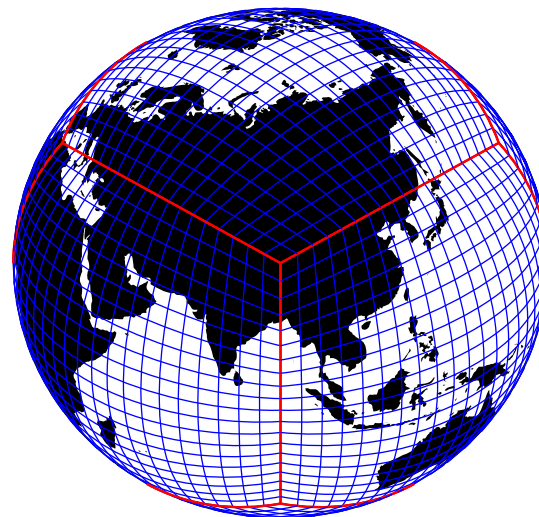
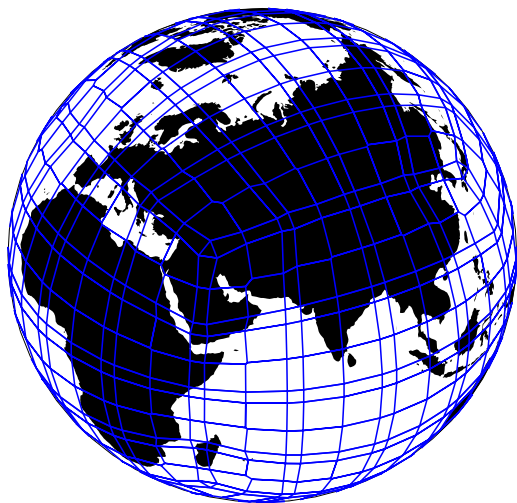


# Physics-dynamics coupling with Galerkin methods: equal-area physics grid

Peter Hjort Lauritzen<sup>§</sup>, M.A. Taylor<sup>\*</sup>,  
S. Goldhaber<sup>§</sup>, R.D. Nair<sup>§</sup>, J.T. Bacmeister<sup>§</sup>

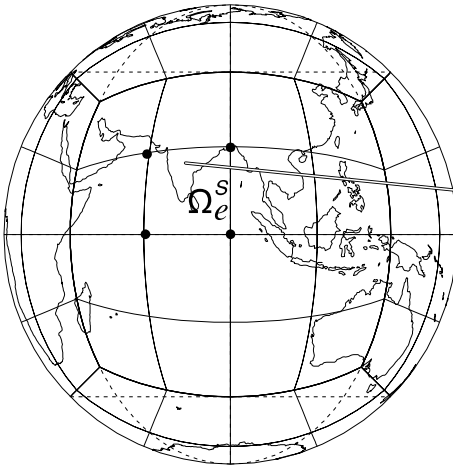


Poster presentation at PDEs on the Sphere, April 10, 2014

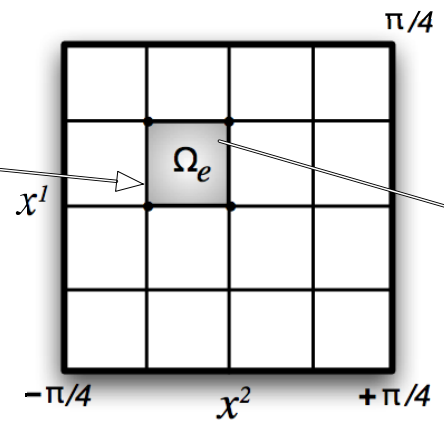
<sup>§</sup> NCAR

<sup>\*</sup> Sandia National Laboratories

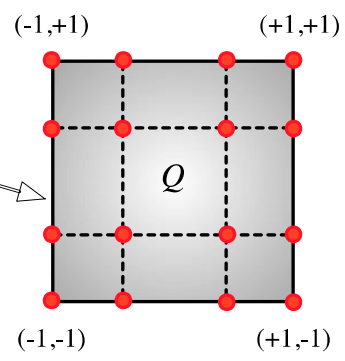
# Consider, e.g., spectral element version of HOMME (dynamical core in CAM-SE)



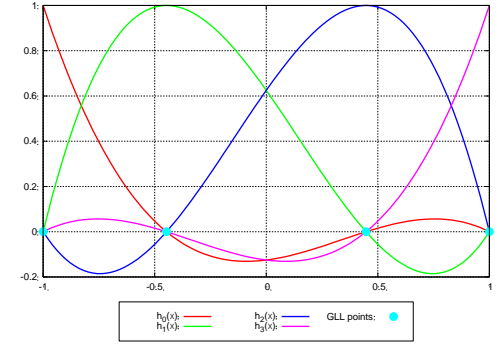
Physical Domain



Computational Domain



GLL Quadrature Grid

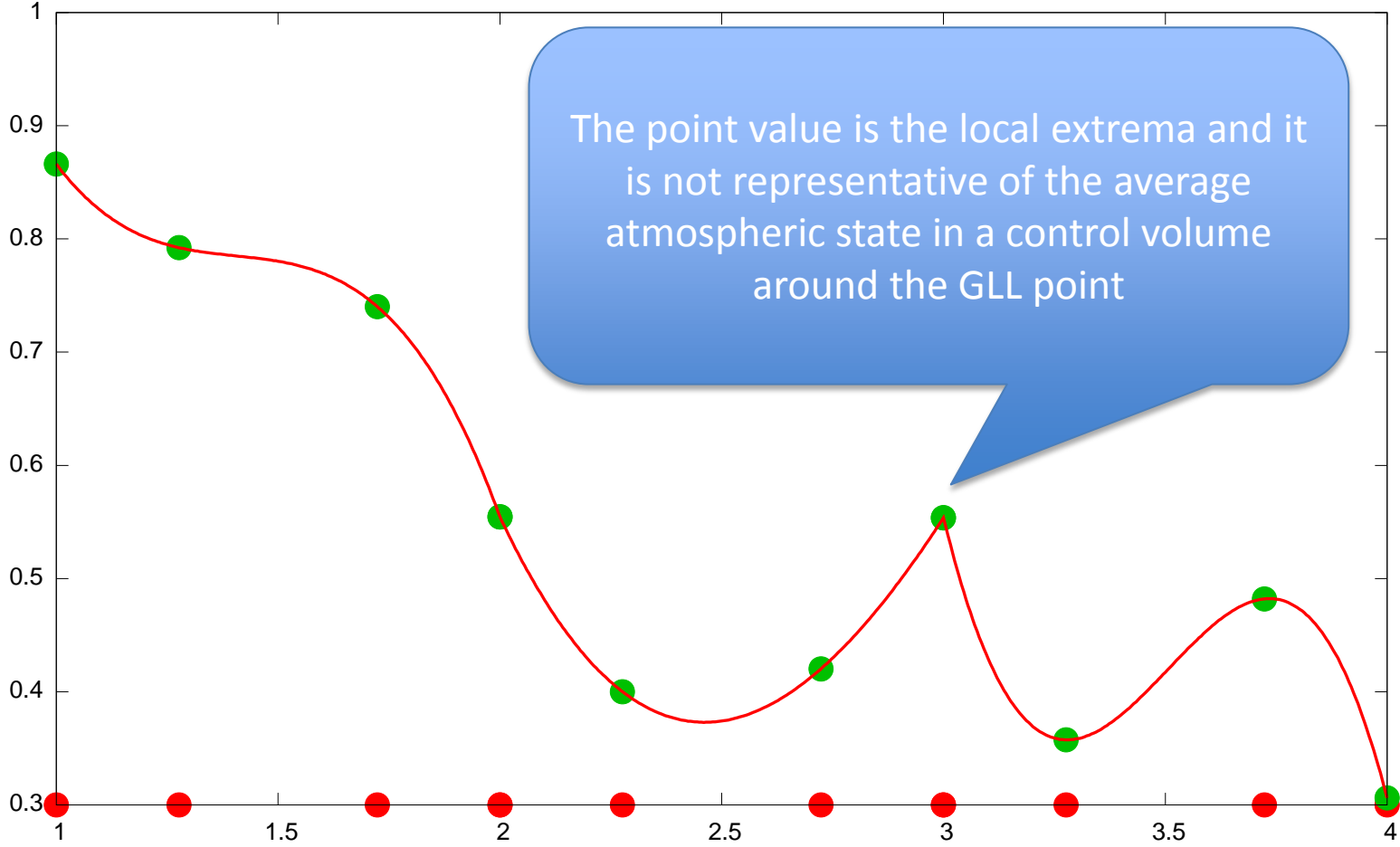


Degree 3 Lagrange polynomial basis

Conventional physics-dynamics coupling: state of the atmosphere passed to physics (a.k.a. sub-grid scale parameterizations) is based on the dynamics grid values (in this case GLL point values)

