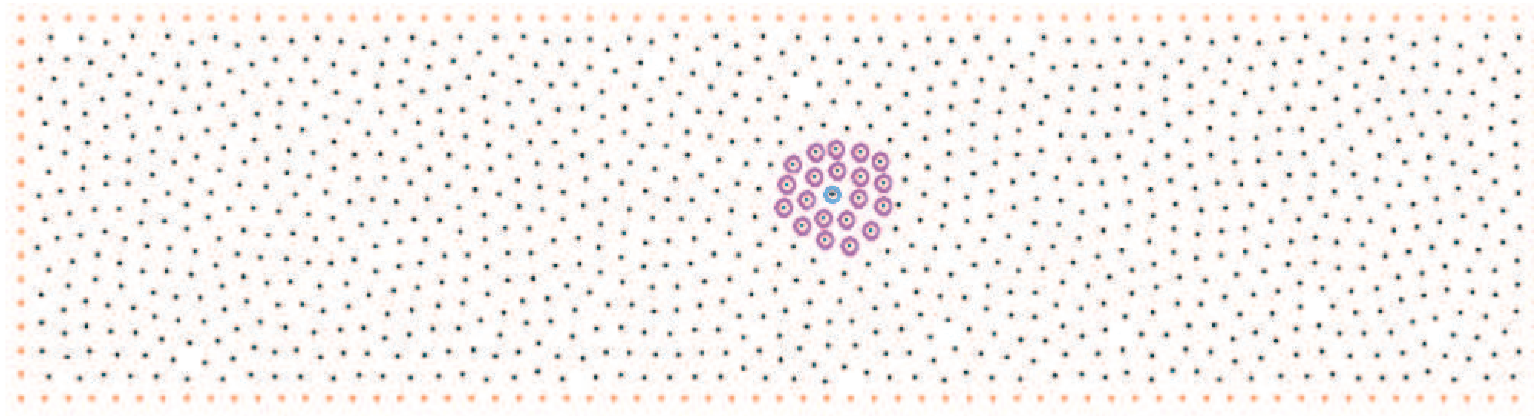
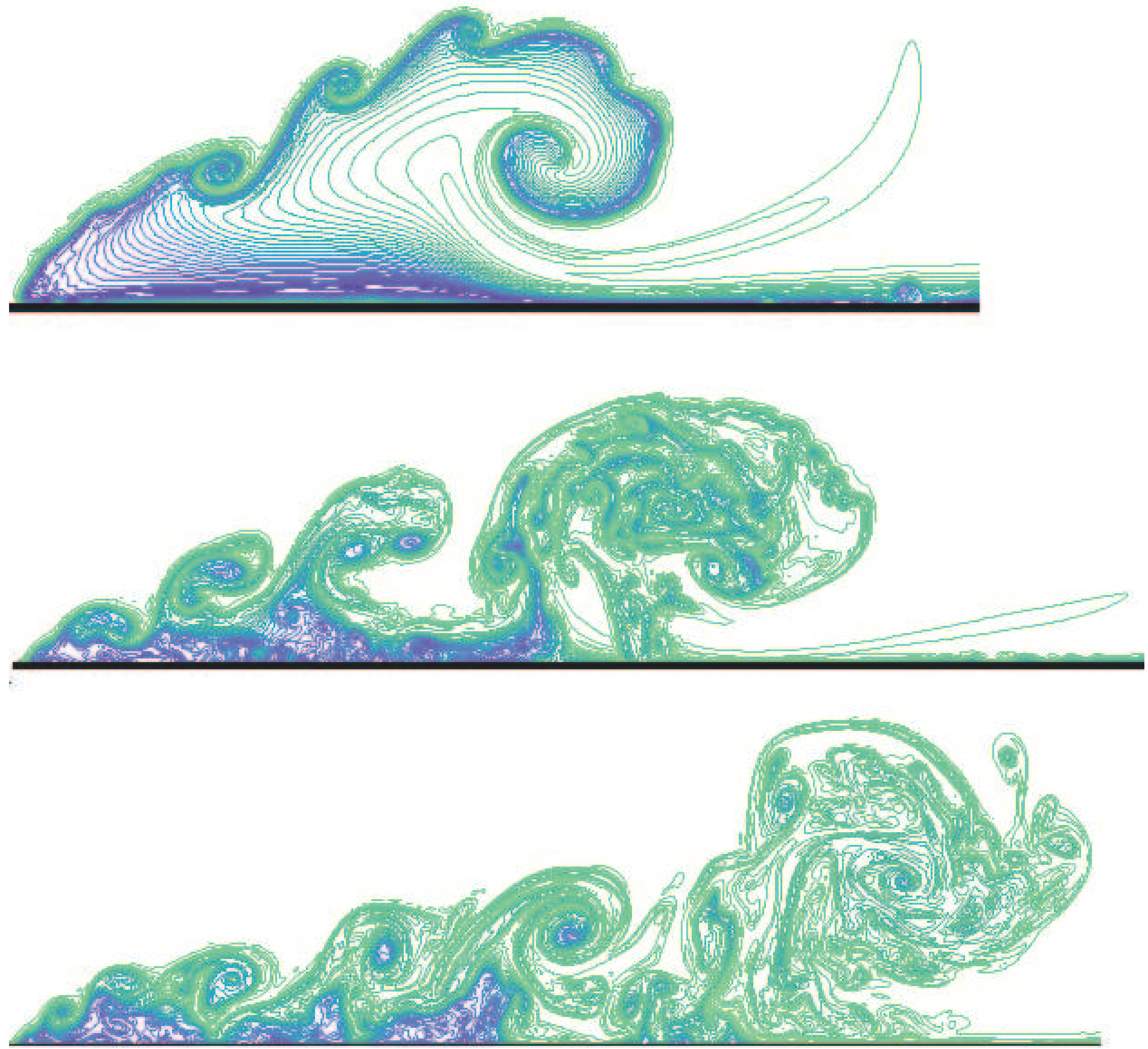
A **Local Polyharmonic Spline RBF Method** for Nonhydrostatic Atmospheric Modeling

