MODELING GLOBAL THUNDERSTORM ELECTRICAL ACTIVITY WITH RADIAL BASIS FUNCTION-generated FD

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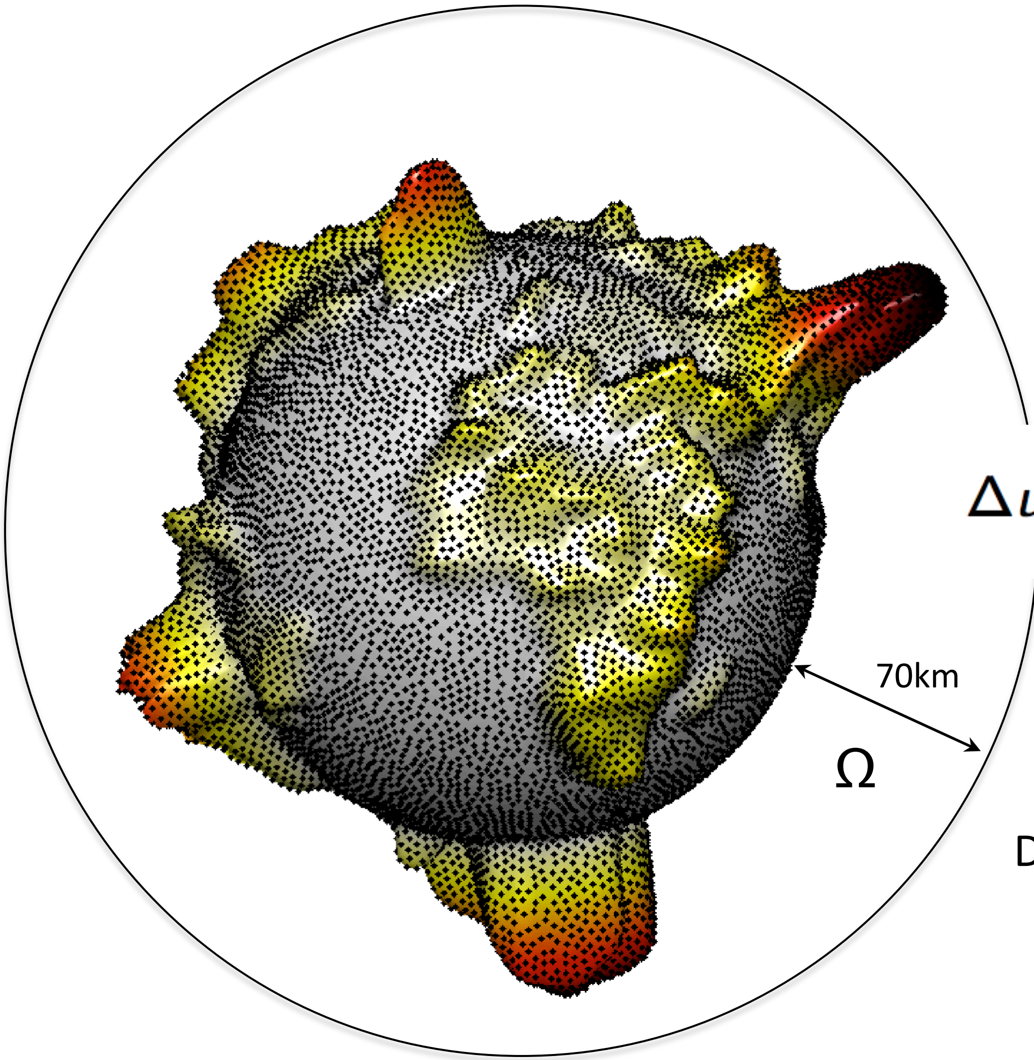


$$\nabla_{3D} \cdot \underbrace{\vec{J}(\vec{x})}_{\text{Electric Current}} = \underbrace{F(\vec{x})}_{\substack{\text{Thunderstorm sources} \\ \text{(measured data)}}} \quad \text{in } \Omega$$