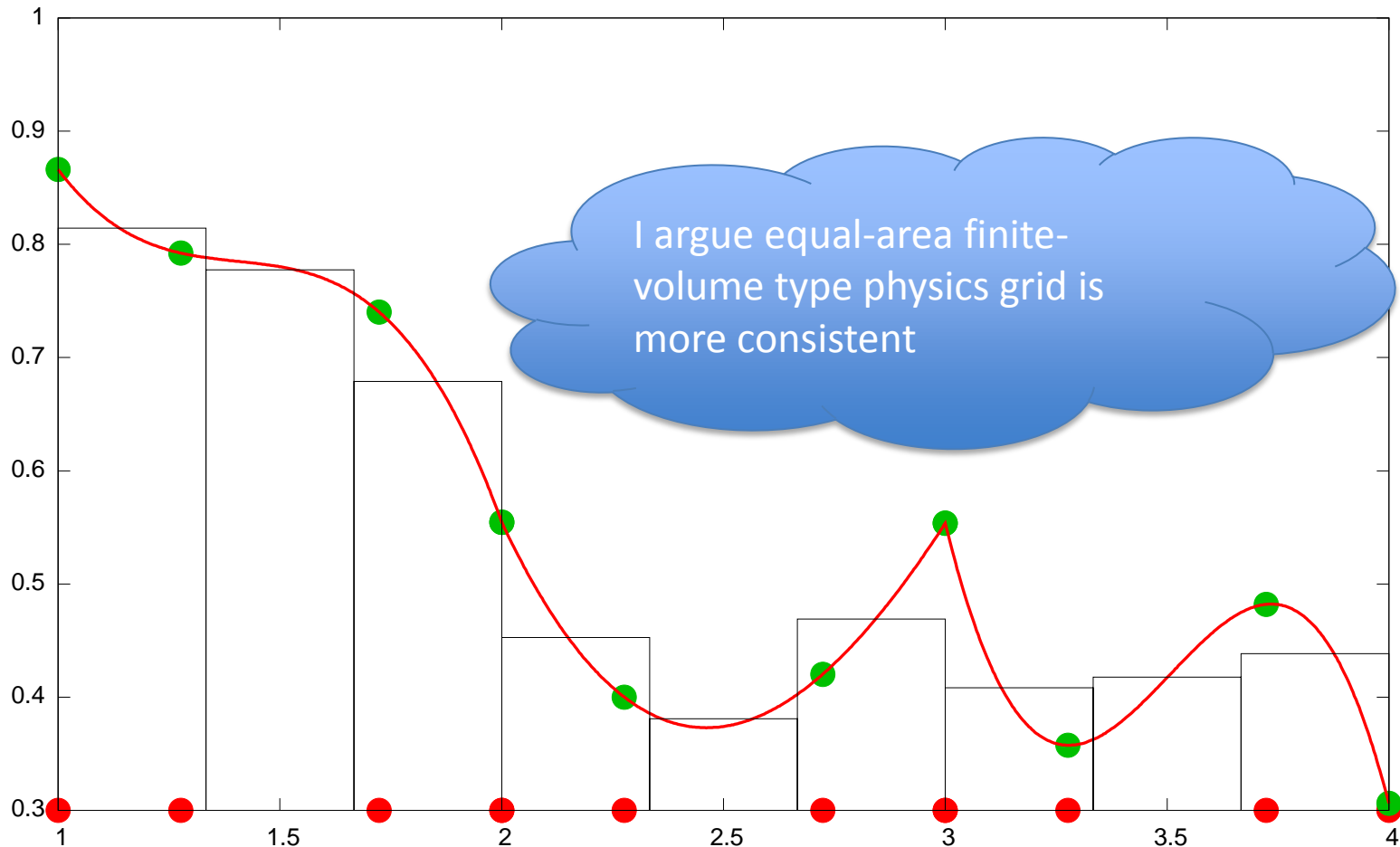
- This grid is not isotropic (gets “worse” with increasing order)
- Many parameterizations (e.g. convection) expect a grid-cell averaged state; the point value may not be representative of a grid cell mean value

**I argue that parameterizations should be given a grid cell mean value for the atmospheric state rather than a (GLL quadrature) point value**

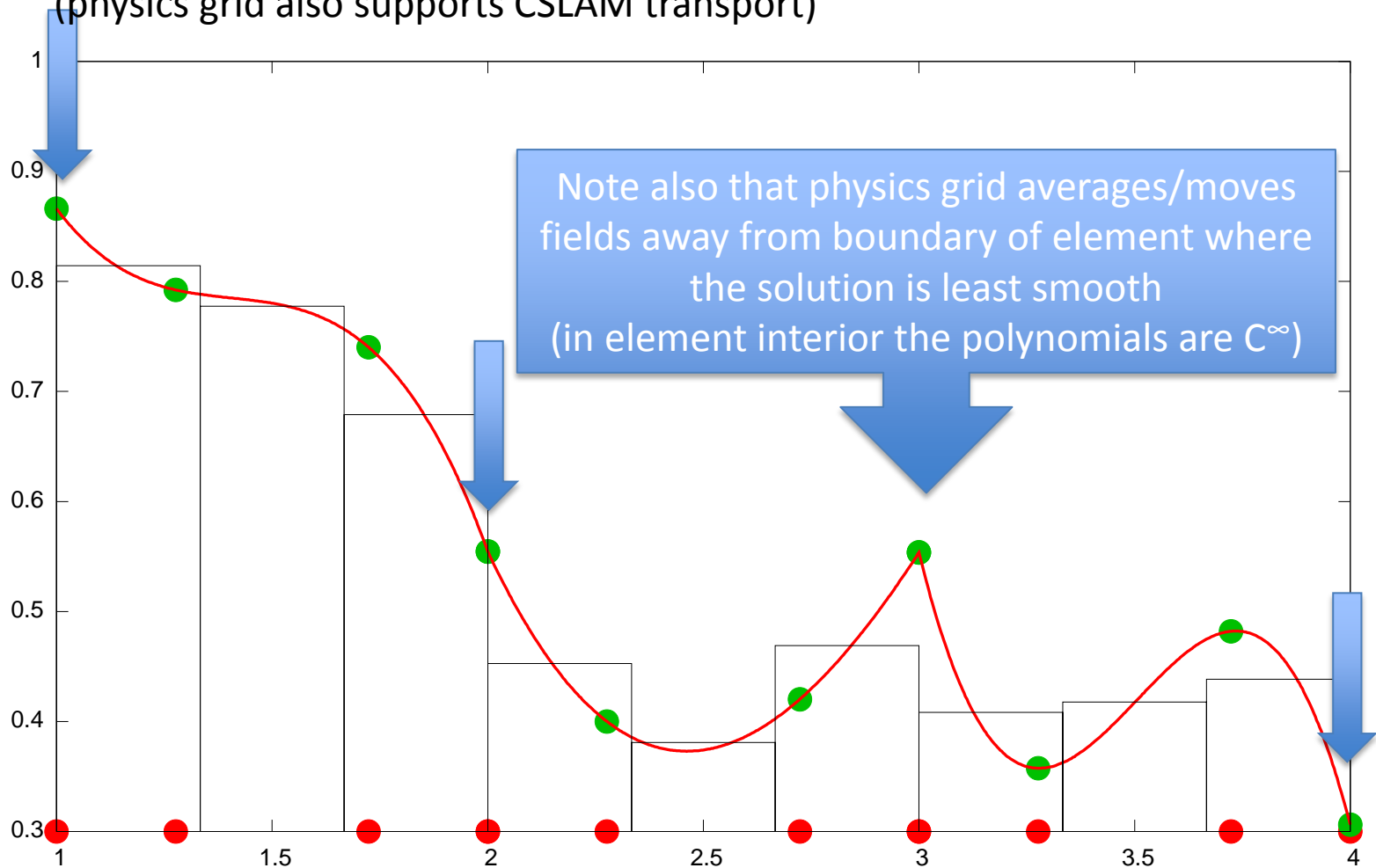


The point value is the local extrema and it is not representative of the average atmospheric state in a control volume around the GLL point

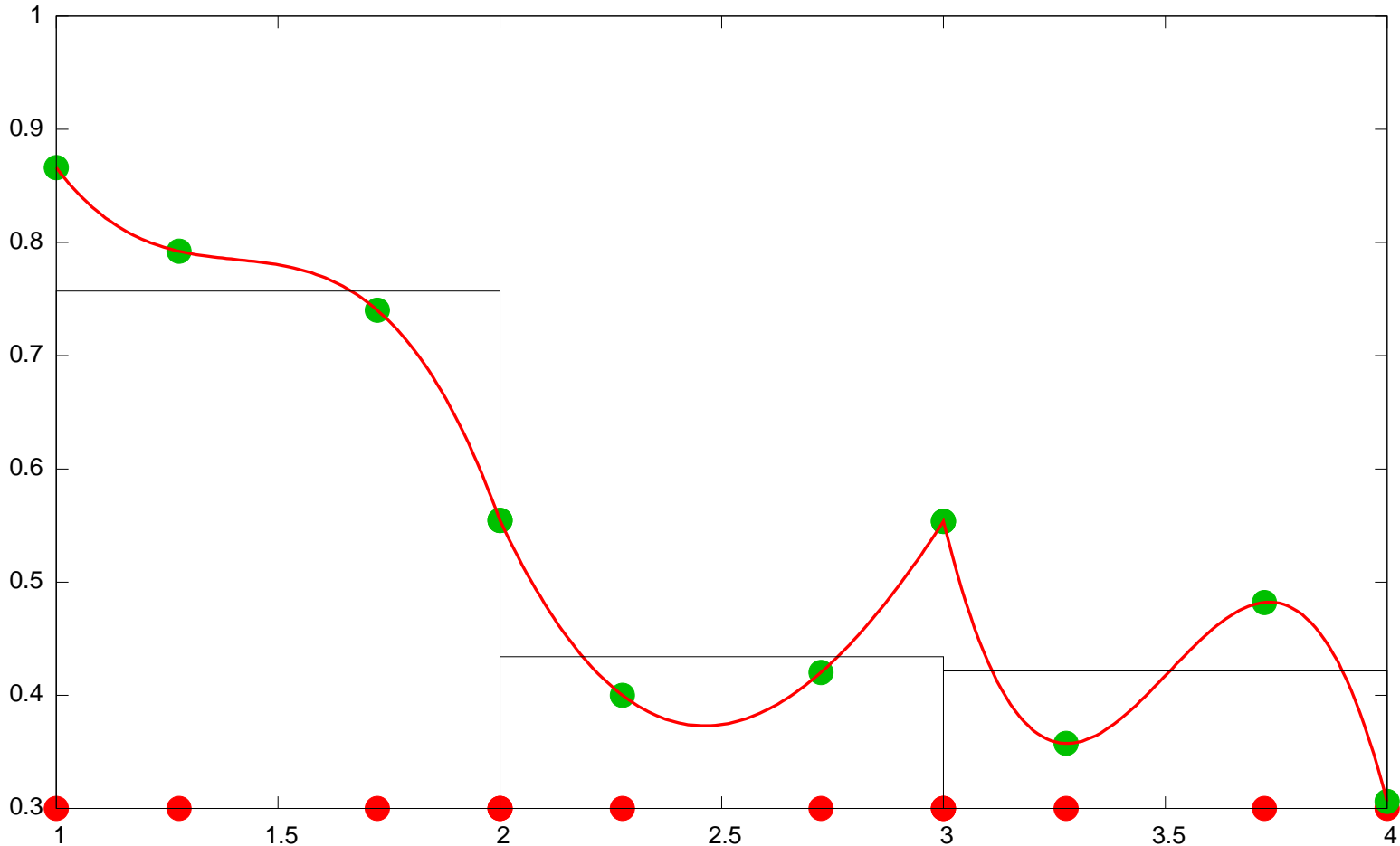
- Integrate polynomial basis over equal-area control volume  
(example: same number of degrees of freedom on both grids)  
(physics grid also supports CSLAM transport)