Natasha Flyer (NCAR), Gregory Barnett (CU), Lou Wicker (NOAA-NSSL)



Cartersian: FD6 or RBF

Scattered: RBF



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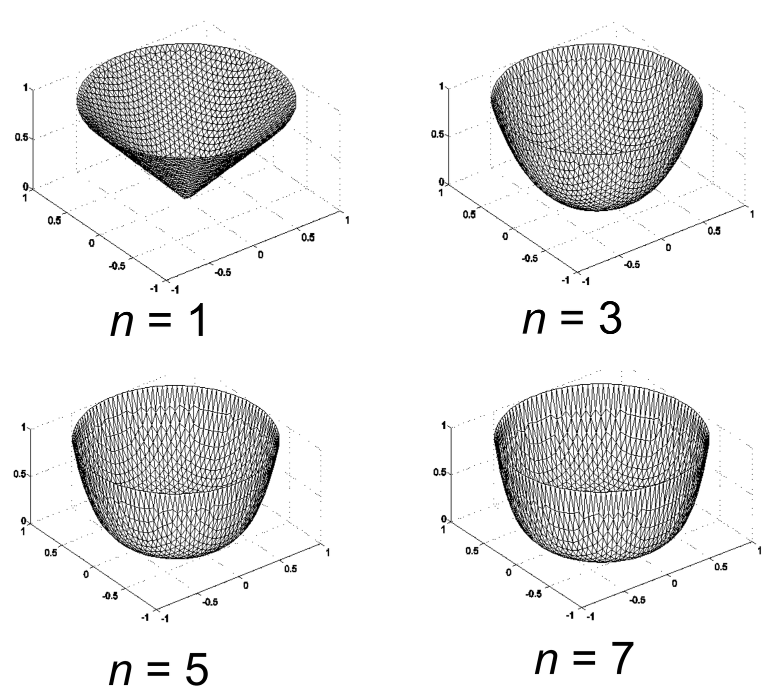
Simplest RBF