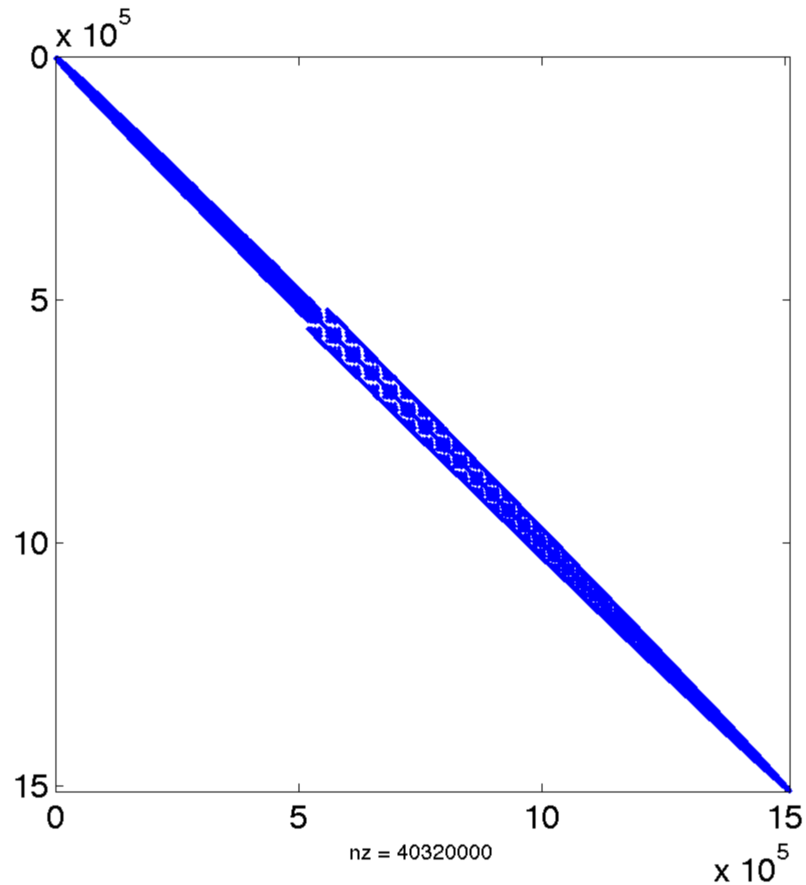
$$\Delta u(\vec{x}) + \nabla \log \sigma(\vec{x}) \cdot \nabla u(\vec{x}) = \frac{F(\vec{x})}{\sigma(\vec{x})}$$

u - electrostatic potential

$\sigma(\vec{x}) > 0$, atmospheric conductivity

Dirichlet BCs at Earth's surface and at 70km

Sparsity pattern of 3D differential operator to invert



- Four order accuracy.
- ≈ 1.5 million of nodes in the domain (Δr≈1km, Δθ≈Δλ≈140 km)
- GMRES or BI-CGSTAB w/ ILU preconditioner
- ≈ 25min on MacBook Pro Intel Core i7 2.7GHz (includes pre-processing)

Modeling Global Thunderstorm Electrical Activity with RBF-FD

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Formulation

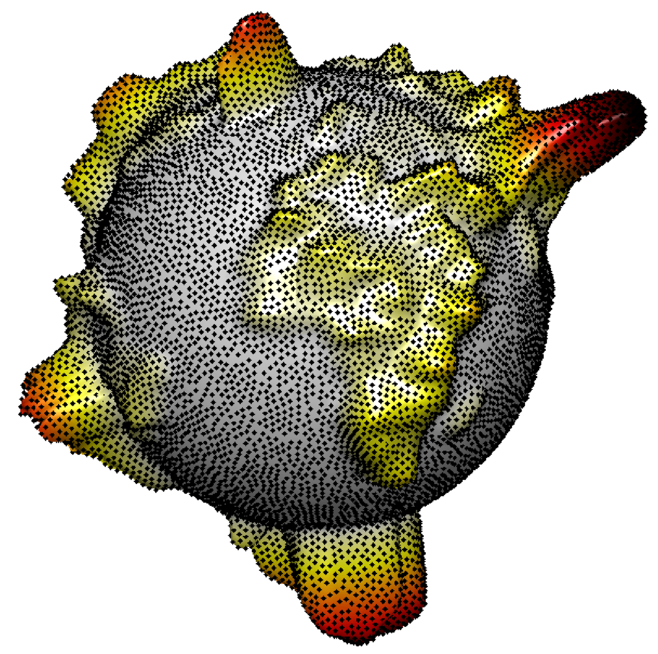
The GEC is modeled by the equation

$$\nabla \cdot [\sigma(\lambda,\theta,r)\,\nabla u(\lambda,\theta,r)] = f(\lambda,\theta,r), \qquad \text{in } \Omega \tag{1}$$