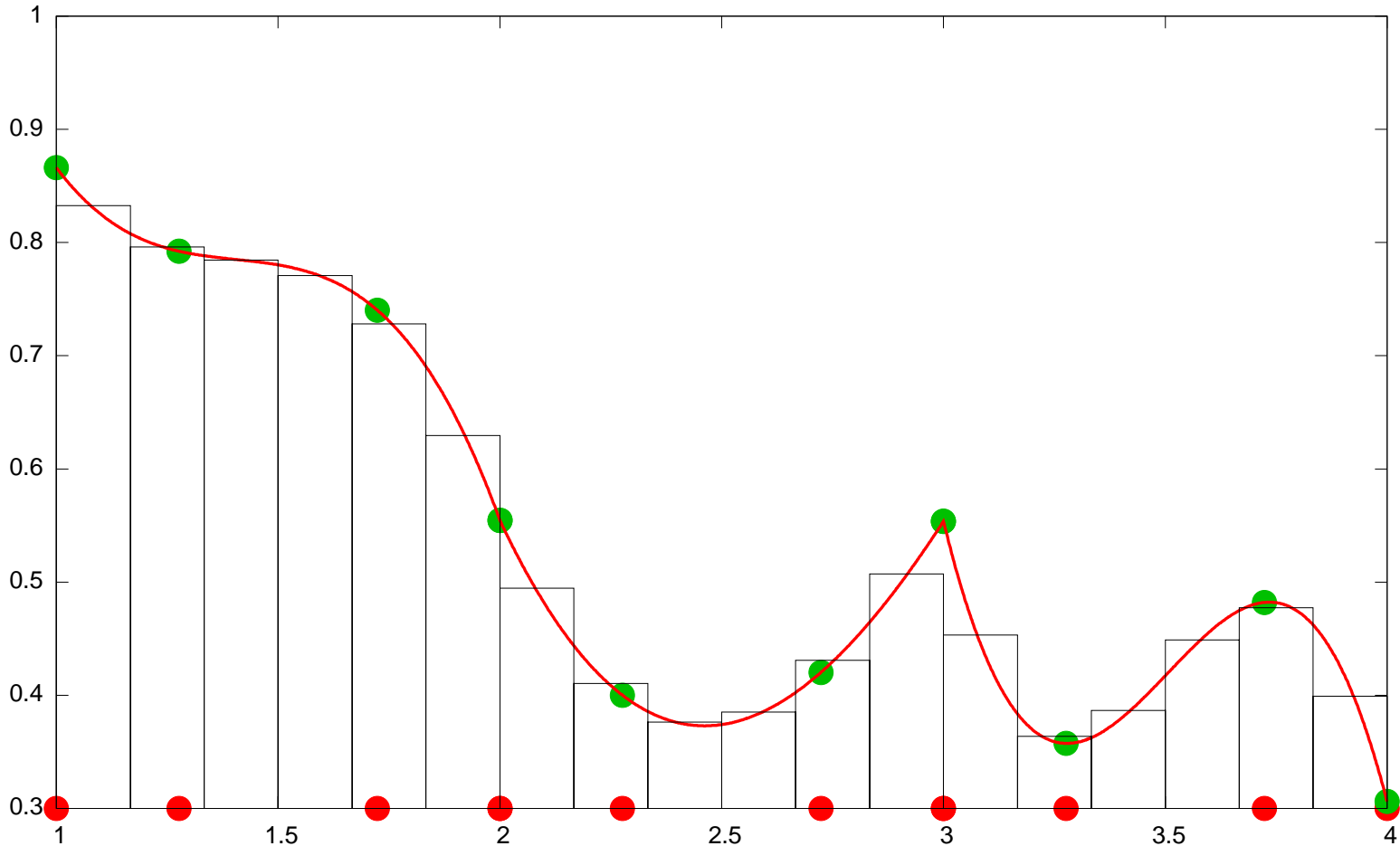
- Integrate polynomial basis over equal-area control volume (example: same number of degrees of freedom on both grids) (physics grid also supports CSLAM transport)



- Coarser resolution physics grid (Williamson, 1999)  
-> according to linear theory the dynamical core has not converged at the grid scale so one could argue that physics should not be passed non-converged scales

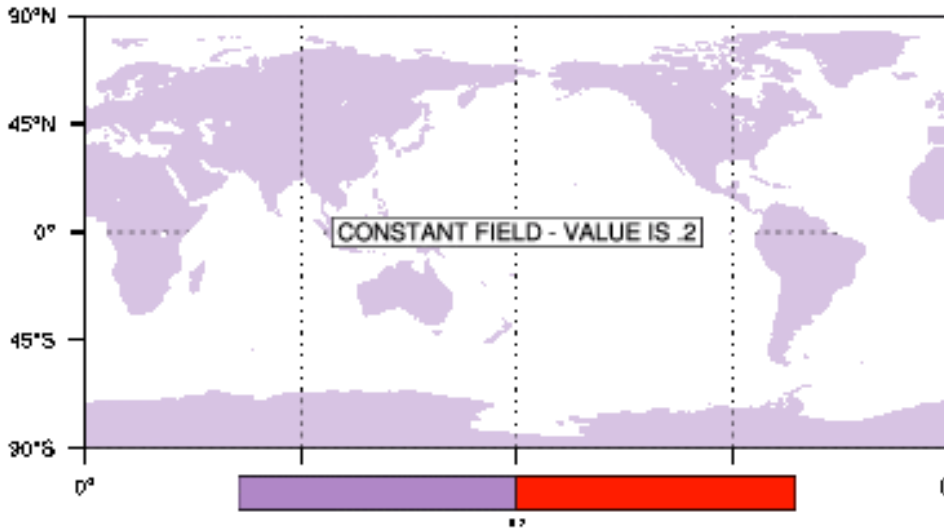


- Finer resolution physics grid
  - > dynamical core makes use of high-order basis function for advancing solution in time; why should that information not be passed to physics?  
(somewhat similar to sub-columns)

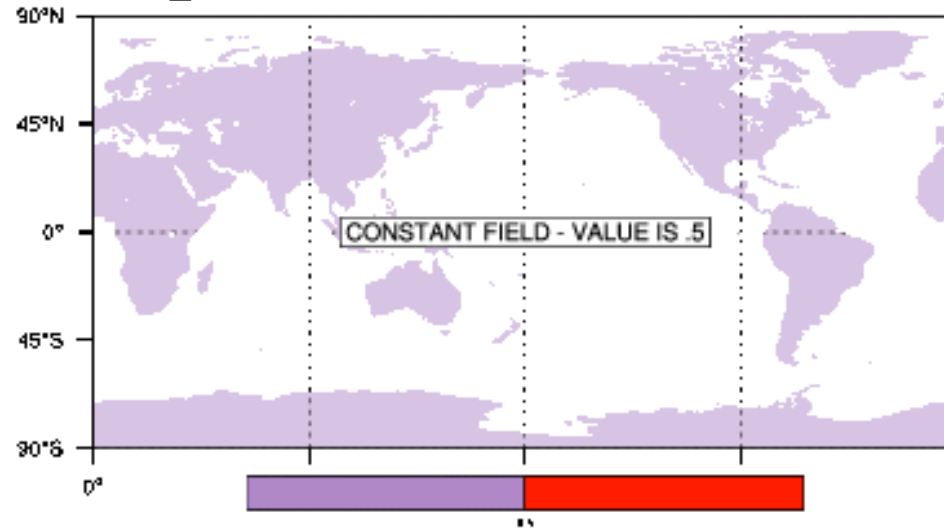




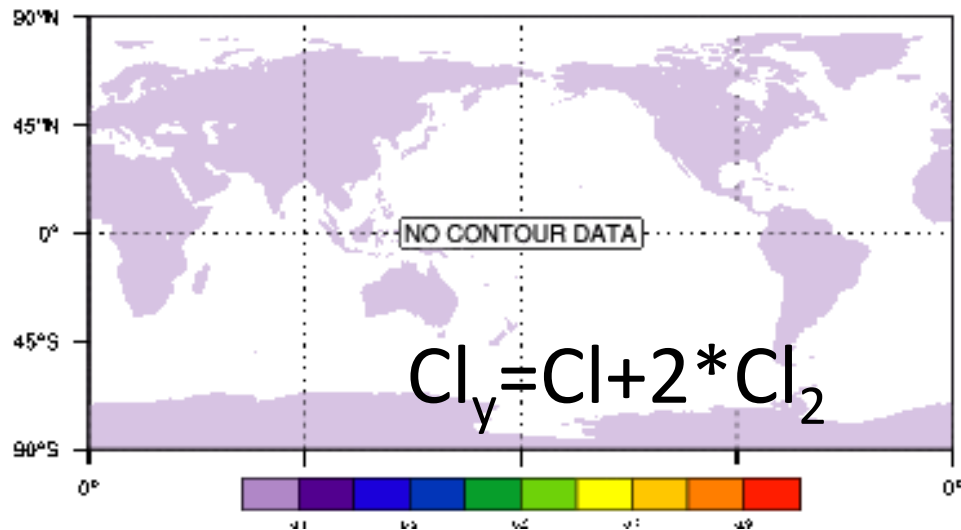
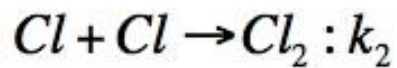
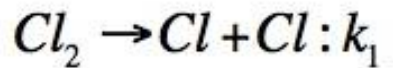
# Cl



