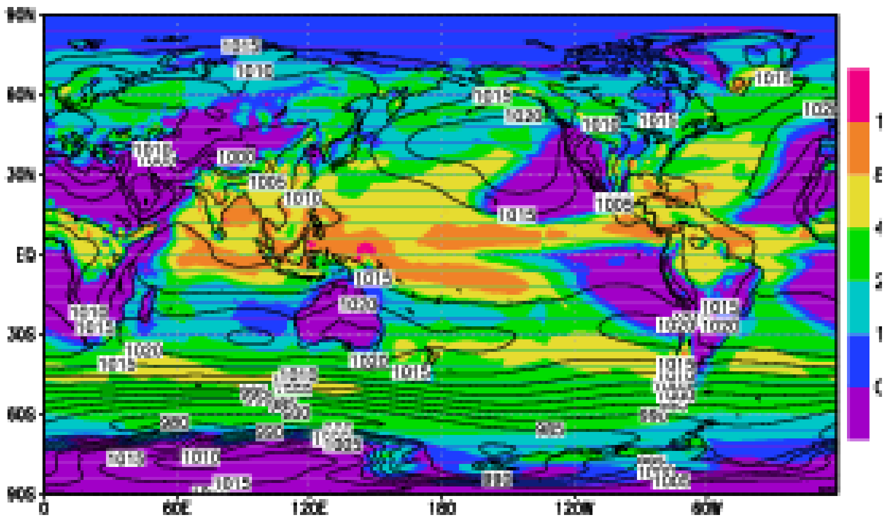
DJF



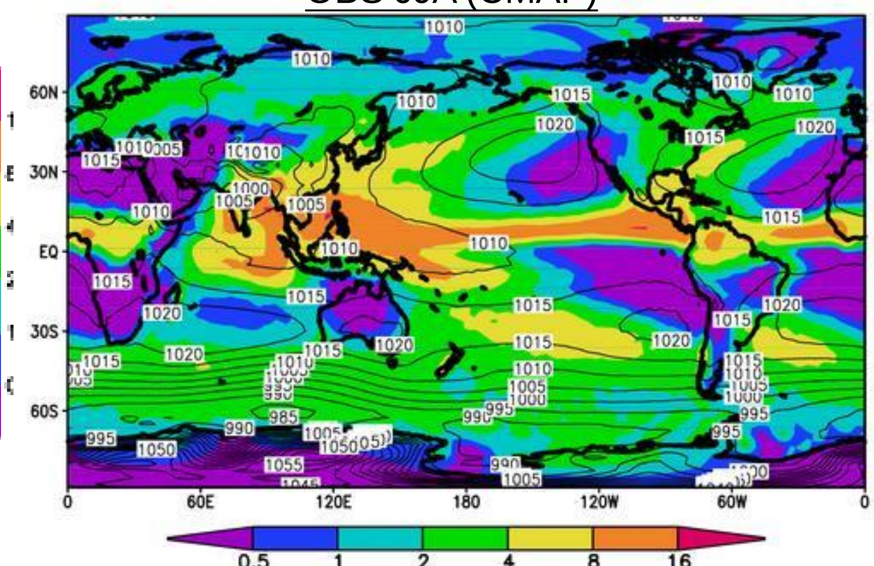
OBS DJF (CMAP)