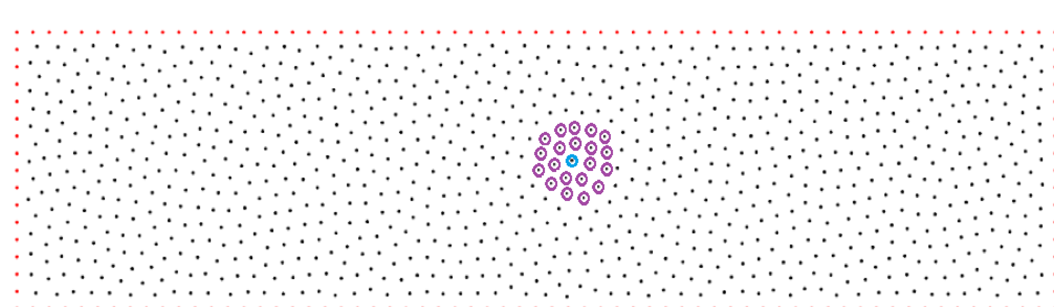
The simplest RBF is the function r, i.e., the l_2 norm or Euclidean distance

Spatial Dimension d	Definition of r
1	$r(x) = \sqrt{x^2} = \lvert x \rvert$
2	$r(x,y) = \sqrt{x^2+y^2}$
3	$r(x,y,z) = \sqrt{x^2+y^2+z^2}$
⋮	⋮

Polyharmonic Spline (PHS) RBF: r^n



Scattered node layout: 400m resolution



Total number of nodes N = 1107

Purple nodes show a local RBF stencil of size n. PHS-RBF, with polynomials, are centered at each of these nodes $\{x_j\}$, $j = 1,\ldots,n$, to calculate differentiation weights for L at the blue node x_c.

An Ex.: $r^3 = \|\mathbf{x}-\mathbf{x}_j\|_2^3$ and up to linear polynomials

$$\begin{bmatrix} \|\mathbf{x}_1-\mathbf{x}_1\|_2^3 & \cdots & \|\mathbf{x}_1-\mathbf{x}_n\|_2^3 & 1 & x_1 & y_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \|\mathbf{x}_n-\mathbf{x}_1\|_2^3 & \cdots & \|\mathbf{x}_n-\mathbf{x}_n\|_2^3 & 1 & x_n & y_n \\ 1 & \cdots & 1 & 0 & \cdots & 0 \\ x_i & \cdots & x_n & \vdots & \ddots & \vdots \\ y_i & \cdots & y_n & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ w_{n+1} \\ w_{n+2} \\ w_{n+3} \end{bmatrix} = \begin{bmatrix} L\|\mathbf{x}-\mathbf{x}_1\|_2^3|_{\mathbf{x}=\mathbf{x}_c} \\ \vdots \\ L(\|\mathbf{x}-\mathbf{x}_n\|_2^3|_{\mathbf{x}=\mathbf{x}_c} \\ L1|_{\mathbf{x}=\mathbf{x}_c} \\ Lx|_{\mathbf{x}=\mathbf{x}_c} \\ Ly|_{\mathbf{x}=\mathbf{x}_c} \end{bmatrix}$$