# Cl<sub>2</sub>



Non-linear  
 “terminator-toy”  
 chemistry:

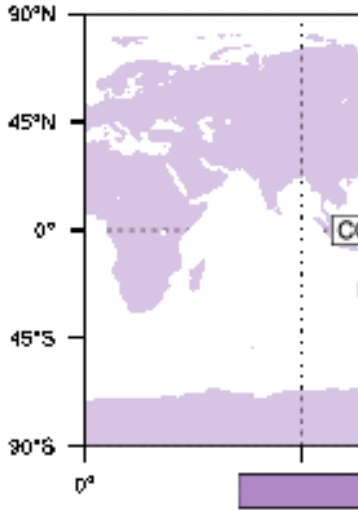


Simulations: CSLAM with shape-preserving limiter





# Cl



Non-linear  
 “terminator-toy  
 chemistry:

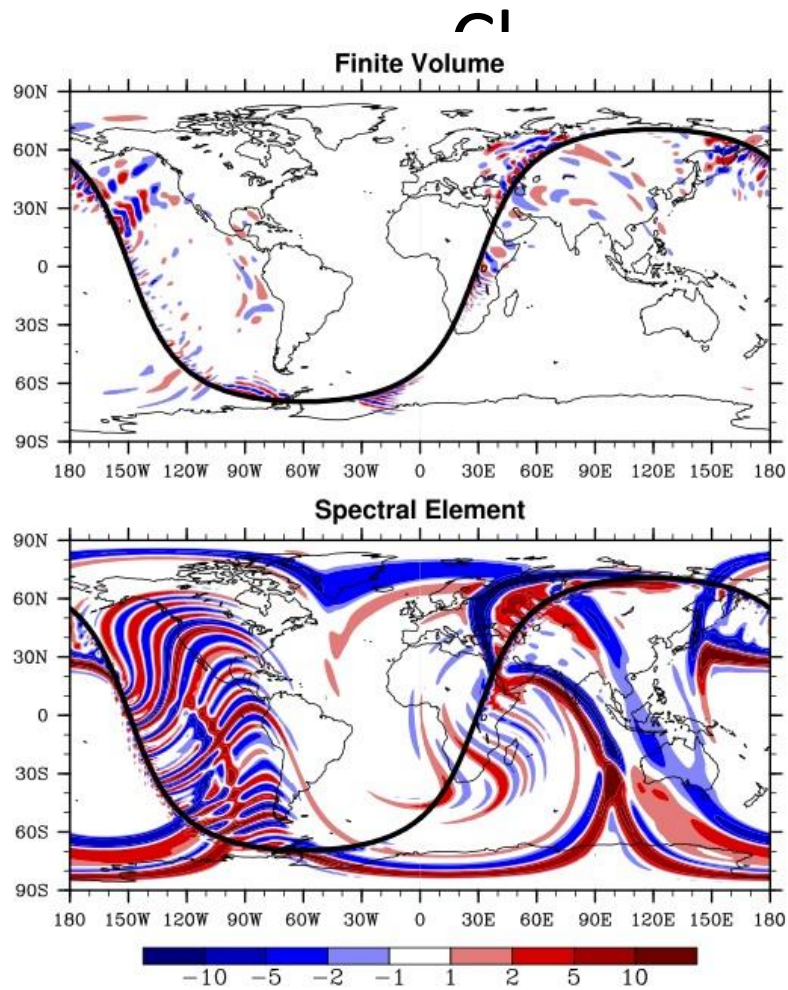
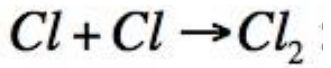
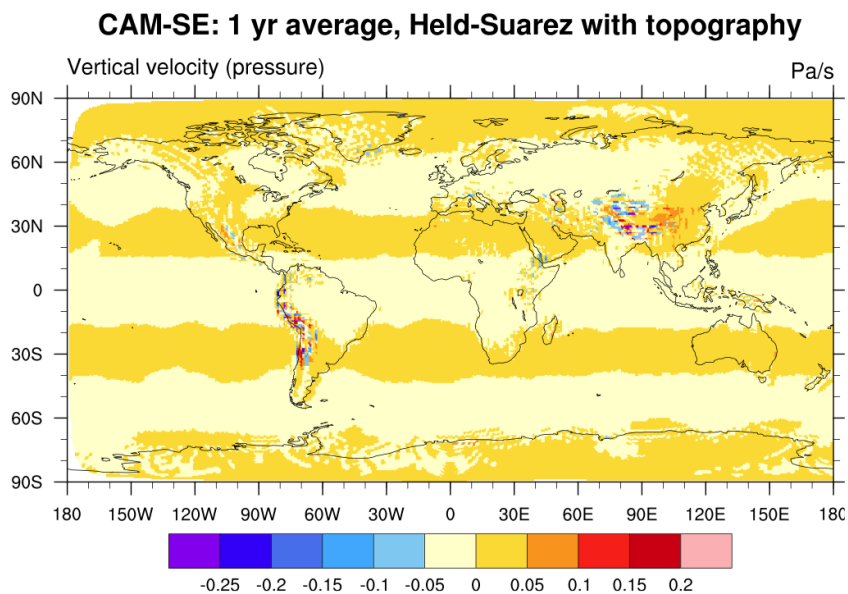


Figure 1. Distribution of the difference (in %) between  $Cl_y$  as simulated by the FV (top) and (bottom) dynamical cores and the value it would have under a perfectly accurate transport. The thick black line indicates the position of the terminator line defined by the fast reaction specified to be similar to a photolysis rate. Results are shown for an instantaneous snapshot of 5 days of simulation.



# Idealized forcing: Held-Suarez



$$\frac{\partial v}{\partial t} = \dots - k_v(\sigma)v$$

$$\frac{\partial T}{\partial t} = \dots - k_T(\phi, \sigma)[T - T_{eq}(\phi, p)]$$

$$T_{eq} = \max \left\{ 200K, \left[ 315K - (\Delta T)_y \sin^2 \phi - (\Delta \theta)_z \log \left( \frac{p}{p_0} \right) \cos^2 \phi \right] \left( \frac{p}{p_0} \right)^{\kappa} \right\}$$

$$k_T = k_a + (k_s - k_a) \max \left( 0, \frac{\sigma - \sigma_b}{1 - \sigma_b} \right) \cos^4 \phi$$

$$k_v = k_f \max \left( 0, \frac{\sigma - \sigma_b}{1 - \sigma_b} \right)$$

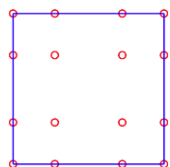
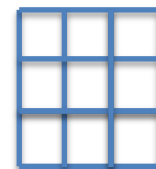


FIGURE 10. One-year average of vertical velocity ( $\omega$ ) using Held-Suarez forcing and ‘real-world’ topography using CAM-SE at approximately  $2^\circ$  horizontal resolution (*ne16np4*). Left plot is based standard CAM-SE setting where the sub-grid scale parameterization are computed on the spectral element quadrature grid and the right plot is based on the physics grid version in which tendencies are computed on a  $3 \times 3$  finite-volume grid inside each element. Note that the physics grid has the same number of degrees of freedom as the quadrature grid in this configuration.



