Two strategies for the mitigation of coordinate singularities of a spherical polyhedral grid

R. James Purser, Miodrag Rancic, Dusan Jovic
IMSG at NOAA/NCEP/EMC, College Park,
and Dragan Latinovic
CPTEC, Sao Paulo, Brazil

Introduction

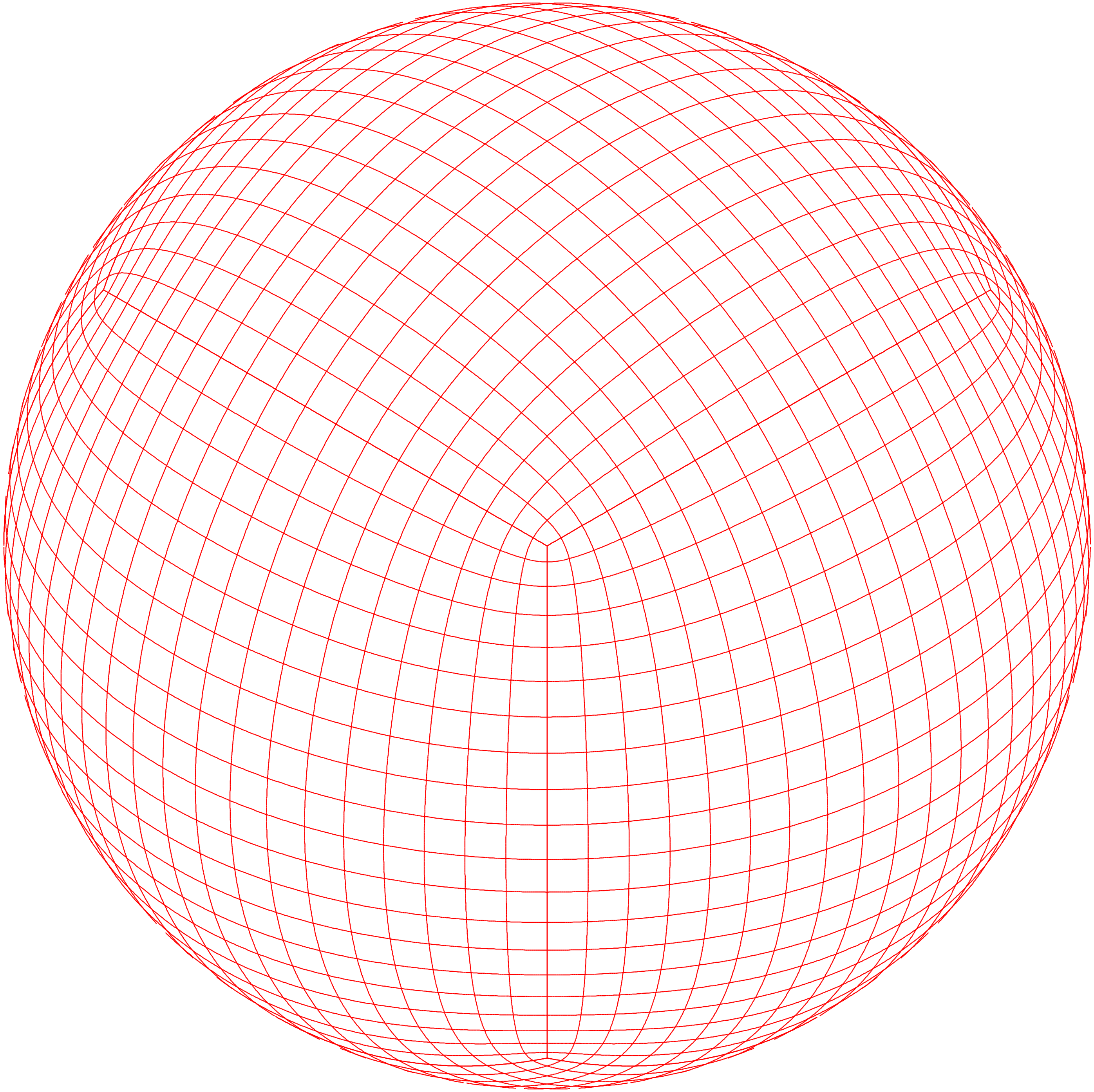
The well-known problems of the latitude-longitude style of gridding the sphere are partially alleviated by employing a grid based upon polyhedral projection.

For example, a cubic spherical gridding does not possess strongly converging longitude lines intersection at polar singularities, but the simplest cubic mapping—derived from central (gnomonic) projection generates discontinuities in derivatives at all the edges. The same applies to the icosahedron.

If we want a smoother mapping, a mathematically ideal and very general approach is to adopt the symmetry-preserving and local angle-preserving conformal mapping.

Now there are no edge discontinuities, but point-singularities (though weaker) reappear at all the corners. The two aspects of each conformal singularity are:

(i) The singularity in the Jacobian
(ii) The singularity in curvature.



We propose two new general strategies to repair these defects.

First, we shall seek an adaptation of the polyhedral grid to remove the jacobian singularity. We do this by “migrating” the initially conformal grid with an irrotational flow field driven by an imposed divergence essentially given (except for a spatially additive part as required by integral consistency) by the negative-logarithm of the initial Jacobian, and applied for a single unit of “time”.

This divergence is attached, like a Lagrangian tracer, to the advecting points, so the actual flow field evolves over the unit of “time”. But the symmetry of the original polyhedral template, being preserved, ensures that the solution for each polyhedron is unique, and the end state is a mapping that is perfectly “equal-area”, and has therefore removed any variation in the Jacobian.

Numerical generation and application of the map-migration flow.