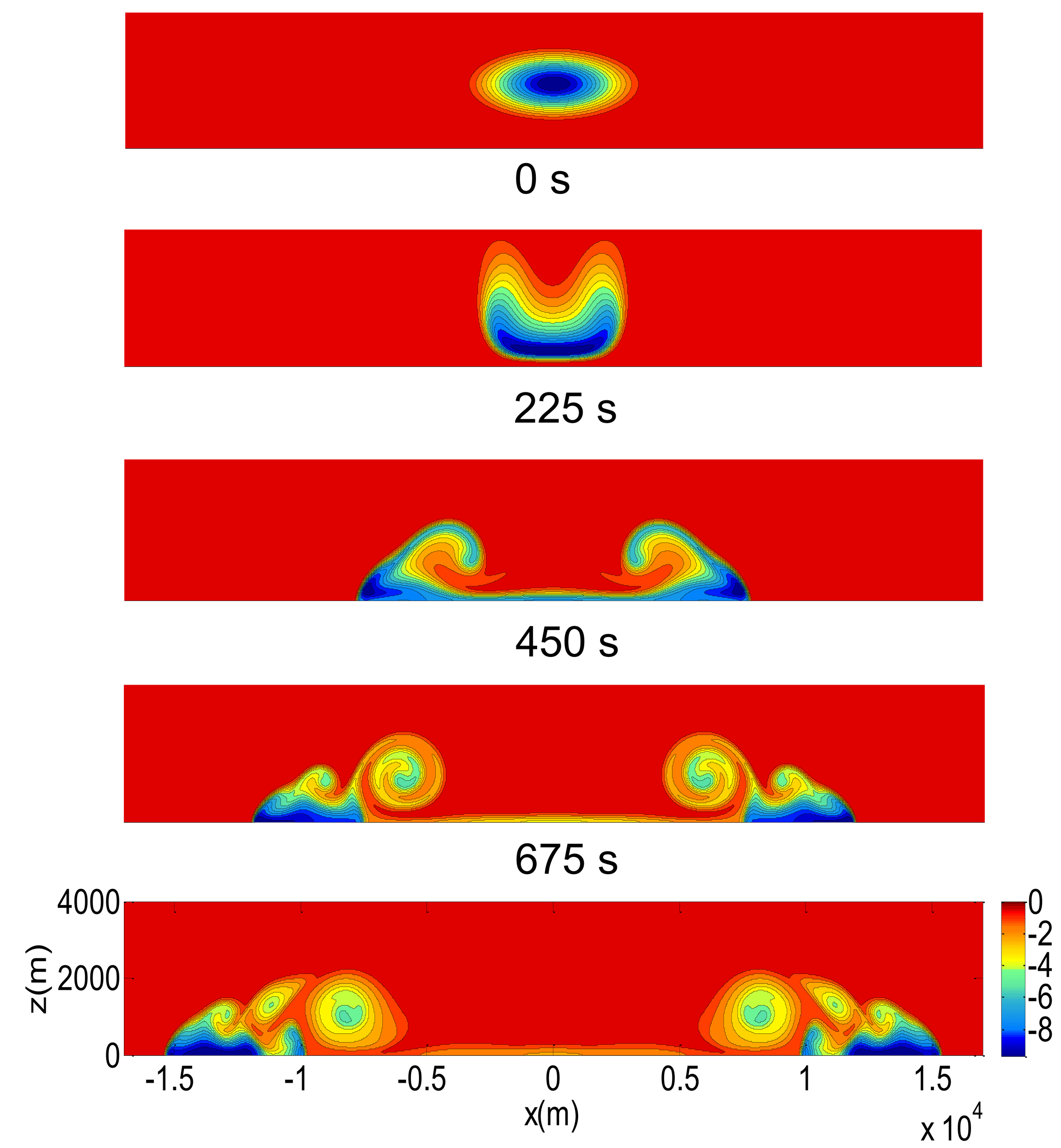
The weights are applied as in classical finite differences. This results in extremely sparse matrices.