# Idealized forcing: Held-Suarez

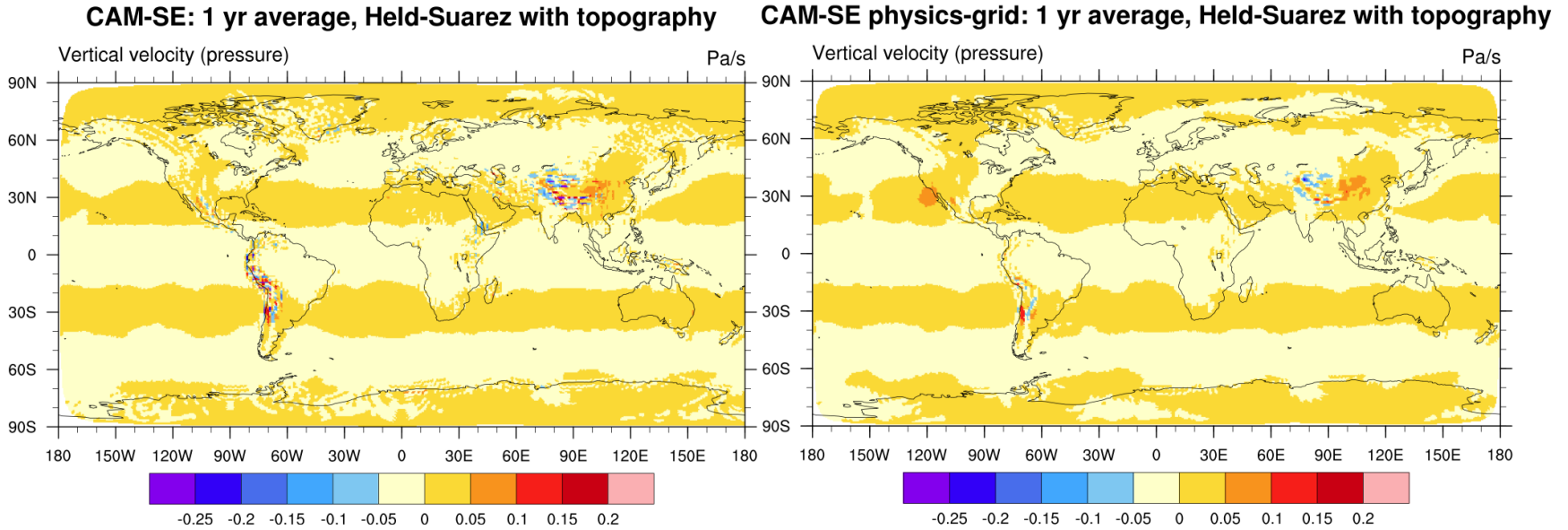
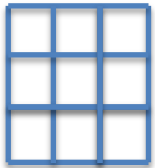
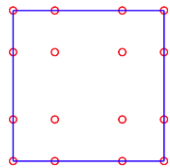
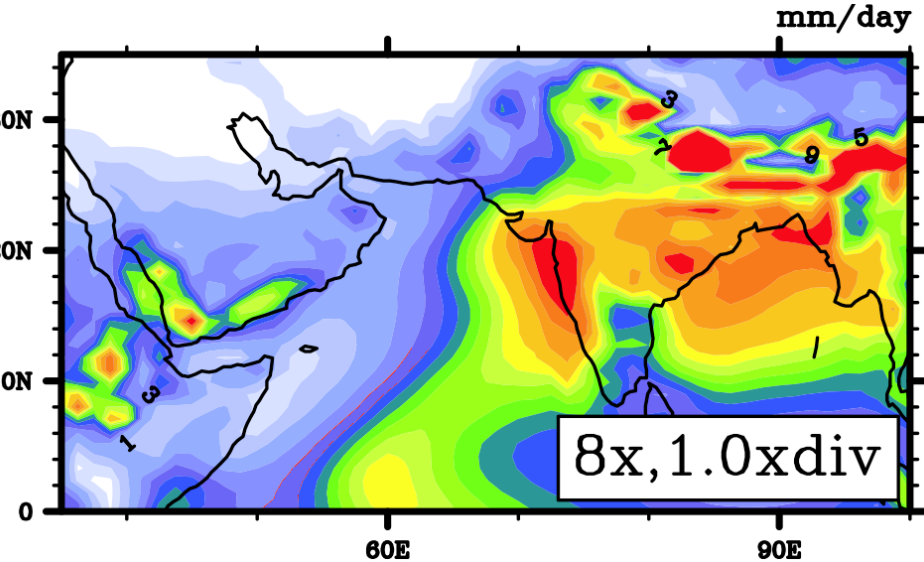
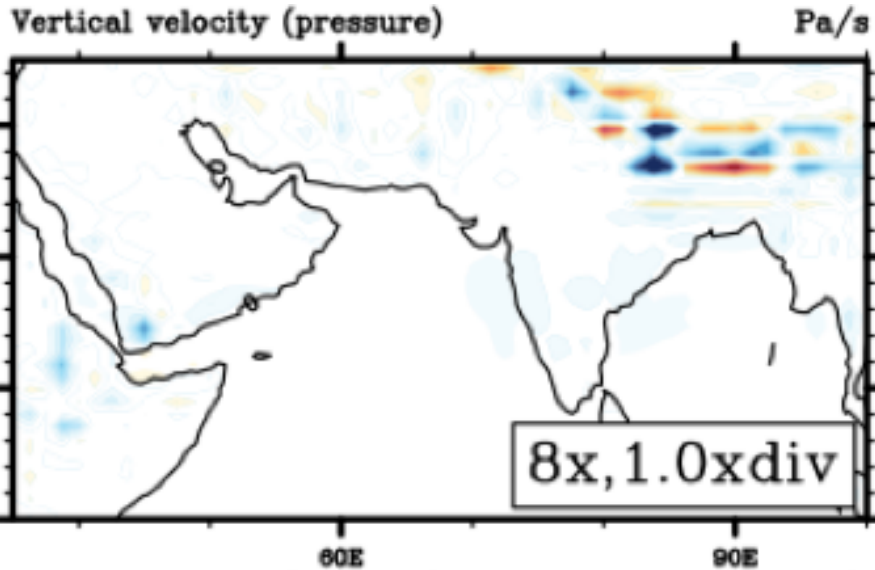


FIGURE 10. One-year average of vertical velocity ( $\omega$ ) using Held-Suarez forcing and ‘real-world’ topography using CAM-SE at approximately  $2^\circ$  horizontal resolution (*ne16np4*). Left plot is based standard CAM-SE setting where the sub-grid scale parameterization are computed on the spectral element quadrature grid and the right plot is based on the physics grid version in which tendencies are computed on a  $3 \times 3$  finite-volume grid inside each element. Note that the physics grid has the same number of degrees of freedom as the quadrature grid in this configuration.





# Noise in rough topography run







# Physics-dynamics coupling with Galerkin methods: equal-area physics grid



Peter H. Lauritzen<sup>1</sup>, Mark A. Taylor<sup>2</sup>, Steve Goldhaber<sup>1</sup>, Julio T. Bacmeister<sup>1</sup>, R.D. Nair<sup>1</sup>  
 National Center for Atmospheric Research, Boulder <sup>2</sup> Sandia National Laboratories, New Mexico

## Introduction

Consider a cubed-sphere tiling of the sphere with quadratic elements on each face. Inside each element there are 4x4 Gauss-Lobatto-Legendre (GLL) quadrature points:

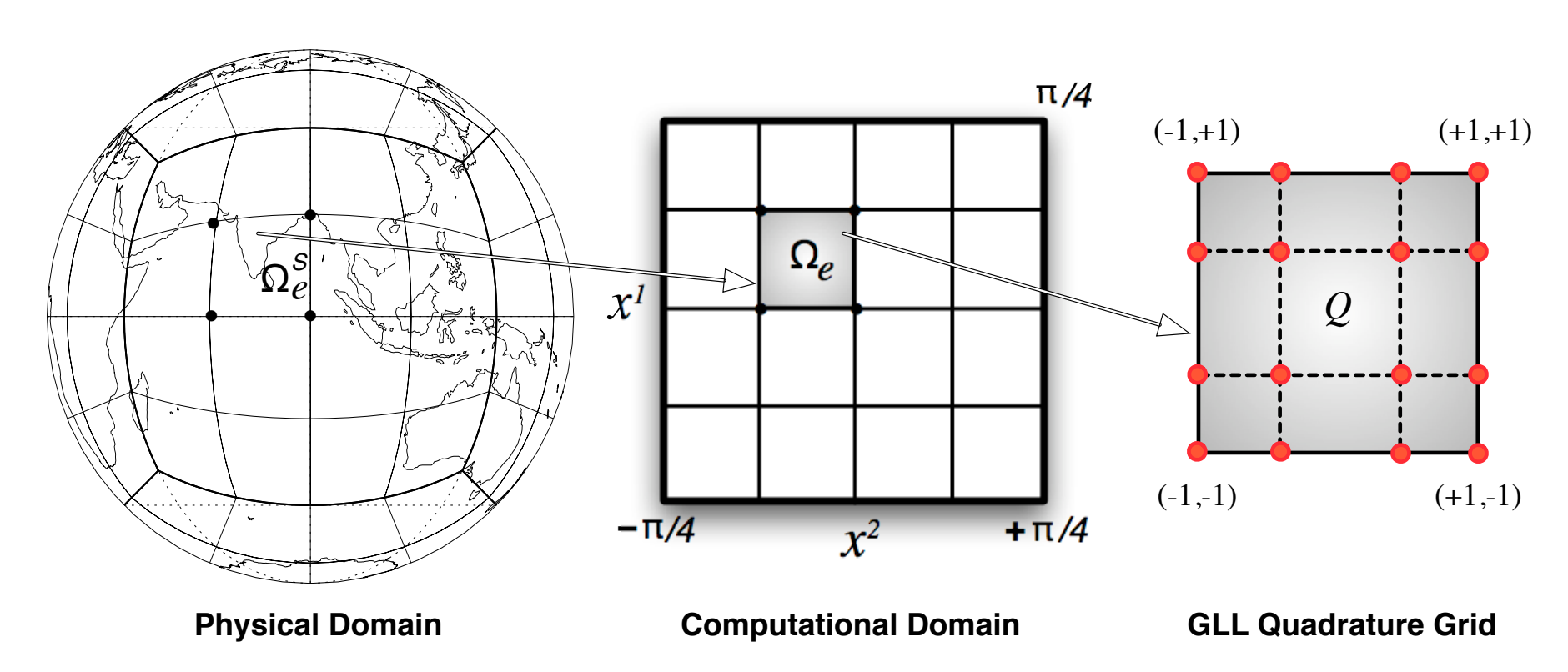


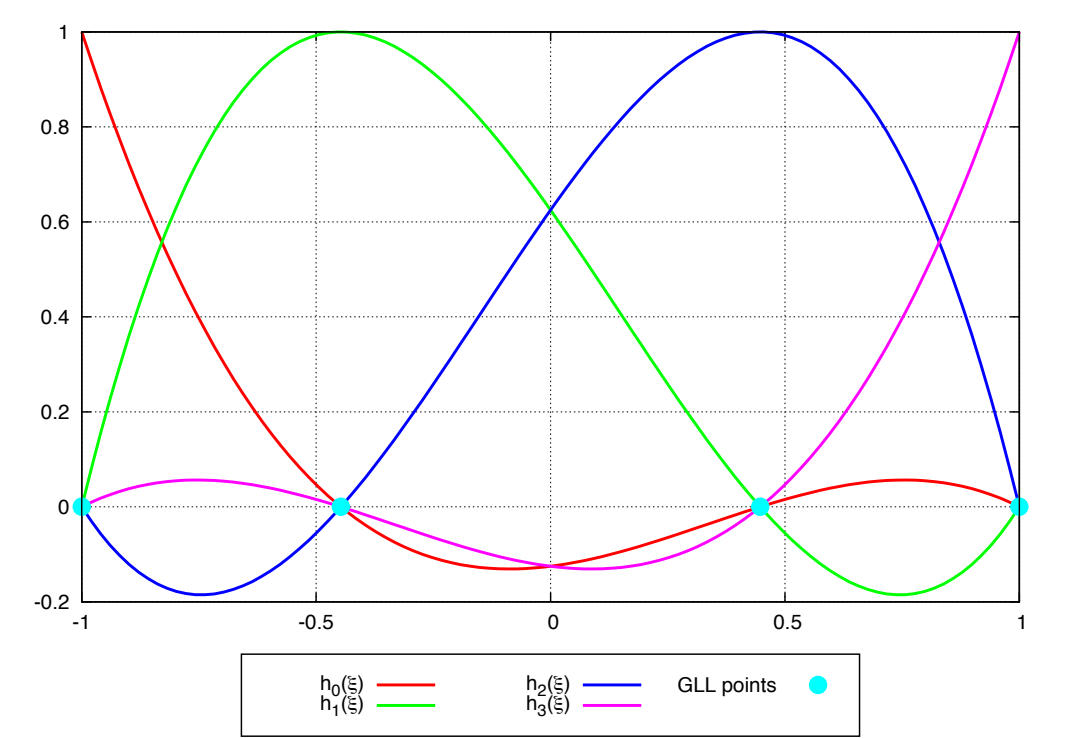
Fig. 9.22 A schematic diagram showing the mapping between each spherical tile (element)  $\Omega_e^s$  of the physical domain (cubed-sphere)  $\mathcal{S}$  onto a planar element  $\Omega_e$  on the computational domain  $\mathcal{C}$  (cube). For a DG discretization each element on the cube is further mapped onto a unique reference element  $Q$ , which is defined by the Gauss-Lobatto-Legendre (GLL) quadrature points. The horizontal discretization of the HOMME dynamical cores relies on this grid system.