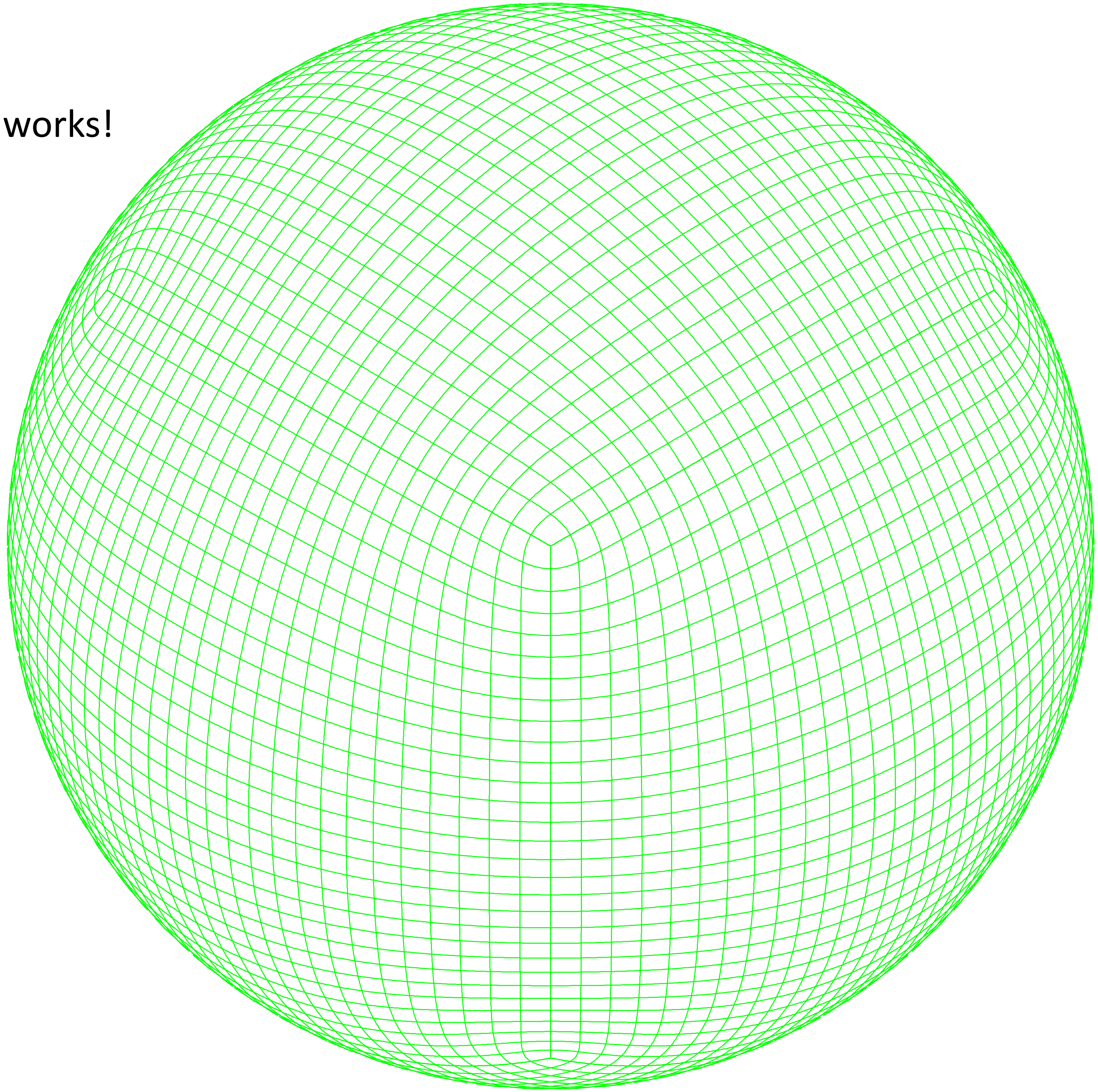
There are two parts to this problem: generating and applying the migration flow field away from the vertex singularities, where it is safe to use conventional gridding techniques; applying an asymptotic approximation near the singularities, where grid techniques are unreliable but where the asymptotic form of the flow is very accurate.

These two approaches are applied simultaneously at every stage, and even afterwards when applying the solution to the production of new computational grids for user applications.

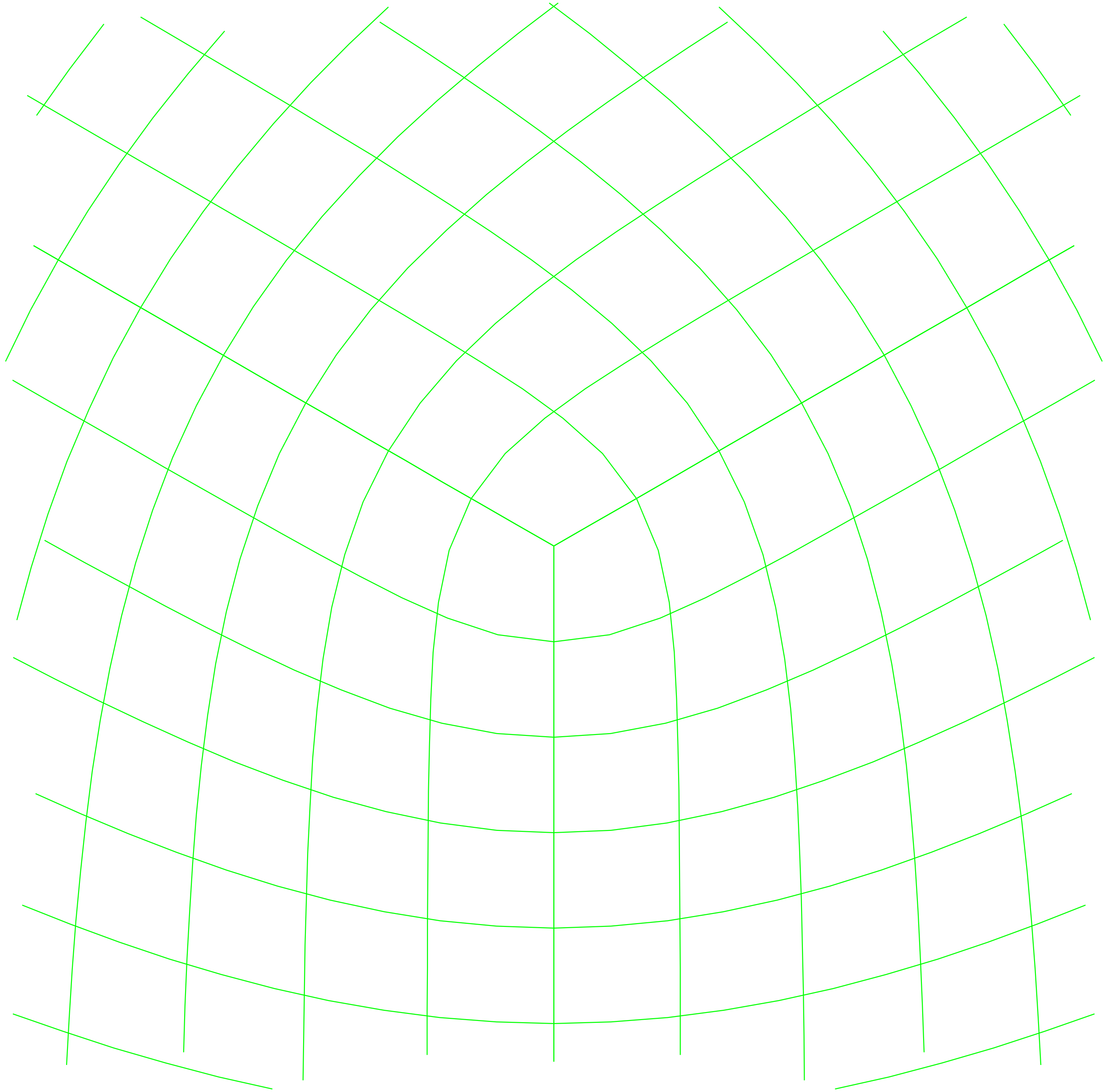
The partial solutions near and away from singularities are blended smoothly over an annular region centered on each vertex.