Governing Equations

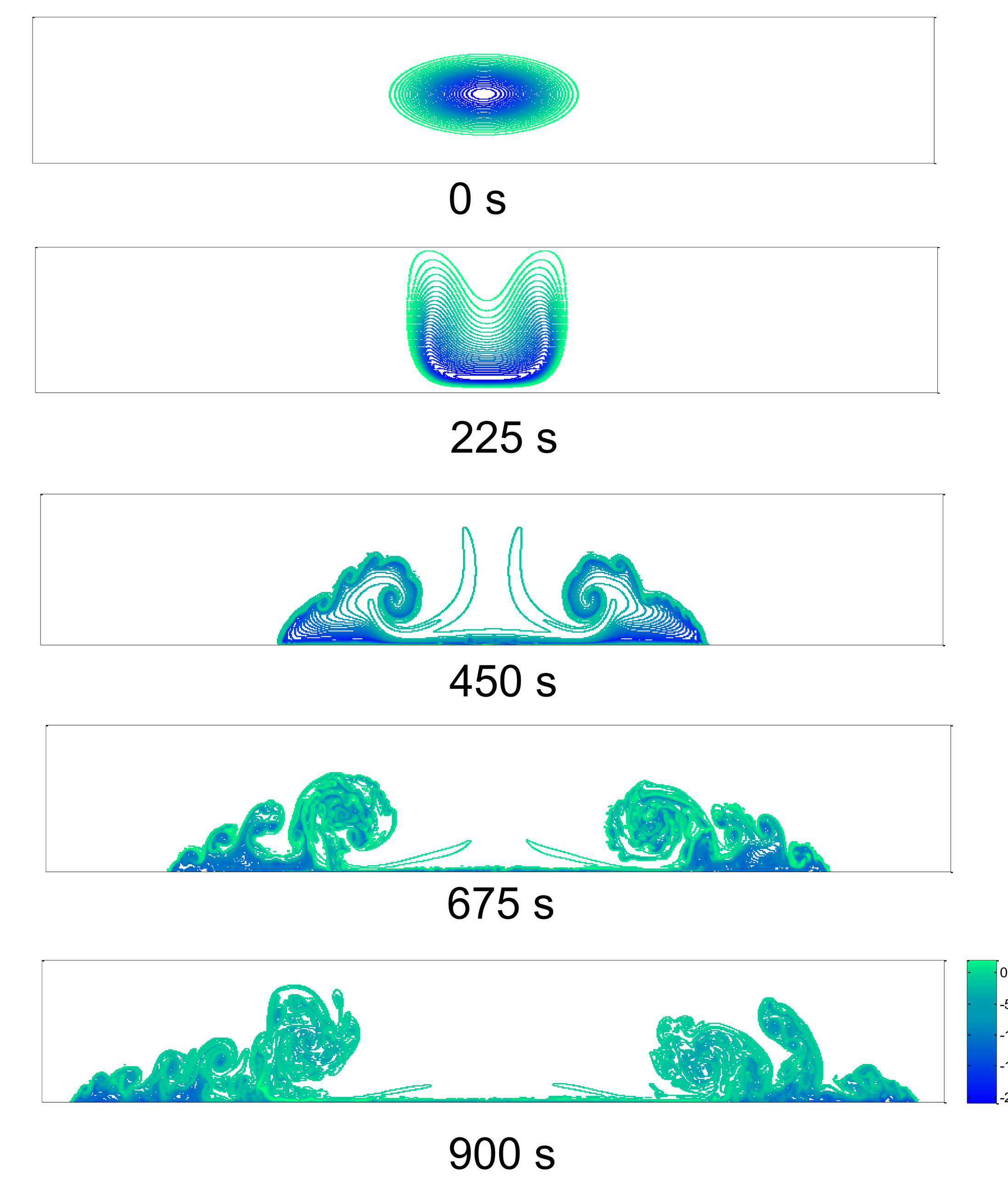
$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial z} - c_p\theta\frac{\partial \pi}{\partial x} + \mu\Delta u,$$
$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial z} - c_p\theta\frac{\partial \pi}{\partial z} - g + \mu\Delta w$$
$$\frac{\partial \theta}{\partial t} = -u\frac{\partial \theta}{\partial x} - w\frac{\partial \theta}{\partial z} + \mu\Delta\theta,$$
$$\frac{\partial \pi}{\partial t} = -u\frac{\partial \pi}{\partial x} - w\frac{\partial \pi}{\partial z} - \frac{R_d}{c_v}\pi\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right),$$