where u is the electric potential, σ the atmopheric conductivity and f the current source. Since $\sigma > 0$, it can be written as

$$\Delta u + (\nabla \log \sigma)(\nabla u) = \frac{f}{\sigma}, \tag{2}$$

leading to a better numerical conditioning. The domain Ω extends from the Earth's surface to the base of the ionosphere at 90 km altitude. At the bottom and top of the domain, Dirichlet BCs are enforced. An exponential change of variable is used to stretch the topography of the Earth and increase the resolution over the ground.



Since we are using RBF-FD methods, the nodes can be easily scattered over the topography.

RBF-FD methods

RBF-FD methods are well-known for its hability to solve PDEs in irregular domains. As in FD, the differential operator is approximated as

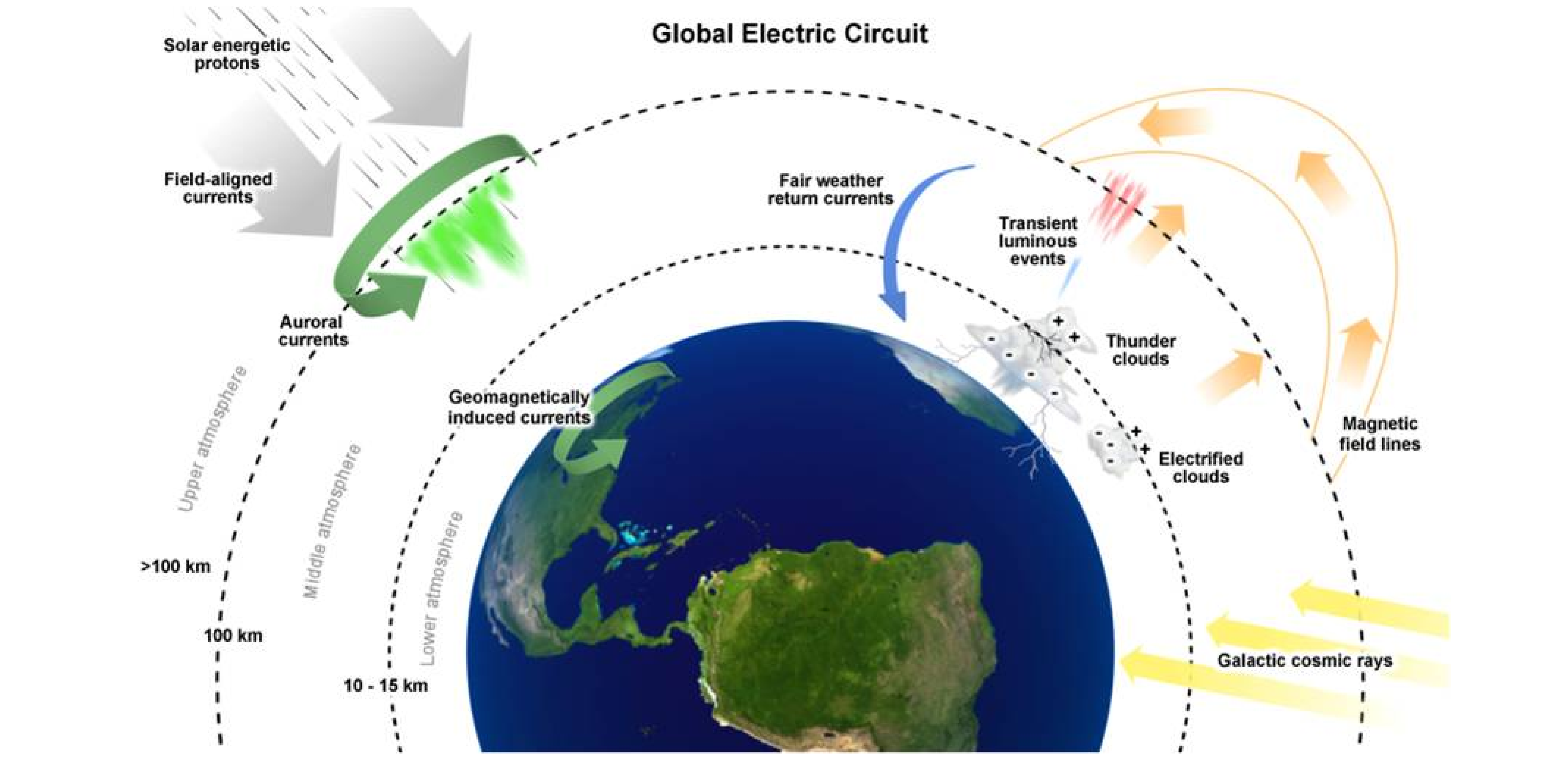
$$\mathcal{L}u|_{x_k} \approx \sum_{i=1}^{n} w_i u_i, \tag{3}$$