Symmetry is fully exploited (computations are done on only 1/48 of the cube, and only on 1/120 of the area of the icosahedron, for example).

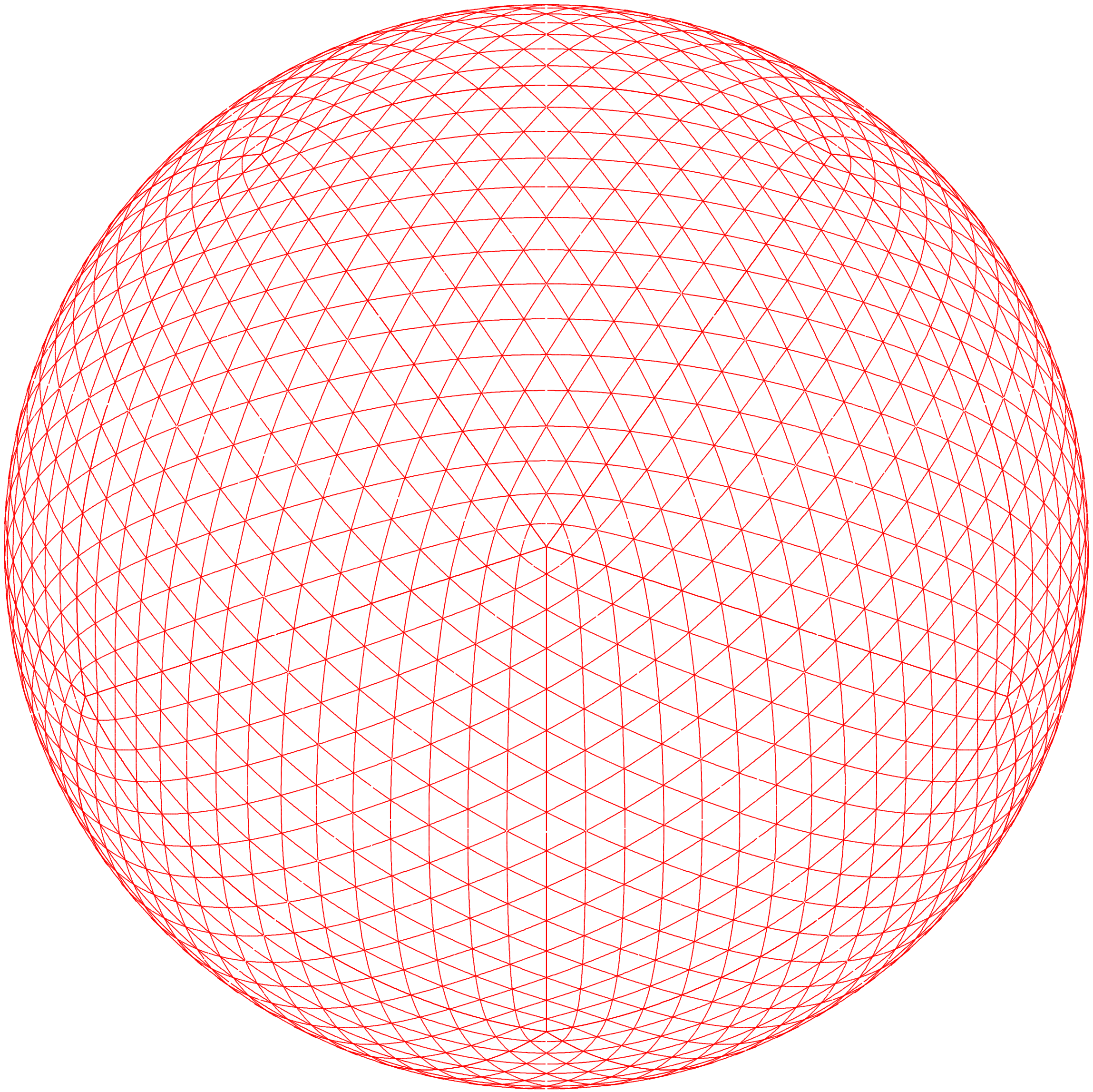
And it works!