Straka Cold Density Current, r^7 and up to 4th order polynomials

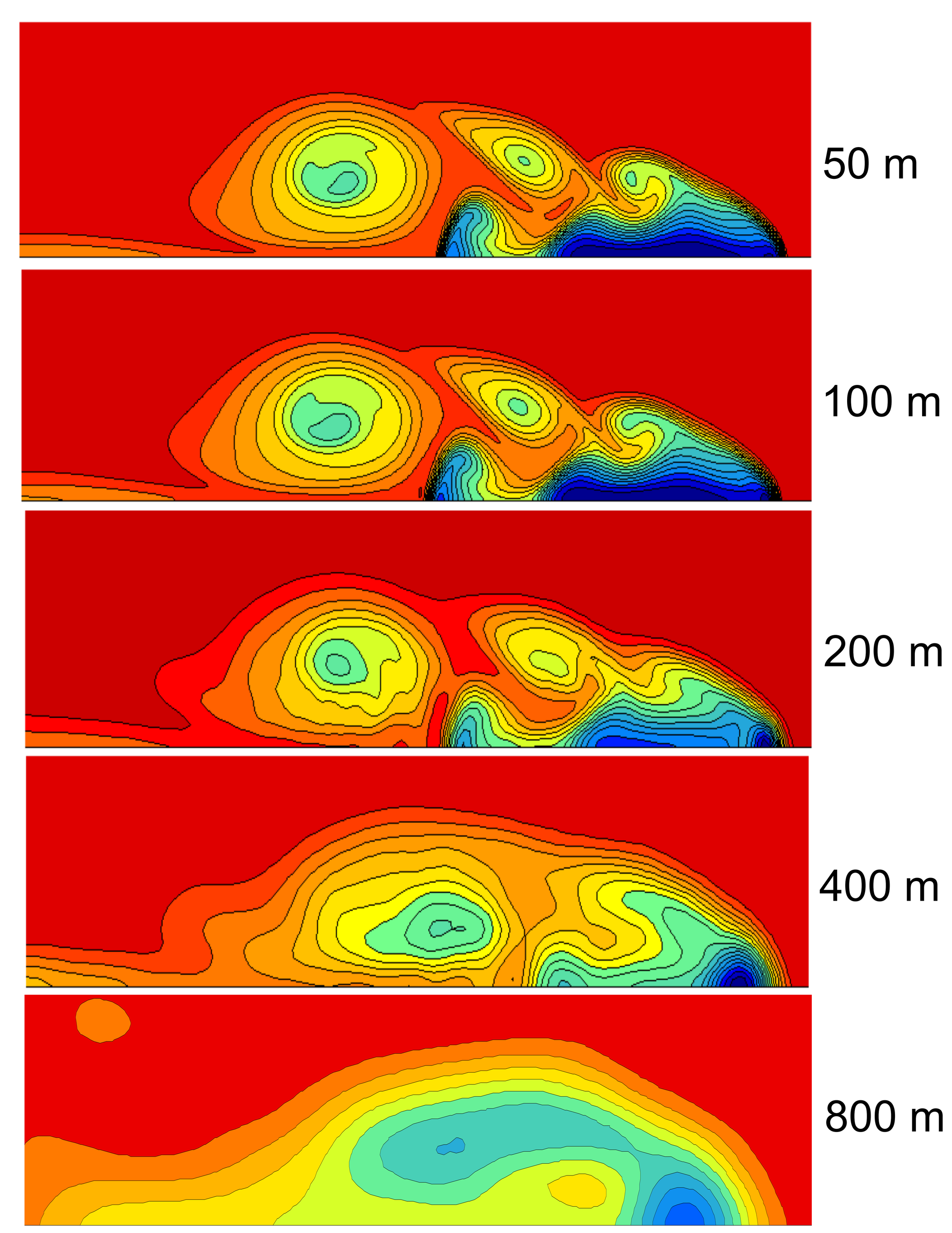
Dynamic viscosity μ = 75 m²/s
Time evolution at 50m resolution