where the stencil is formed by the n-th closest nodes, but the weights w_i are computed imposing (3) to be exact for RBFs, $\phi_{ij} = \phi\left(\|\underline{x}_i - \underline{x}_j\|_2\right)$. It leads to the system of equations

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} \mathcal{L}\phi\left(\|\underline{x} - \underline{x}_1\|_2\right)|_{x_k} \\ \mathcal{L}\phi\left(\|\underline{x} - \underline{x}_2\|_2\right)|_{x_k} \\ \vdots \\ \mathcal{L}\phi\left(\|\underline{x} - \underline{x}_n\|_2\right)|_{x_k} \end{bmatrix}$$

which is guaranteed to be non-singular, no matter how the nodes (assumed to be distinct) are scattered in any number of dimensions. The corresponding weights result into a highly sparse differentiation matrix, which may be solved using an iterative solver such as BI-CGSTAB or GMRES.

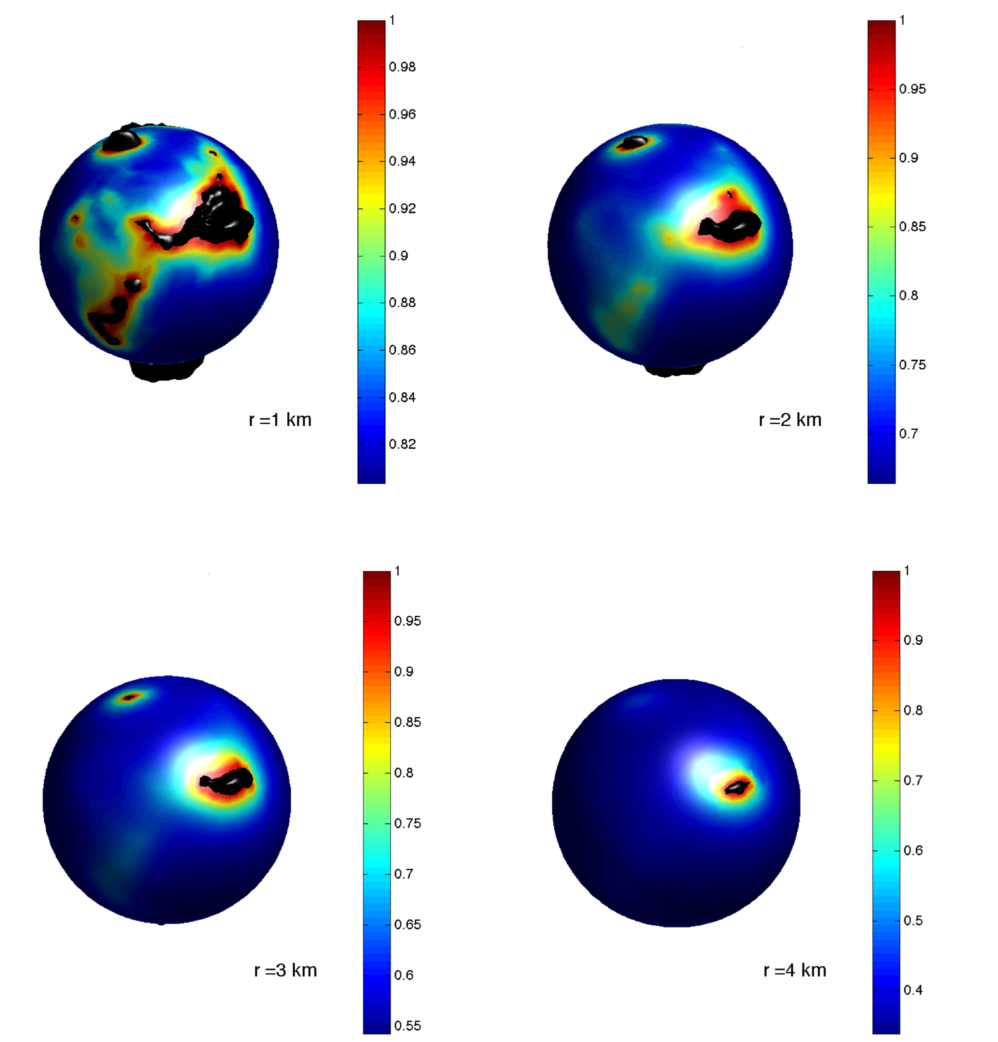
Global Electric Circuit



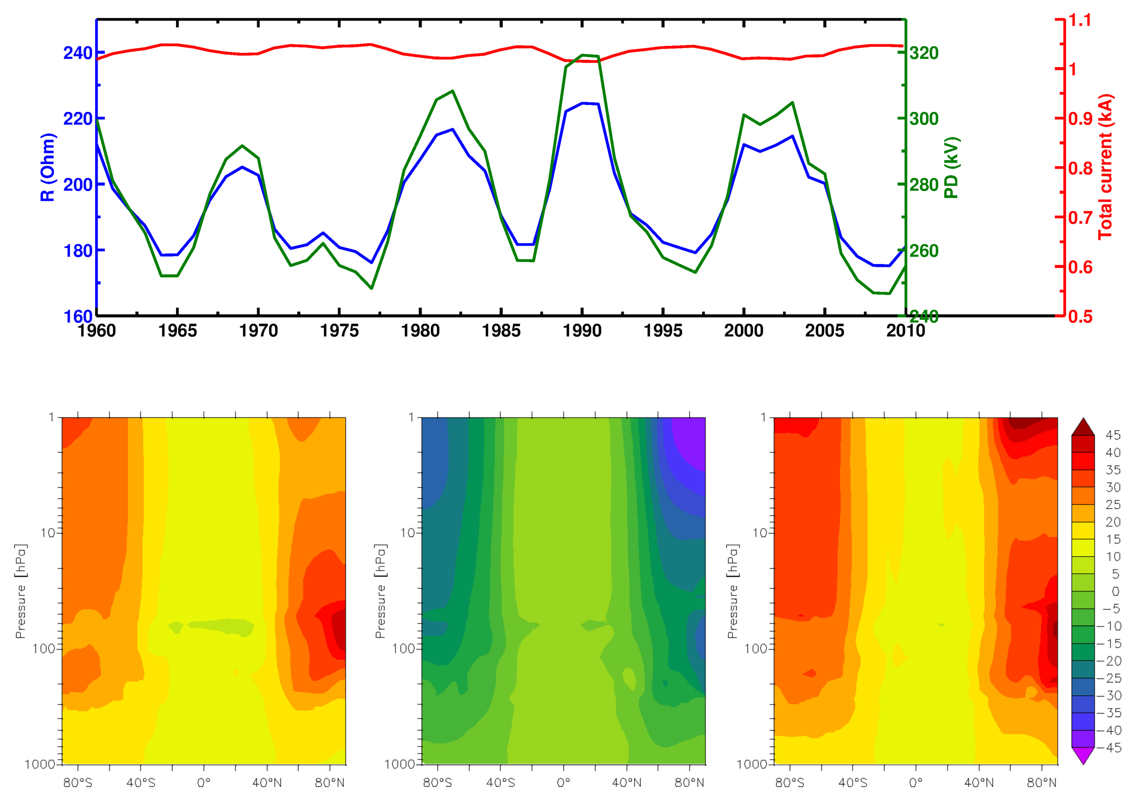
Electrical linkages within the atmosphere are often discussed in terms of a "Global Electric Circuit" (GEC). It extends from the Earth's surface to the base of the ionosphere defined to be 90 km altitude. The basic idea is that thunderclouds and other highly electrified clouds produce an upward current ($1000 - 2000$ A) that maintains the ionosphere at a quasi-static potential of order 240 ± 40 kV with respect to the ground. A downward return current density is distributed over the rest of the globe in so-called "fair-weather regions". The downward, fair-weather, current density varies considerably with space and time due to variations in atmospheric conductivity, which in turn is dependent upon atmospheric chemistry, aerosols, solar radiation, fluxes of GCRs, radiation belt relativistic electrons and energetic solar protons. The diverse physical phenomenas than play a major role in supplying current to the global circuit makes the electric system a frontier research problem for earth system studies.