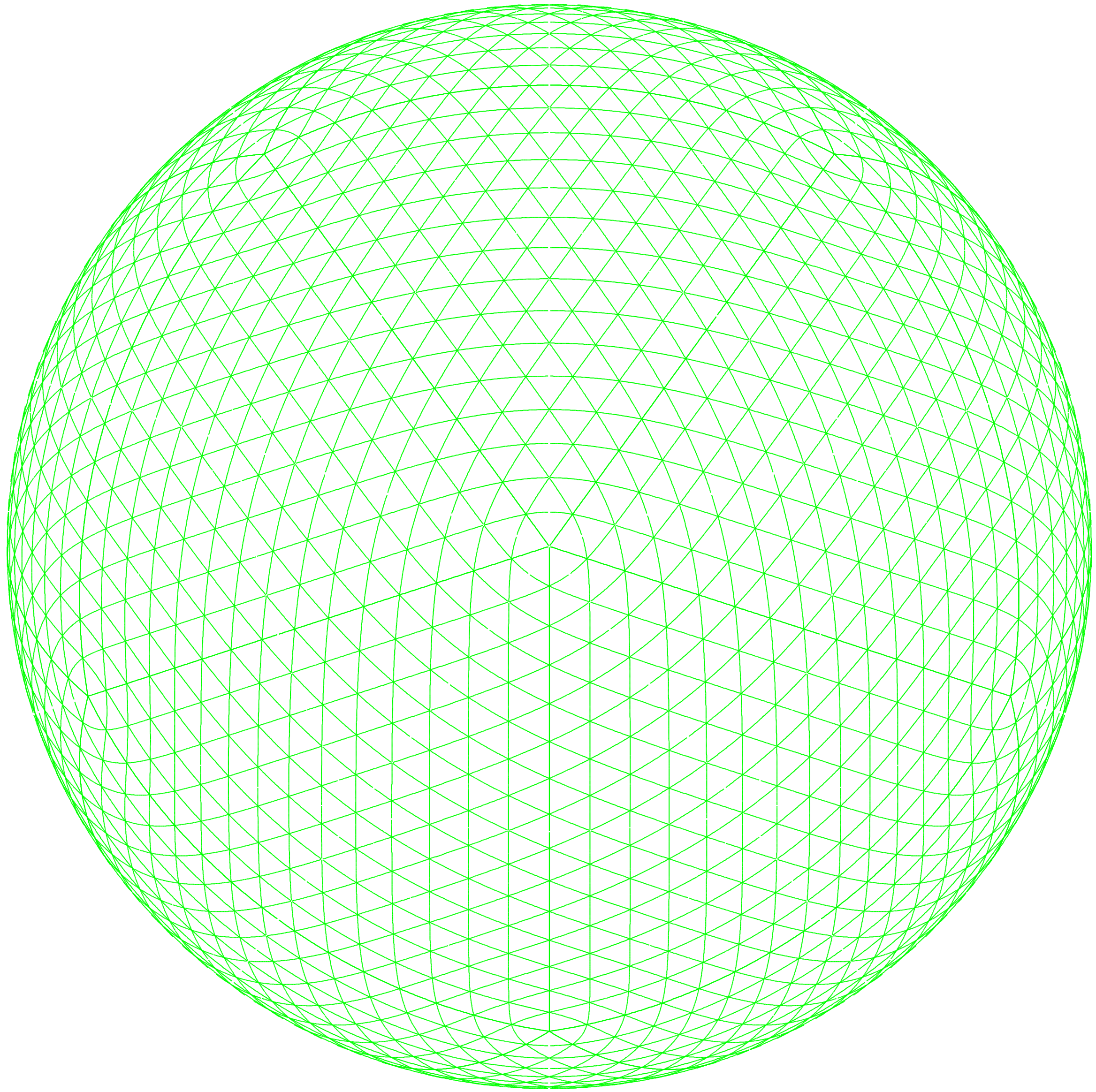
Detail near a vertex for the equal-area cubic mapping



For the icosahedron, whose conformal mapping grid looks like:



The equal-area icosahedral grid looks like:



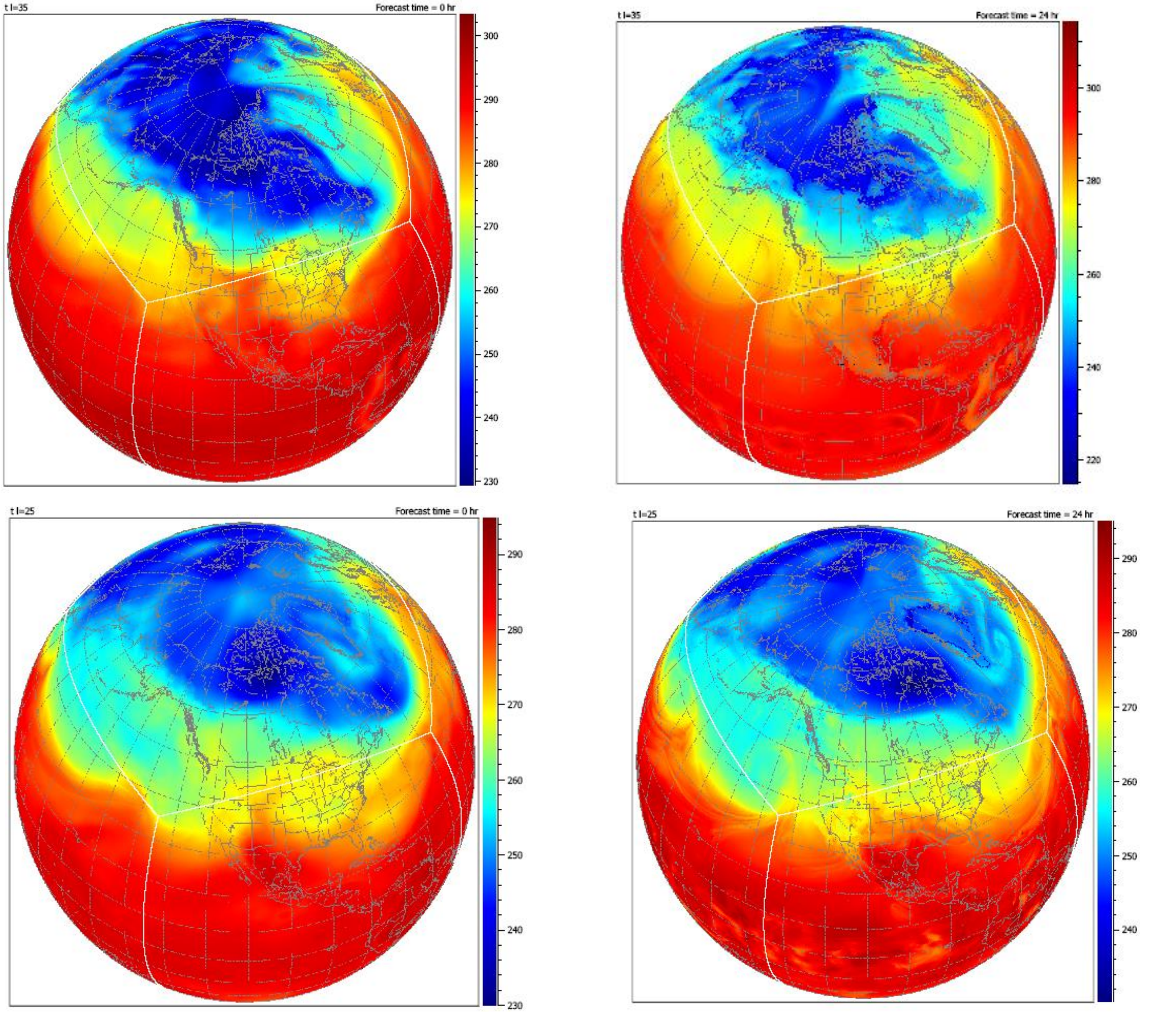
Model used for testing new grid

- A grid-point model emerged from a globalization of the regional Eta model (Zhang and Rancic 2006)
- Capable of operating on both cubic and various octagonal grids
- Presently used in CPTEC, Brazil, for research of seasonal prediction in relation to ENSO
- A distant relative of NMMB
- Operates with 38 vertical levels
- Modernized lately at ESSIC/UMD in transition toward NMMB, thanks to NSF grant 0739518
- A novel scale-selective treatment of horizontal advection
- Tests were done on 401x401x6 horizontal scalar grid points (resolution of about 24 km)
- Temperature fields and short wave radiation reaching the ground are shown as an preliminary illustration of model ability to perform.