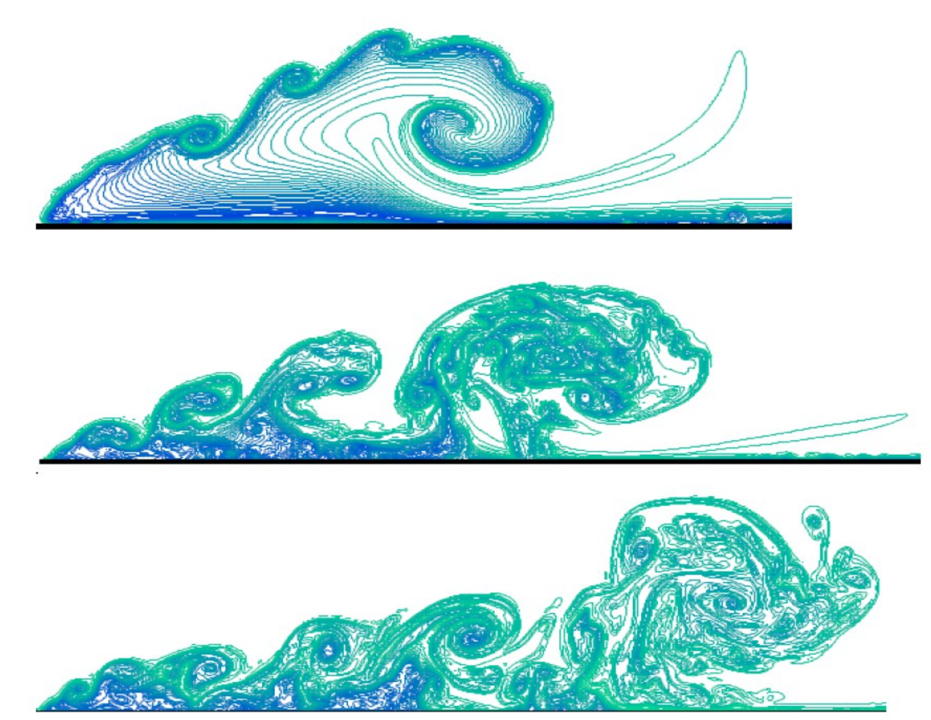
Dynamic viscosity of air $2(10)^{-5}$ m²/s
Time evolution at 25m resolution



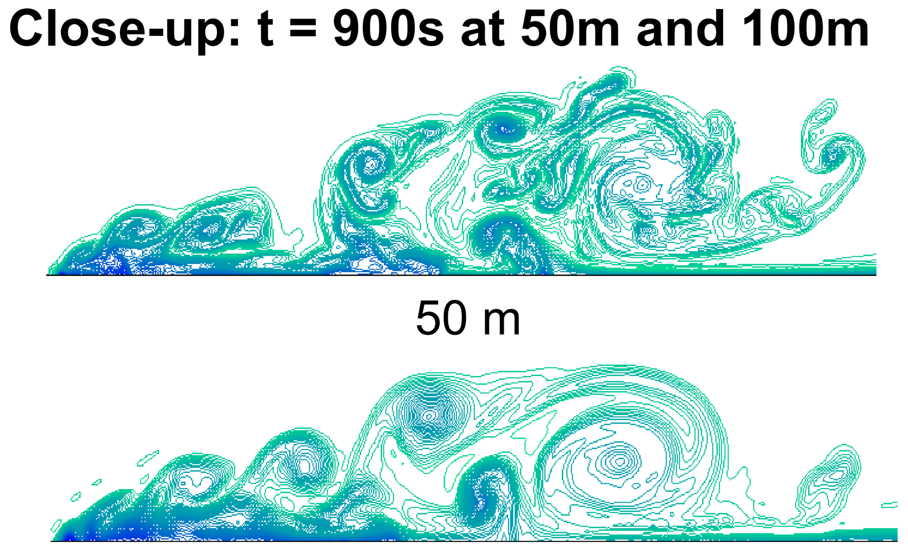
Solution at 900s: 50m to 800m resolution