(Figure and caption from Nair et al., 2011)

Assume a nodal basis set constructed using Lagrange polynomials  $h_k(\xi)$ ,  $\xi \in [-1, 1]$ :

$$h_k(\xi) = \frac{(\xi - 1)(\xi + 1)P'_N(\xi)}{N(N+1)P'_N(\xi_k)(\xi - \xi_k)}$$

where  $P_N(\xi)$  is the Legendre polynomial of degree  $N$  and  $P'_N(\xi)$  is the derivative of  $P_N(\xi)$ . With 4 GLL points there are 4 Lagrange basis functions ( $k=0,1,2,3$ ):

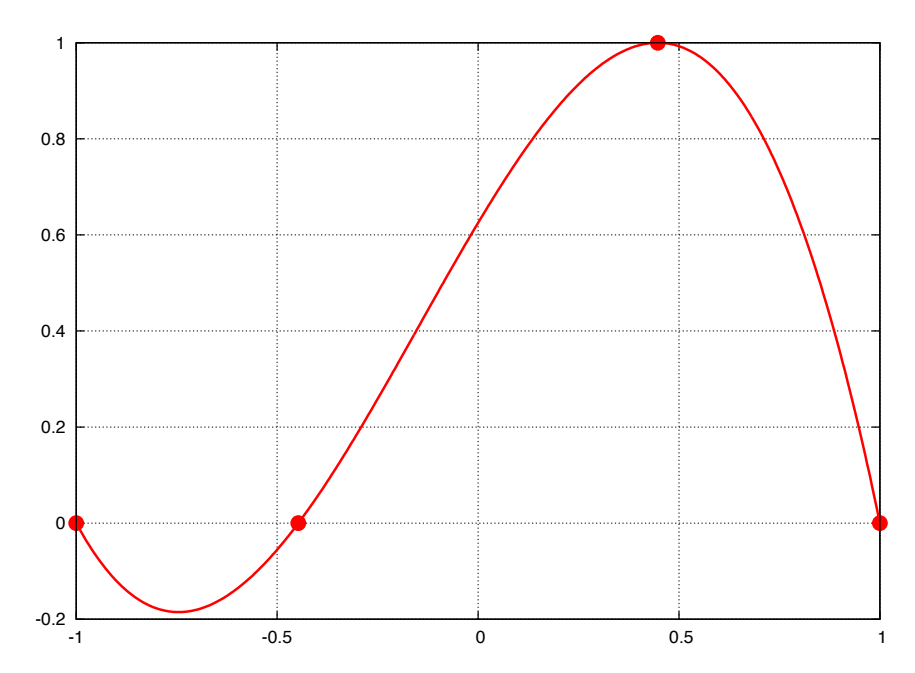


The solution  $U$  at time  $t$  inside element  $j$  is given by

$$U_j(\xi, t) = \sum_{k=0}^3 U_{j,k}(t) h_k(\xi), \quad \xi \in [-1, 1],$$

where  $U_{j,k}(t)$  is the known value at the  $k^{\text{th}}$  GLL point. Note that the solution is expressed as a Lagrange interpolation polynomial.

Given GLL point values,  $U_{j,k}(t) = \{0, 0, 1, 0\}$  for  $k=0, \dots, 3$ , the Lagrange "reconstruction" is shown on the Figure below:



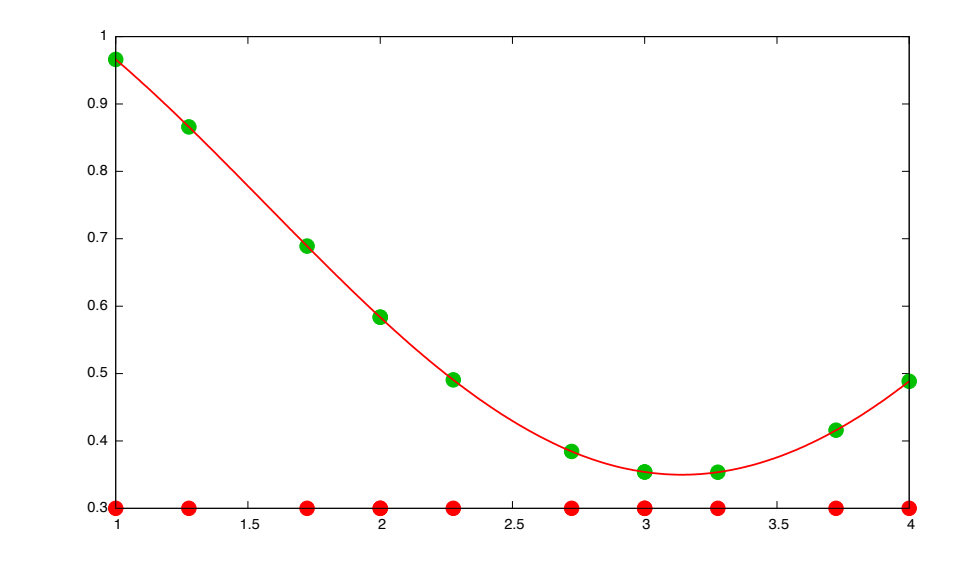
For simplicity we show only 1D examples; the 2D basis set can be constructed with a tensor product of the 1D basis functions:

$$U_h(\xi, \eta, t) = \sum_{l=0}^3 \sum_{m=0}^3 U_{lm}(t) h_l(\xi) h_m(\eta), \quad \text{for } -1 \leq \xi, \eta \leq 1,$$

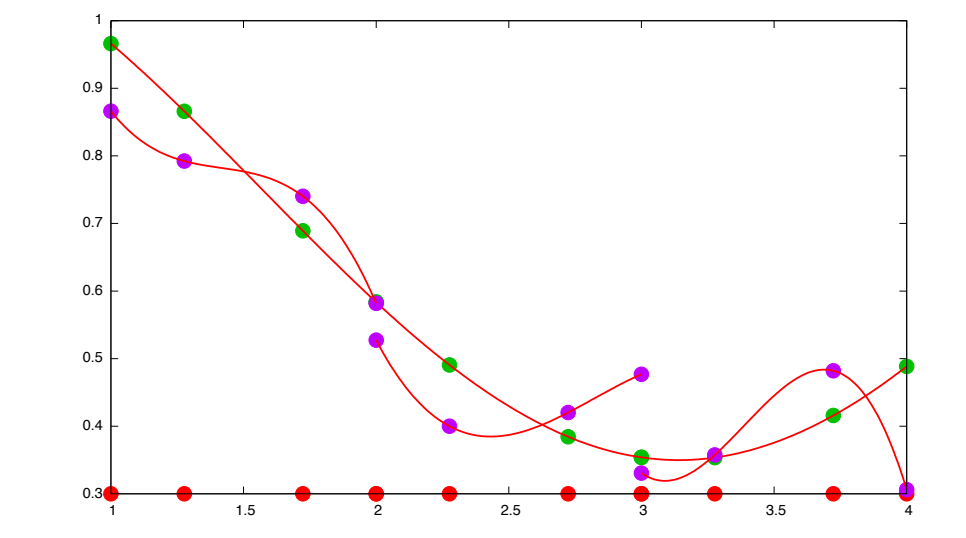
## Physics-dynamics workflow

Consider the continuous Galerkin finite-element method used in CAM-SE (NCAR's Community Atmosphere Model – Spectral Elements).

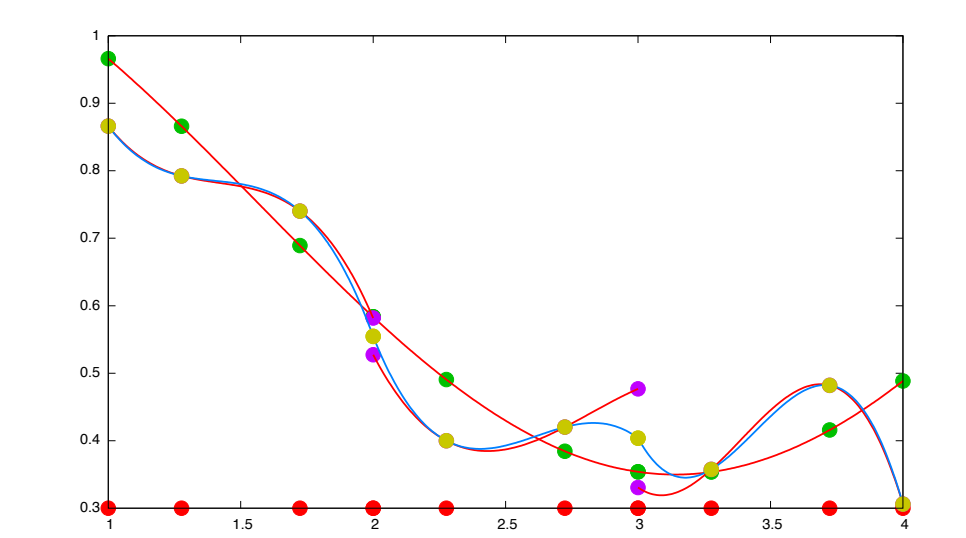
For simplicity consider a domain of 3 elements in 1D and let the initial condition be a "global" degree 3 polynomial (which can be represented exactly by the polynomial basis). Note that GLL points at element edges are shared between neighboring elements:



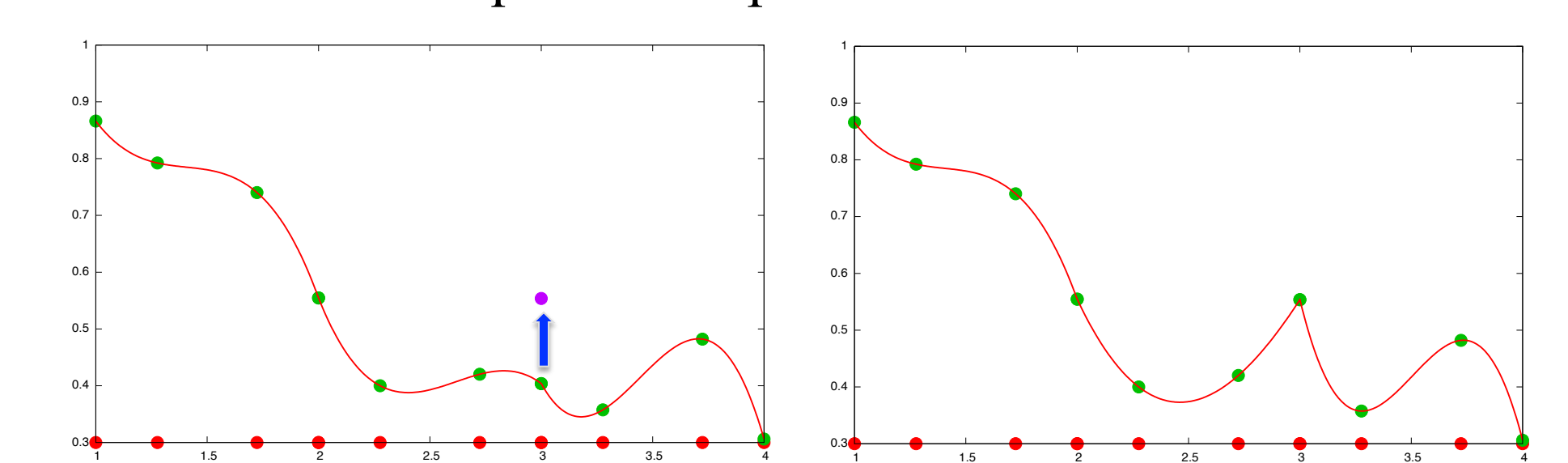
The solution is advanced one Runge-Kutta step inside each element:



The solution is projected onto a  $C^0$  basis (GLL point values at element edges are averaged – blue curve below):



This process is repeated for each Runge-Kutta step. Now the physical parameterization suite is called which, based on the atmospheric state at the GLL point values, computes tendencies at the quadrature points:



Assume that there is only a physics point located at  $x=3$  (see left Figure above). After physics has updated the atmospheric state at the GLL point(s), the polynomial "reconstruction" is shown on the Figure to the right (above).

**Note that the solution is only  $C^0$  at element boundaries! This is typically where noise appears!**

## Grid-scale forcing and noise

The spectral-element "reconstruction" is least smooth at the element boundaries where the  $C^0$  constraint is enforced; in climate simulation with CAM-SE noise in topographically forced flow typically appears near element boundaries (see Figures below).

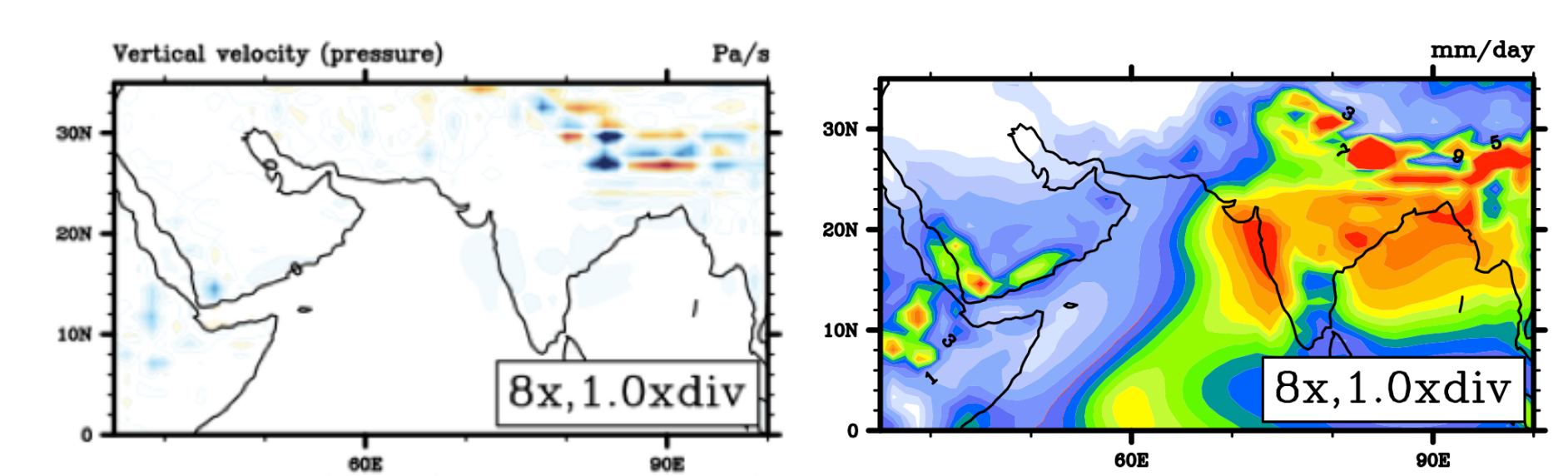
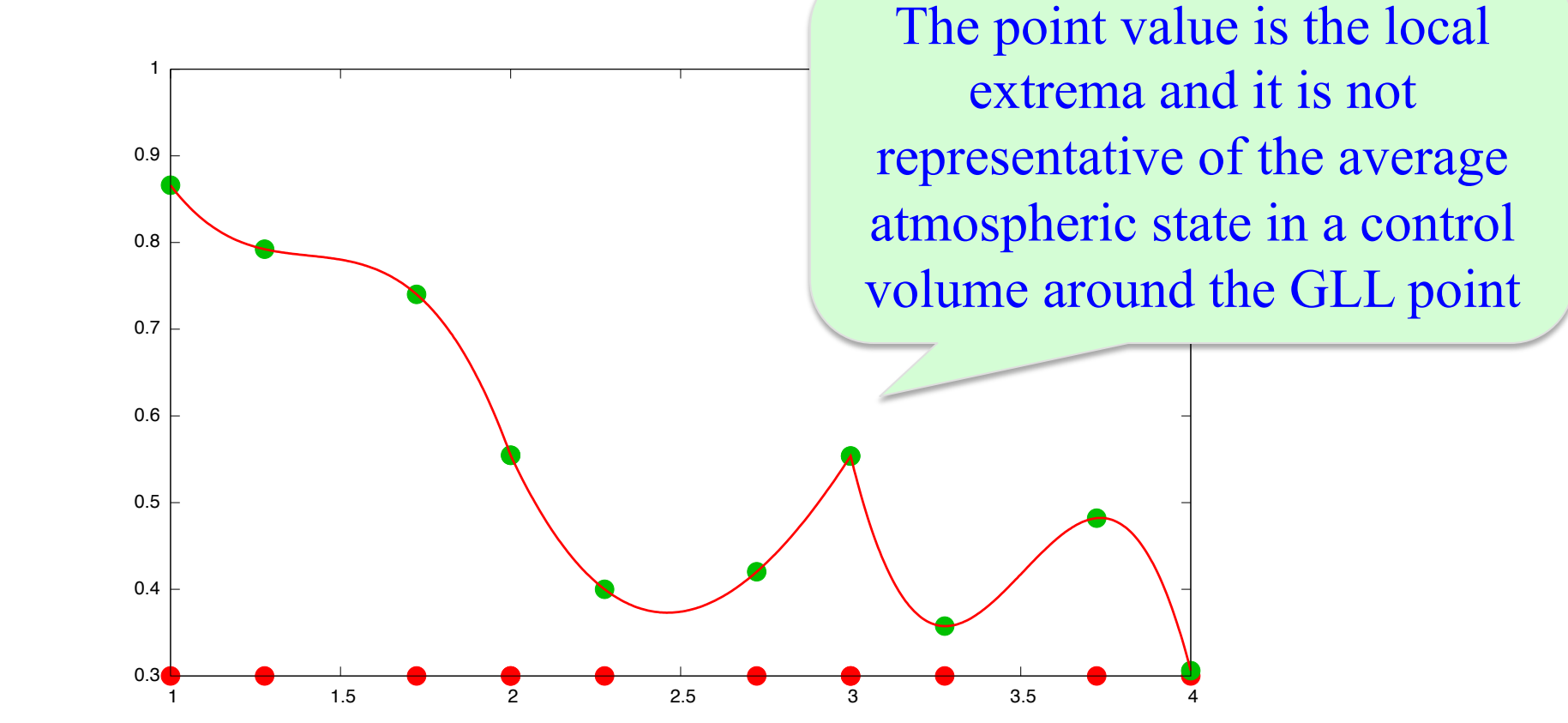