Time=00 Time=24

Lev=35

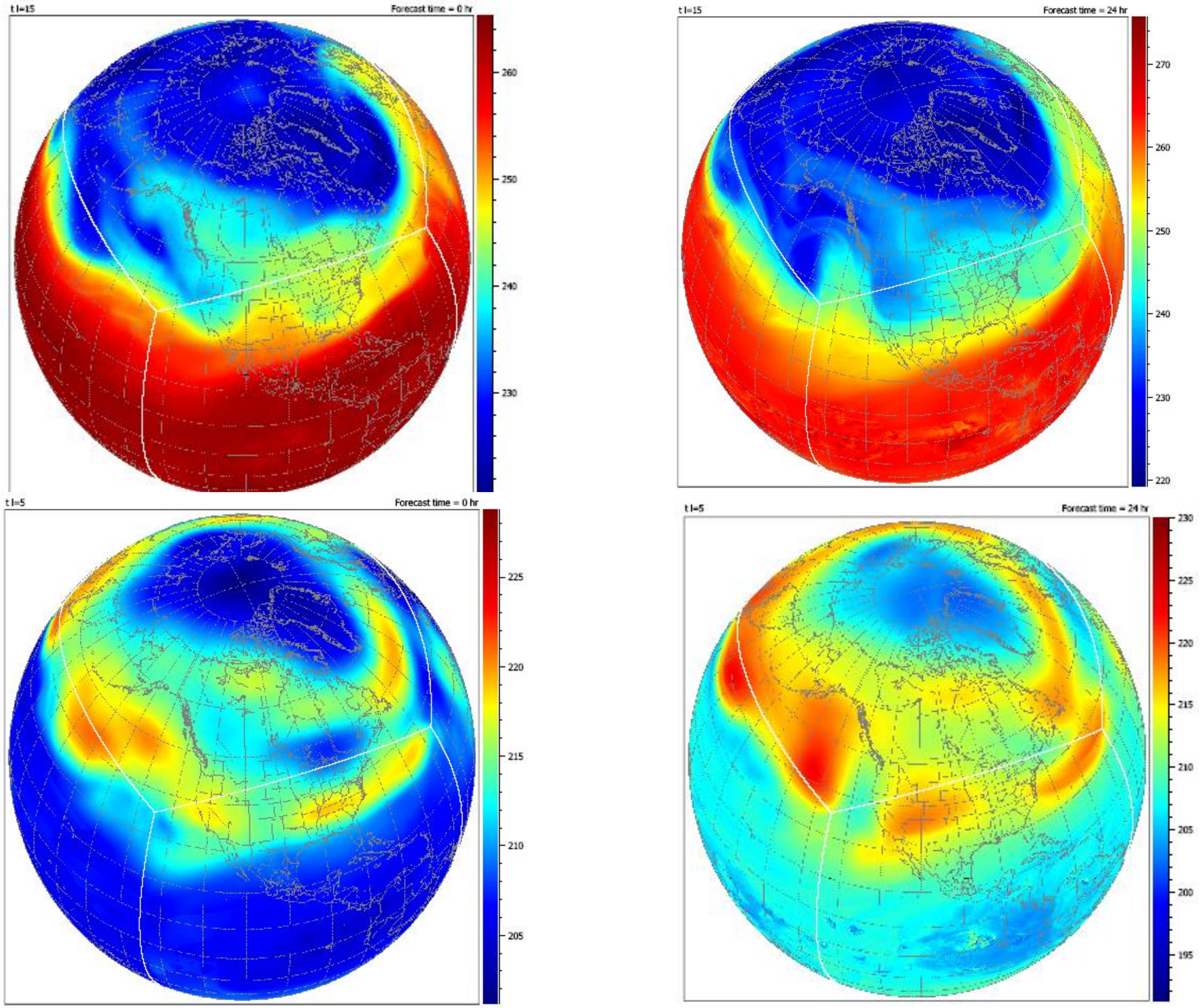
Lev=25

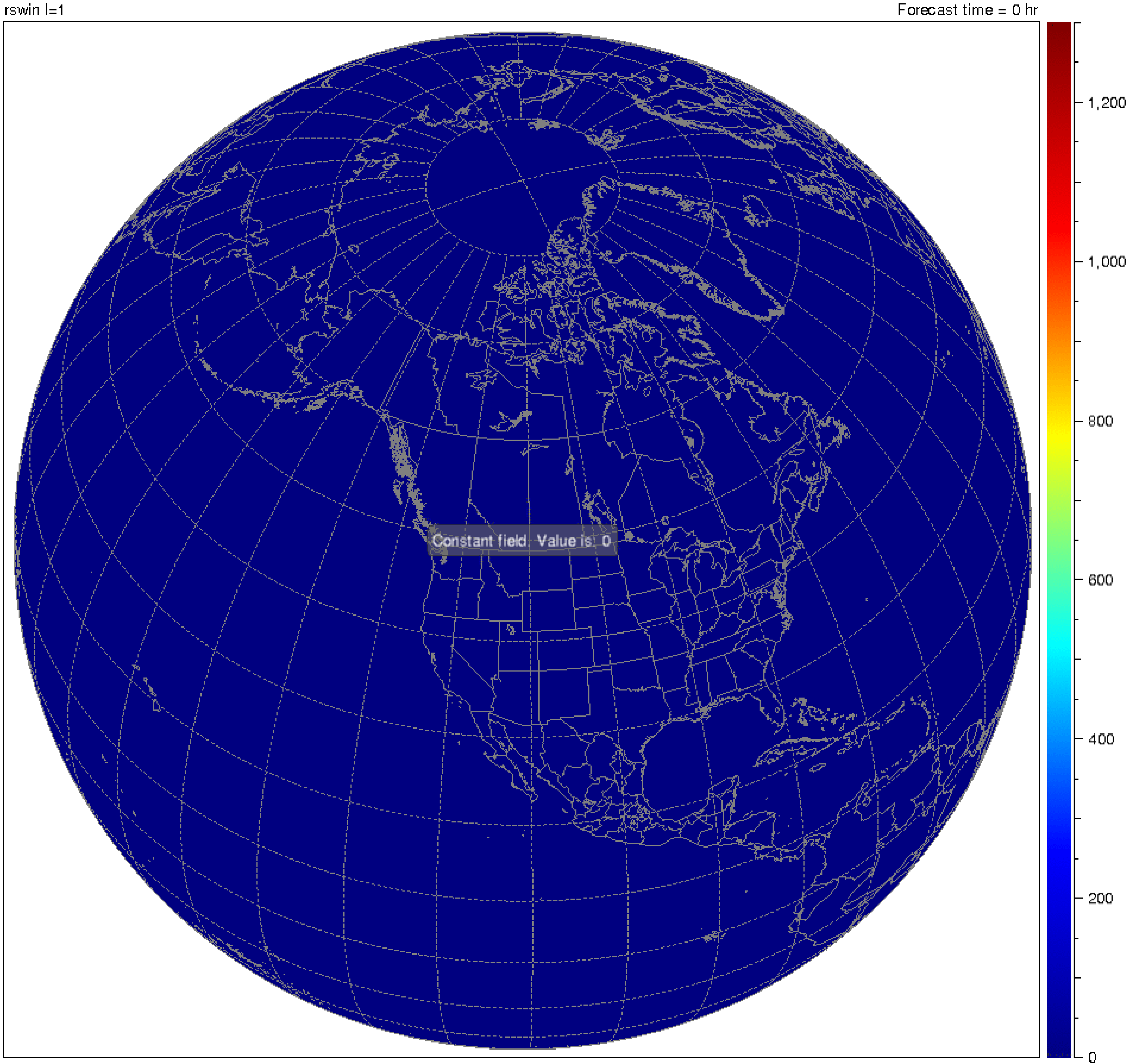


Time=00 Time=24

Lev=15

Lev=05





rswin l=1
Forecast time = 0 hr
Constant field: Value is: 0
1,200
1,000
800
600
400
200
0

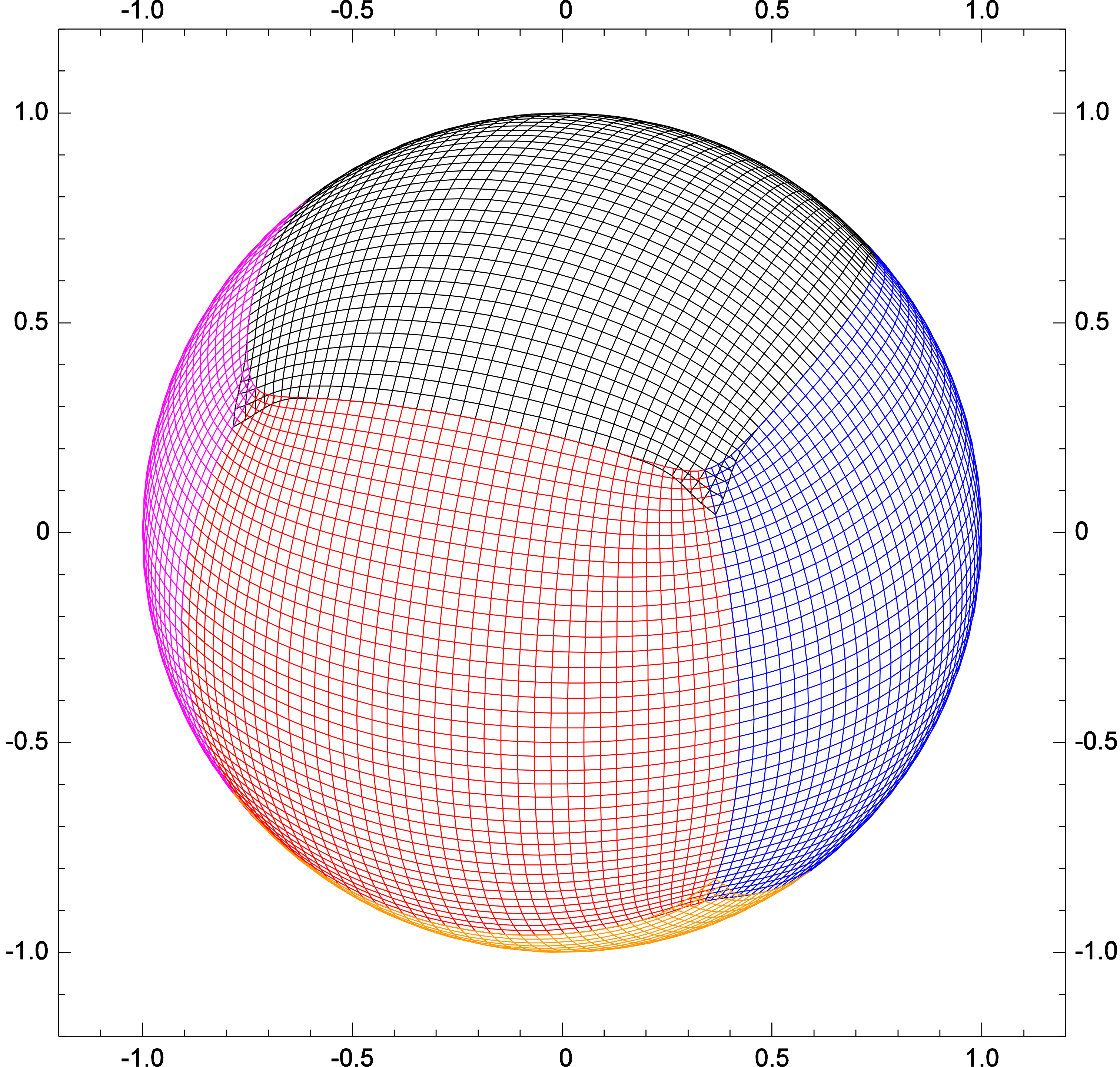
The map migration method addresses only the Jacobian aspect of the singularity; the curvature singularity of the grid is still present and still has numerical effects.

We also lose the original conformal grid's orthogonality.

The second strategy for mitigating the singularity effects involves a completely different approach, removing the singularities completely, retaining conformality (and therefore the numerically desirable orthogonal grid property), but we are required to incise the grid near the original corners, and create oversets at each of these locations.