Charged Earth

As test problem, the Laplace's equation over a ficticious charged Earth is solved. The figure below shows the electric potential at different altitudes from the Earth's surface. The Earth's topography is in black.



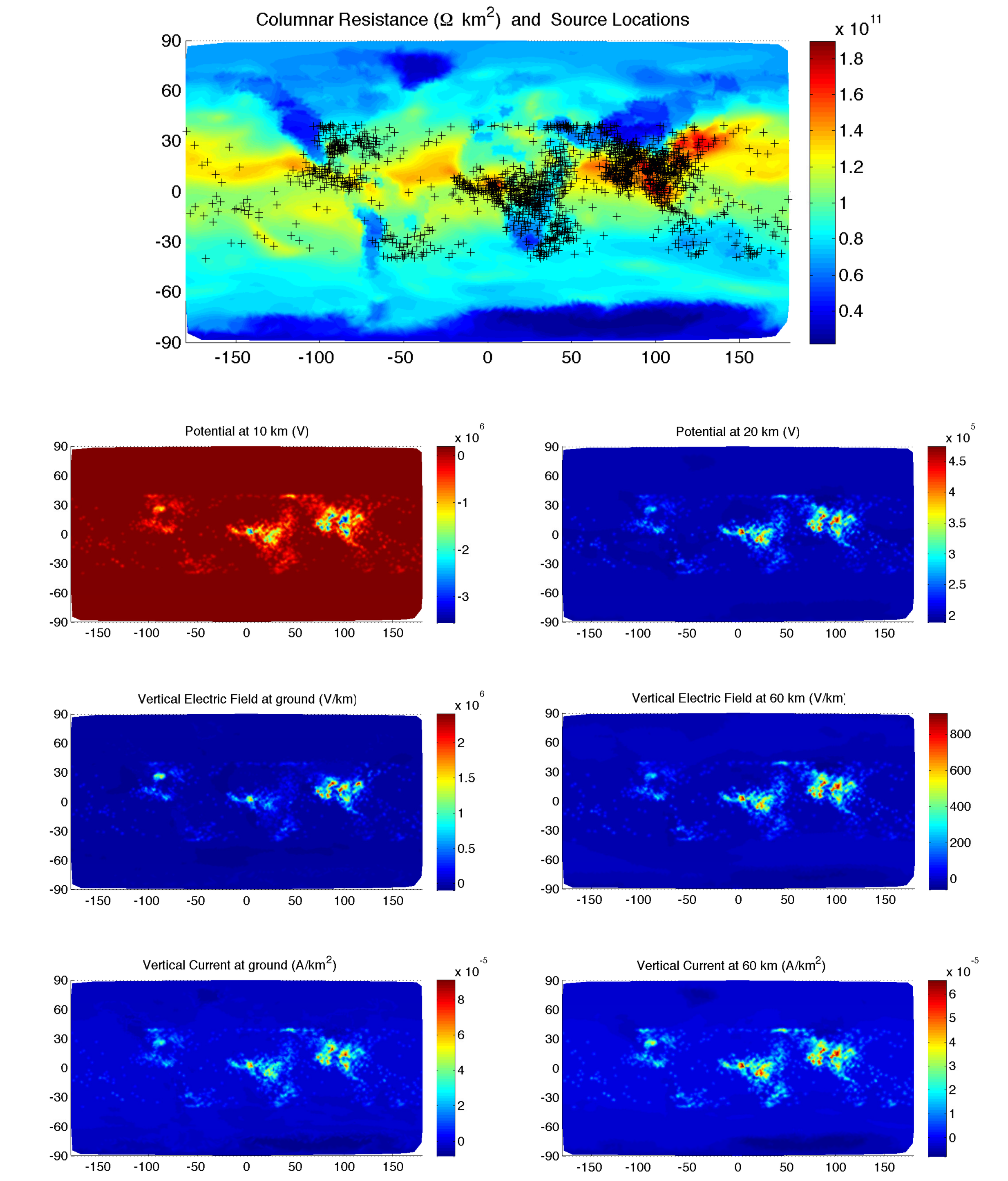
Application I: Solar variability



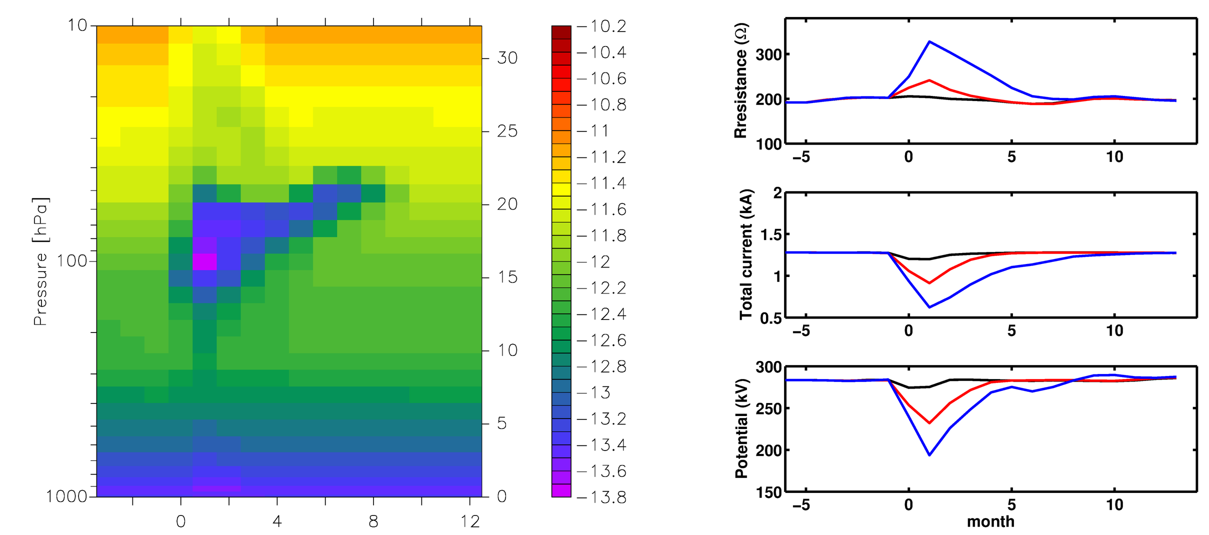
Solar variability: The GEC solver was used to calculate the total global conductivity (blue), global mean PD (green) and generator current (red) from the changes in conductivity due to solar variability as a function of years (top figure). Conductivity for years 1997 (left), 2003 (middle), 2008 (right) is shown in the bottom figure.

Real Data Simulations

The following figures show the potential, electric field and current density computed at different distances from the ground using the column resistance and source locations of the graph below.



Application II: Volcanic Eruption



Volcanic eruptions: The GEC solver was used to calculate the effects of a volcanic eruption on the electrostatic potential of the atmosphere. The left figure shows the time evolution in months of the conductivity due to a volcanic eruption at the equator. The right figure depicts the corresponding resistance (top), total current (middle) and ionospheric potential (bottom) as a function of time.

Acknowledments

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