Close-up: t = 450s, 675s, and 900s at 25m



Close-up: t = 900s at 50m and 100m

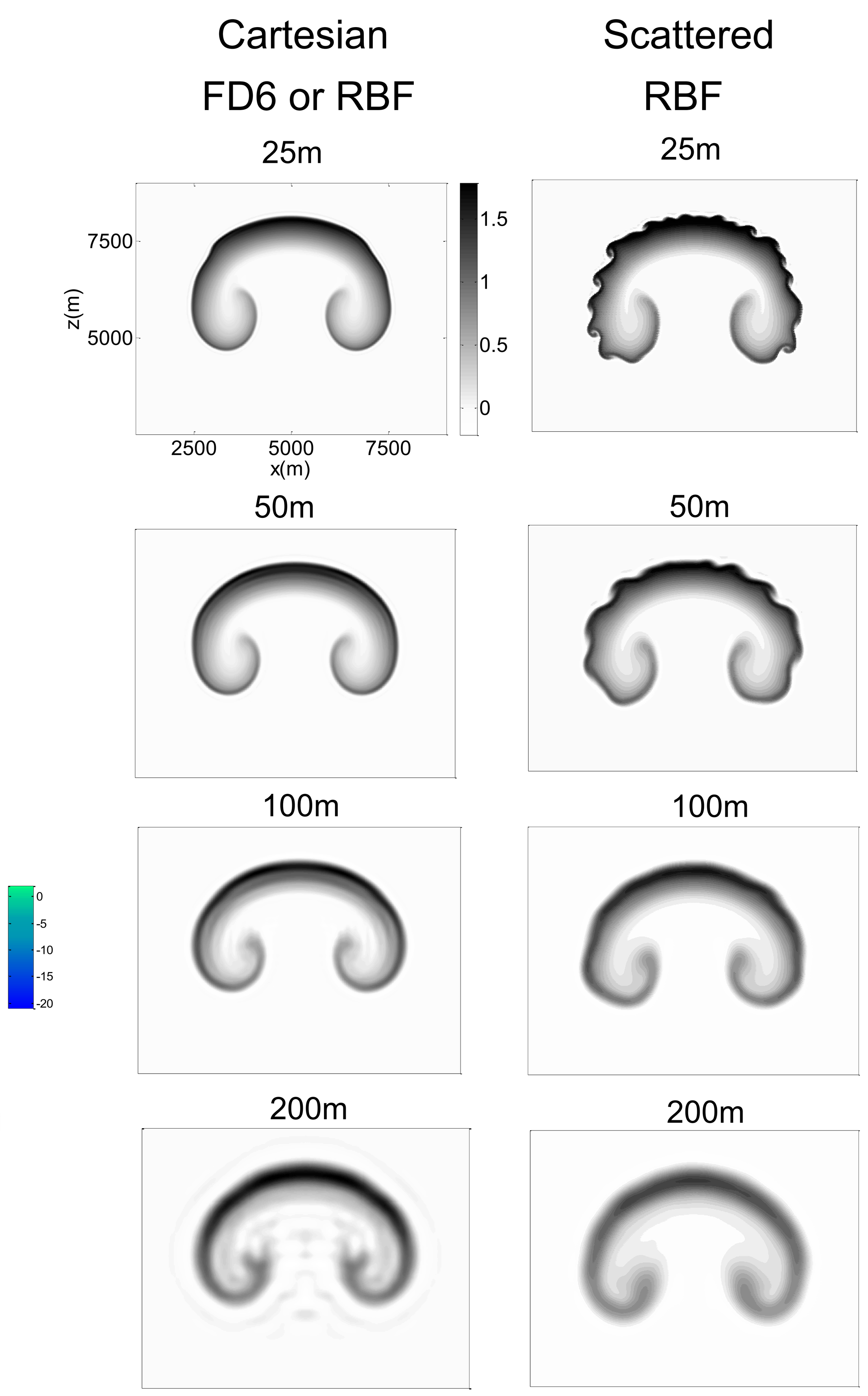


Timings on ASUS laptop with an Intel i7-4700HQ CPU @ 2.4 GHz

Resolution	Time step	Wall-clock time
800 m	5/3	1.09 sec
400 m	1	5.40 sec
200 m	5/9	48.6 sec
100 m	5/18	8.30 min
50 m	5/36	1 hr. 24 min.

Large C^0 Rising Thermal Bubble

Dynamic viscosity of air $2(10)^{-5}$



Small C^1 Rising Thermal Bubble

Dynamic viscosity of air $2(10)^{-5}$ m²/s