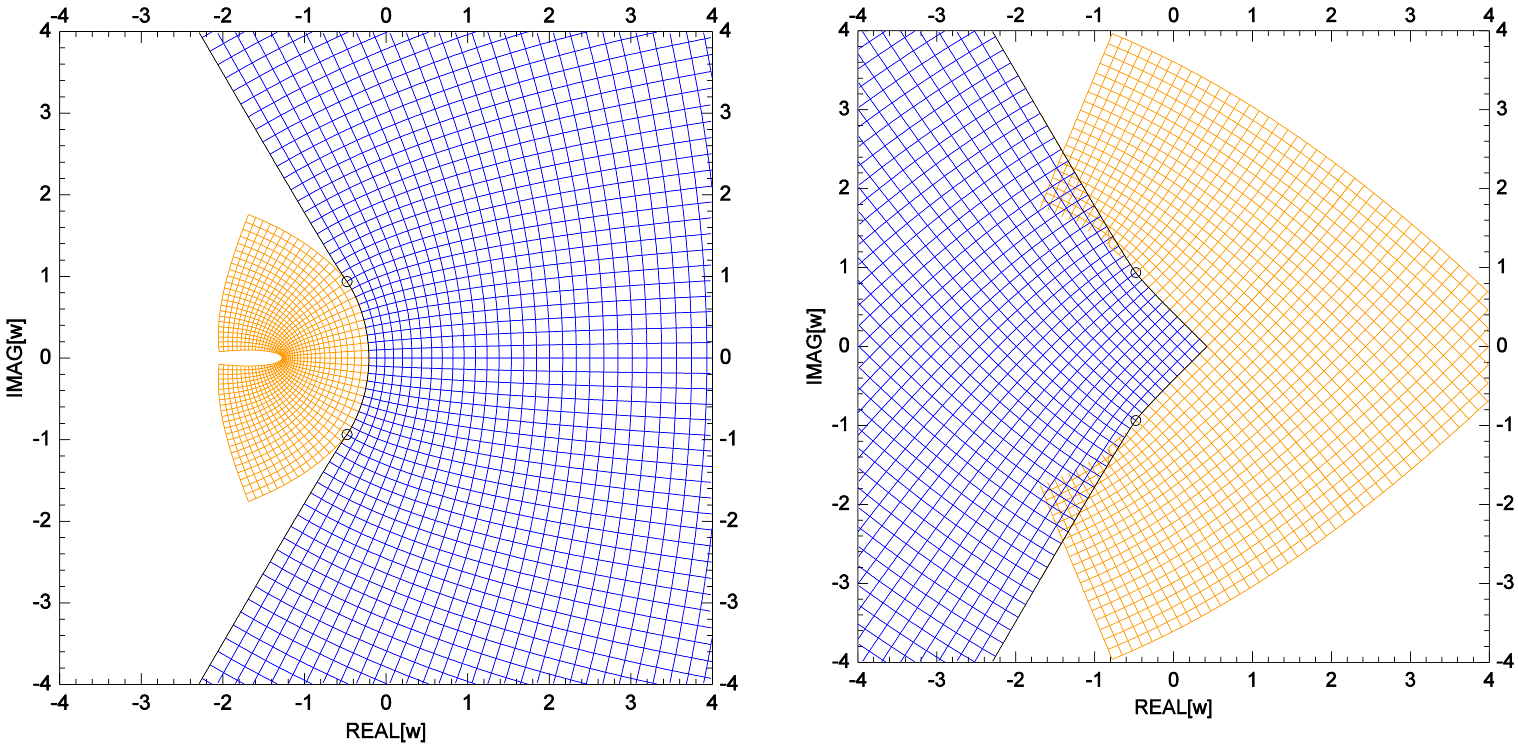
We do this by adopting special (customized) complex analytic functions that run over self-overlapping Riemann Surfaces winding twice around each branch point.

This approach allows us to construct, for example, conformal Yin-Yang grids adapted from the original cubic formulation, but with the overset regions confined to very close to the original corners. Or we can generate the "barrel grid" configuration.

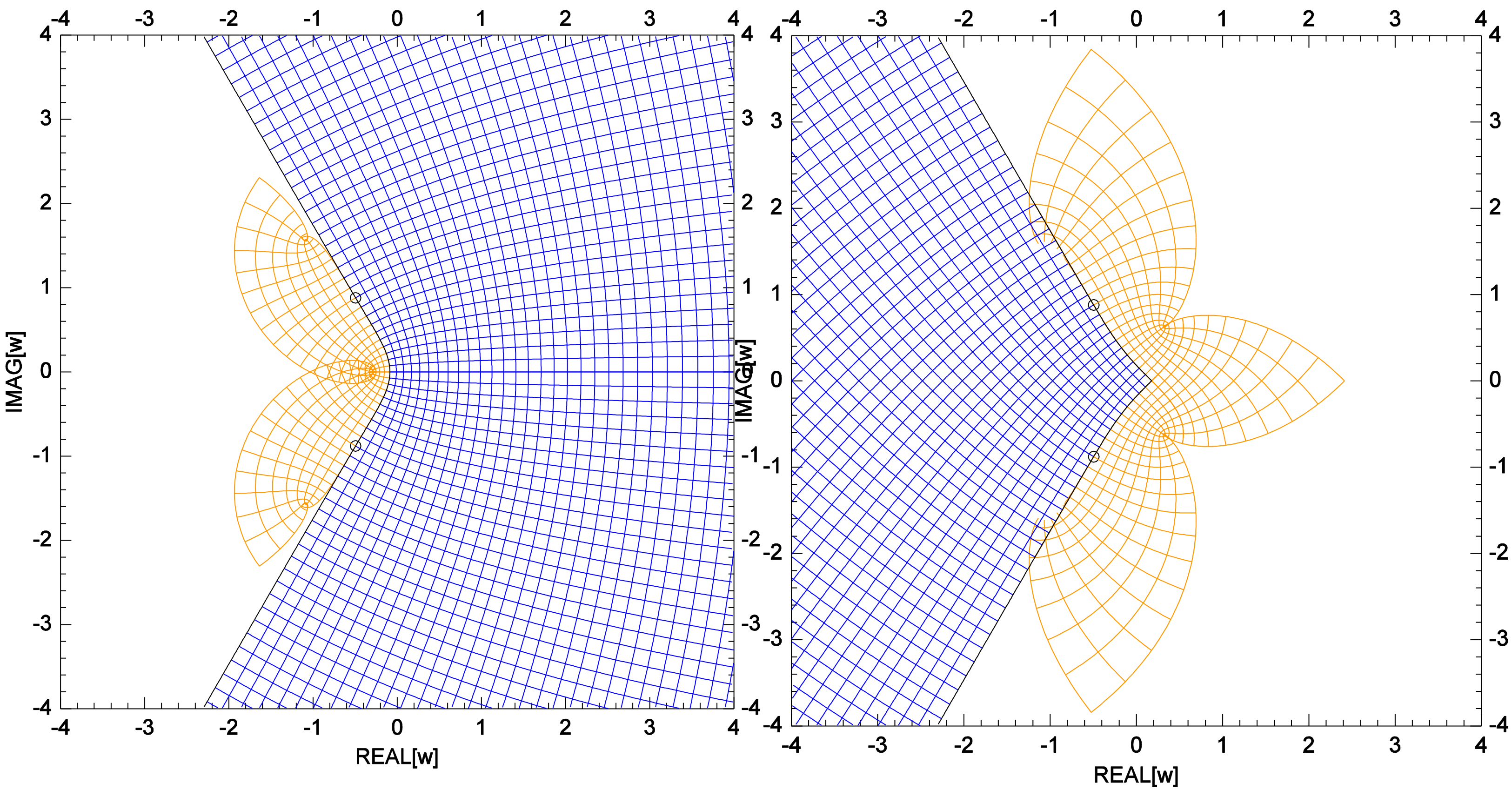


-1.0
-0.5
0
0.5
1.0

First derivatives are continuous at branch points (small circles)



Higher order versions allow a number of derivatives to remain continuous. But eventually, the half-odd integer term in the power law expansion about The branch points comes into effect.