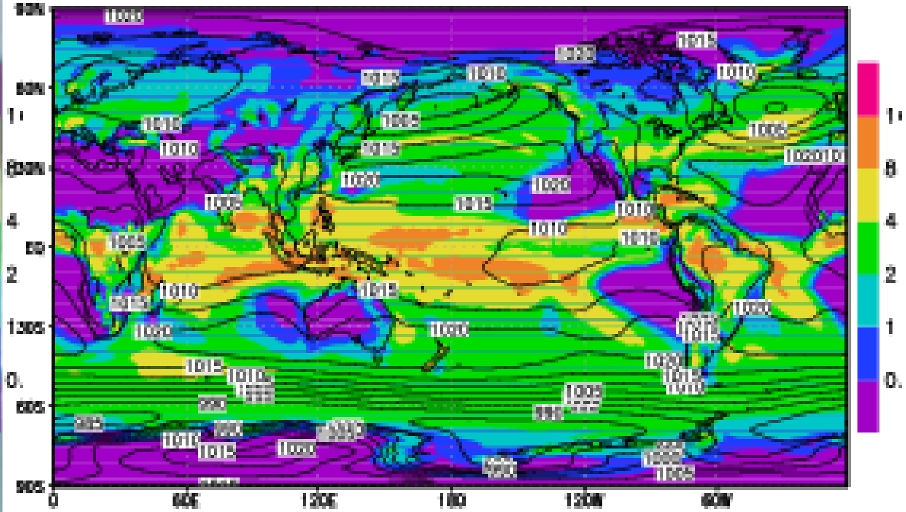
JJA



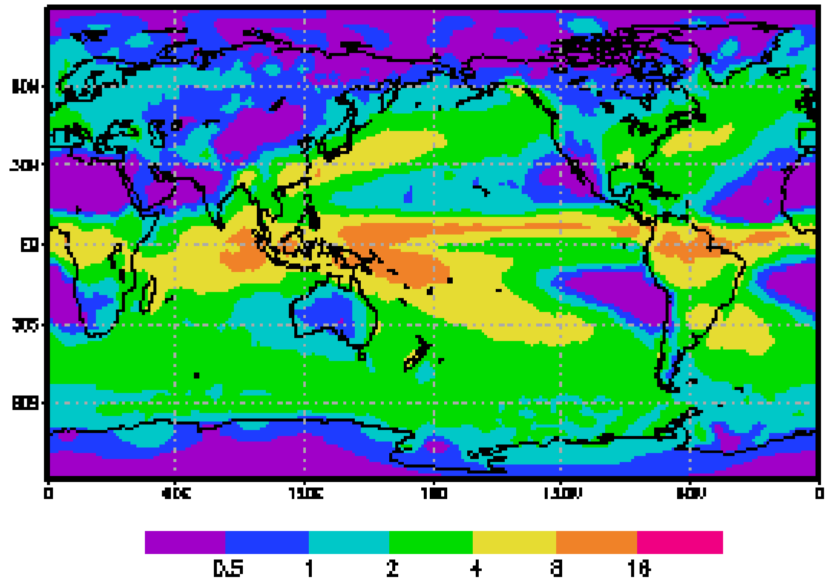
OBS JJA (CMAP)



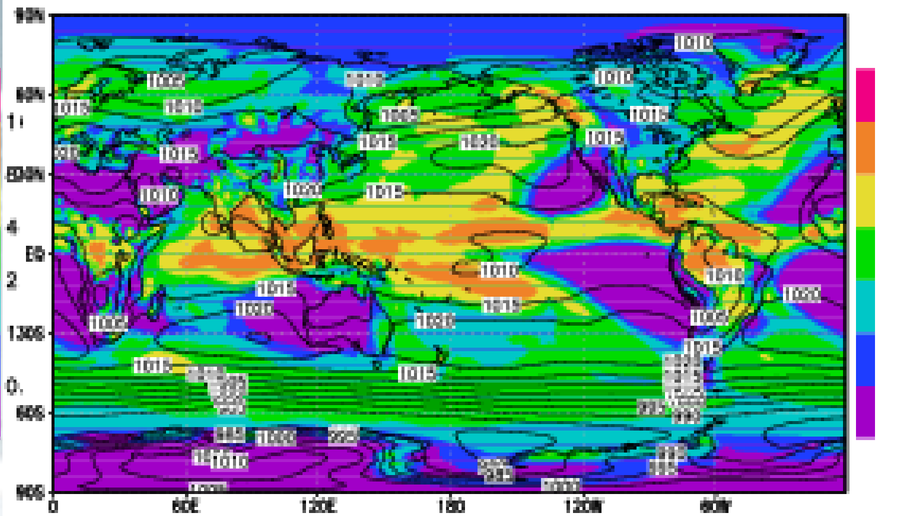
MAM



obs MAM



SON



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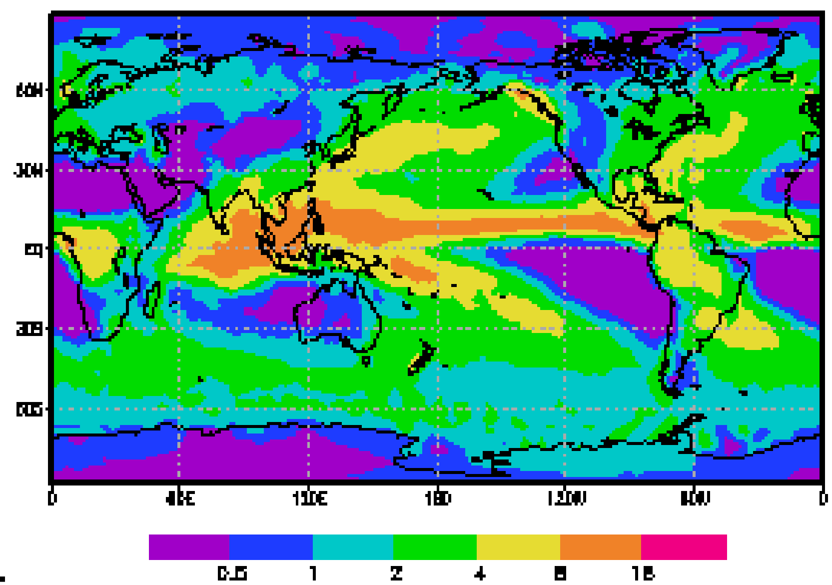

Comparisons of VCAM and CCAM

VCAM advantages

- No Helmholtz equation needed
- Includes full dhi_dx terms (no T linearization needed)
- Mass and moisture conserving
- No semi-Lagrangian resonance issues near steep mountains
- Simpler MPI ("computation on demand" not needed)

VCAM disadvantages

- Restricted to Courant number of 1, but OK since grid is very uniform
- Nonhydrostatic treatment will be more complicated than CCAM

Thank you!