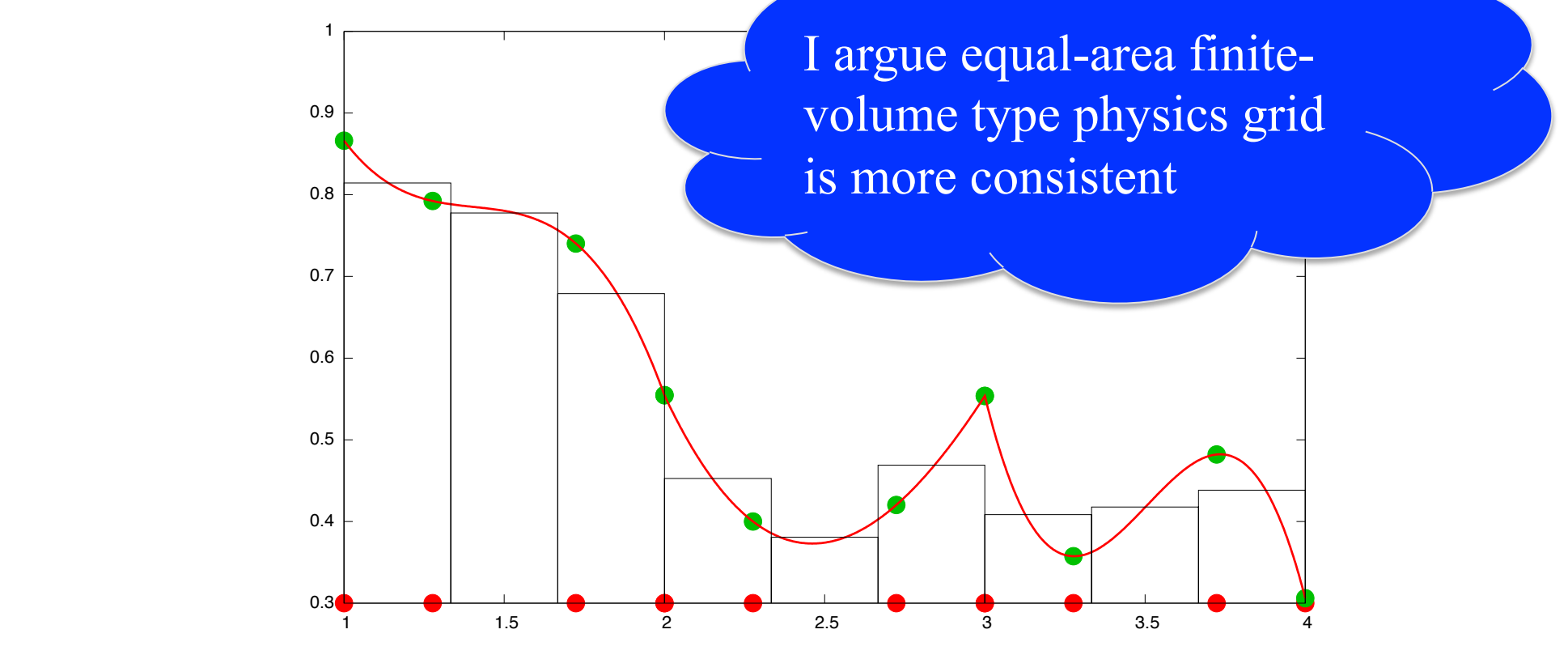
Figure: (left) 30 year average vertical pressure velocity for AMIP run using rough topography and no extra divergence damping. (right) Same as (left) but for precipitation rate.

## State from dynamical core passed to physics

I argue that parameterizations should be given a grid cell mean value for the atmospheric state rather than a (quadrature) point value.



Definition of physics grid: Define equal-area physics grid in each element by dividing each element into equi-distant control volumes and integrate Lagrange basis over finite-volumes.



Note that physics grid averages/moves fields away from boundary of elements where the solution is least smooth (in element interior the polynomials are  $C^\infty$ )

## Held-Suarez forcing with "real-world" mountains

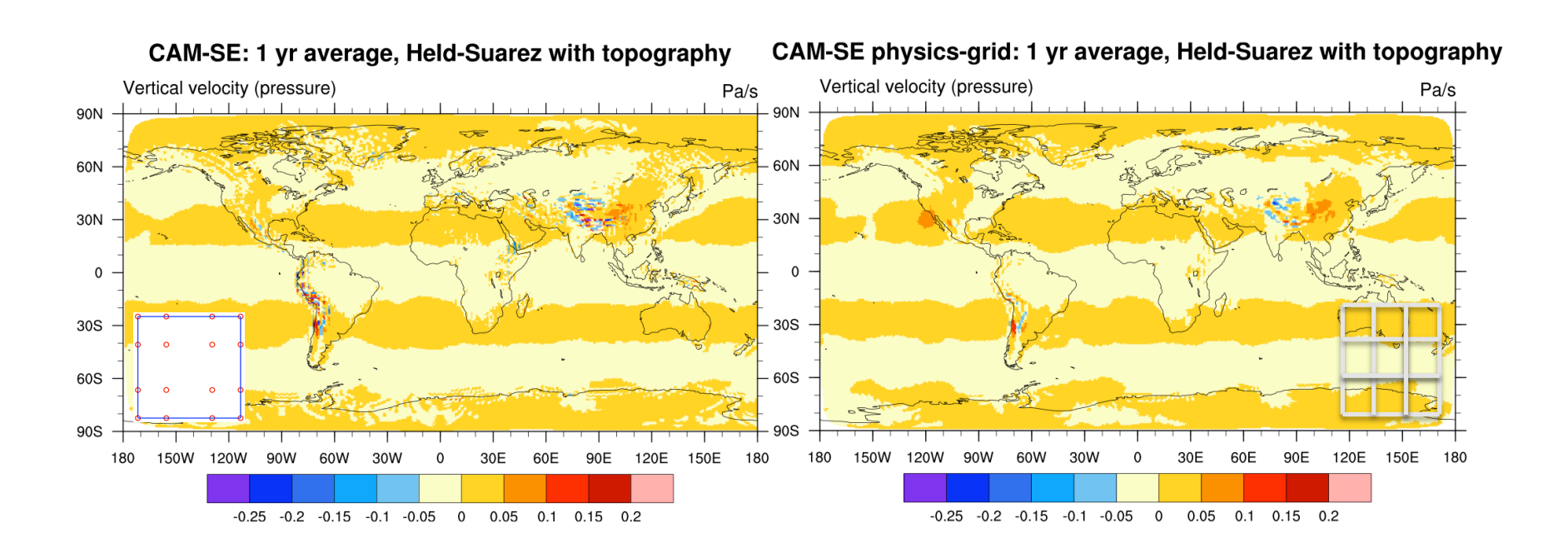


FIGURE 10. One-year average of vertical velocity ( $w$ ) using Held-Suarez forcing and "real-world" topography using CAM-SE at approximately  $2^\circ$  horizontal resolution ( $nc16np4$ ). Left plot is based standard CAM-SE setting where the sub-grid scale parameterization are computed on the spectral element quadrature grid and the right plot is based on the physics grid version in which tendencies are computed on a  $3 \times 3$  finite-volume grid inside each element. Note that the physics grid has the same number of degrees of freedom as the quadrature grid in this configuration.

Note: in this experiment bilinear interpolation was used for moving variables to and from physics-dynamics grid.

## Transferring variables from physics grid to dynamics grid

Moving variables from dynamics to physics through basis function integration is likely the most consistent/accurate approach; going the other way is less obvious:

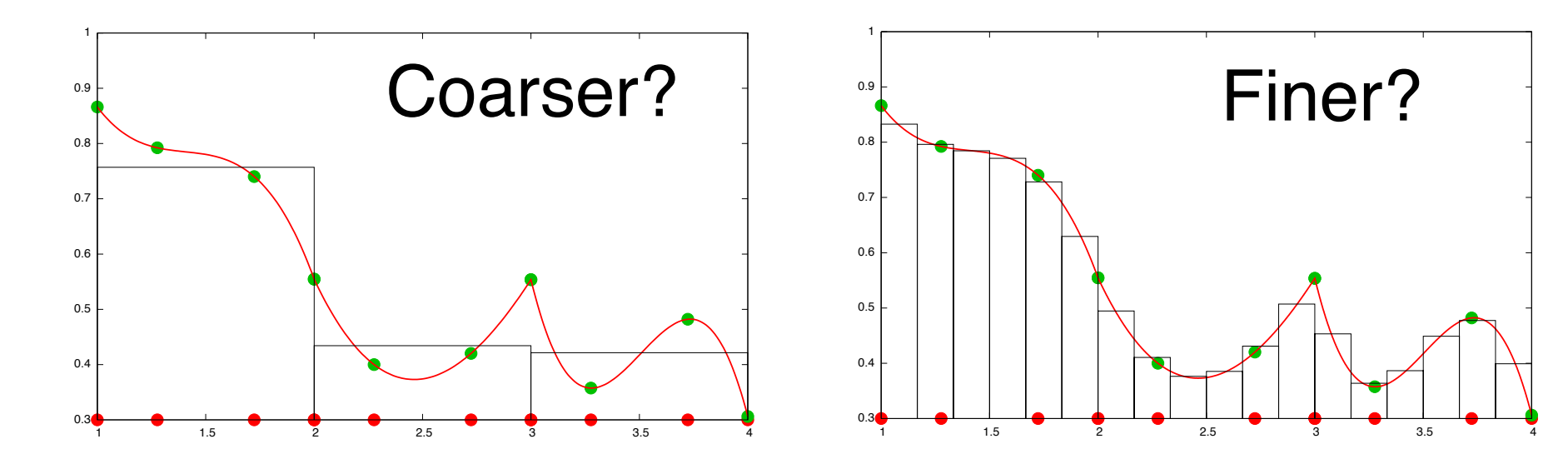
We propose to reconstruct a polynomial  $\hat{\psi}_k(x)$  that satisfies the mass-conservation constraint in all physics grid finite-volumes in element  $k$ :

$$\int_{x_{j-1/2}}^{x_{j+1/2}} \hat{\psi}_k(x) dx = \bar{\psi}_j \Delta x,$$

where  $j=1, \dots, nc$  ( $nc$  is the number of physics grid finite-volumes in element  $k$ ). This polynomial is then evaluated at the GLL points to provide physics tendencies to the dynamical core.

**Note: If dynamical core uses polynomial order  $N$  and  $nc=N+1$  then  $\hat{\psi}_k(x)$  will be identical to the dynamical core Lagrange basis!**

What should resolution of physics grid be?  $nc=N-1$ ?



## Reference

Nair, R.D., M.N. Levy and P.H. Lauritzen, 2011: *Emerging numerical methods for atmospheric modeling* Lecture Notes in Computational Science and Engineering, Springer, Vol. 80, pp.251-311.

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