The construction of these solutions relies on a suitable variant of the "Schwarz" method, using circles, or analytically deformed circles, to infer the power-law expansion coefficients (via Fourier transformation around the circuit formed by these paths).

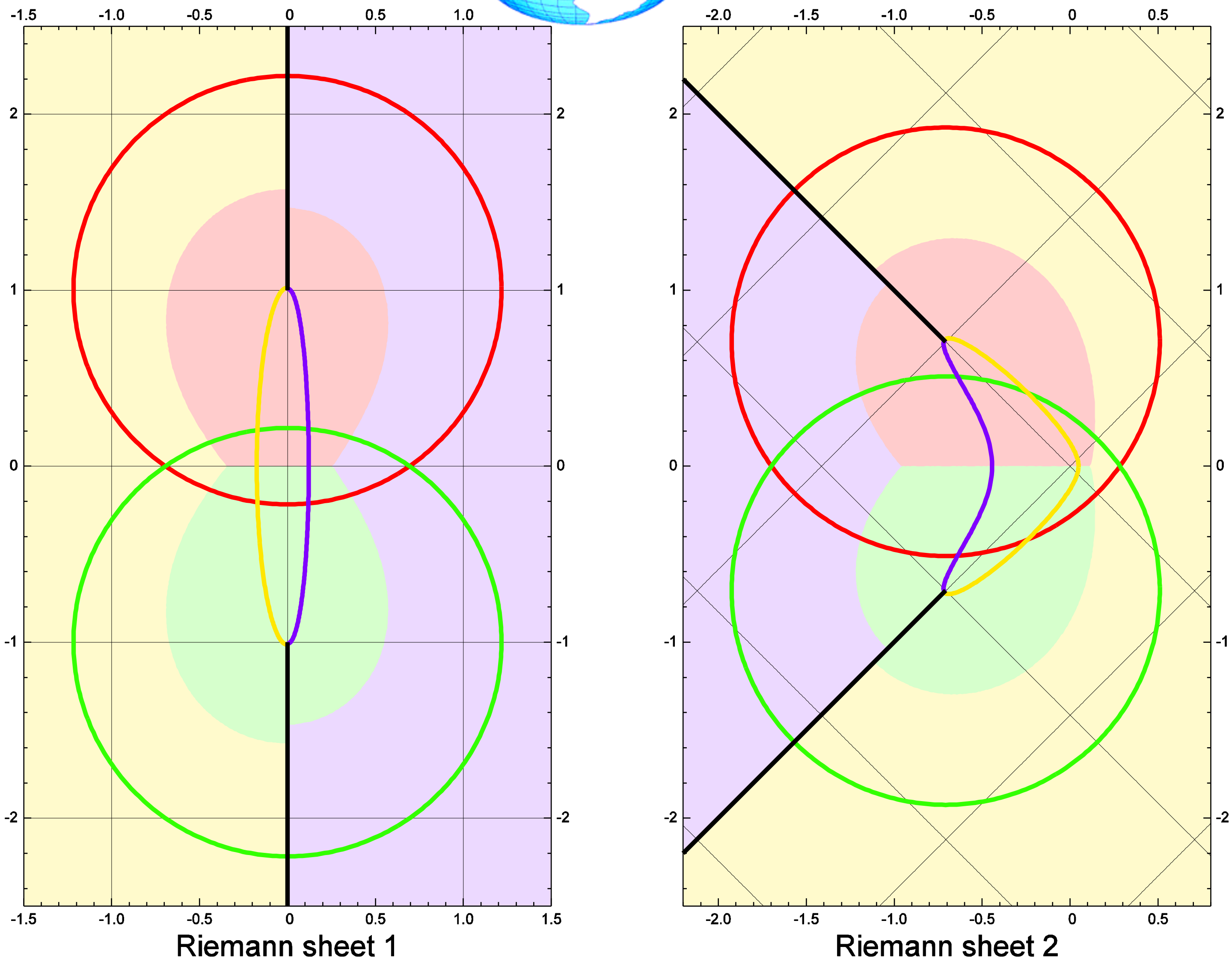
Very simple versions of these techniques are what we use (and described in Rancic et al 1996) to construct all the ordinary polyhedral conformal Mapping.

The introduction of the self-overlapping Riemann surfaces introduces a new level of practical and computational complexity, but does not alter the same basic principle we have been exploiting now for many years.

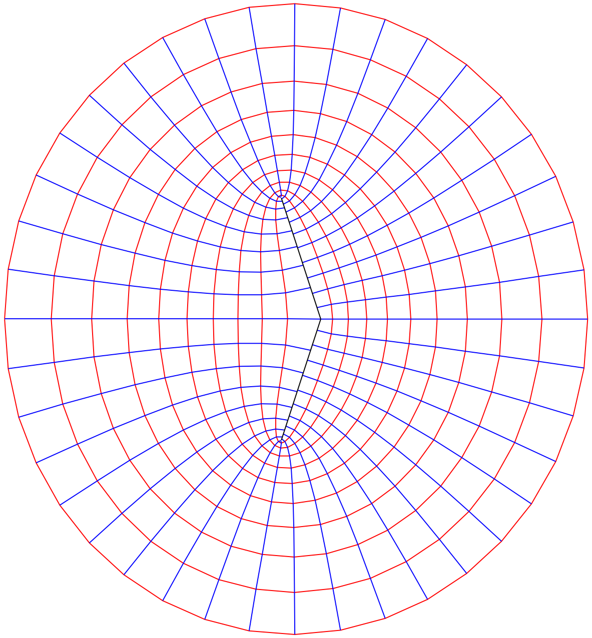
In the following diagram, we show the two sheets of the Riemann surface, together with four distinct smooth circuits (colored curves) threading around and between the two branch singularities in different ways.

The functional form of the mapping is determined by the far-field behavior, and the degree of continuity at the branch points (centers of the 720-degree "circles").

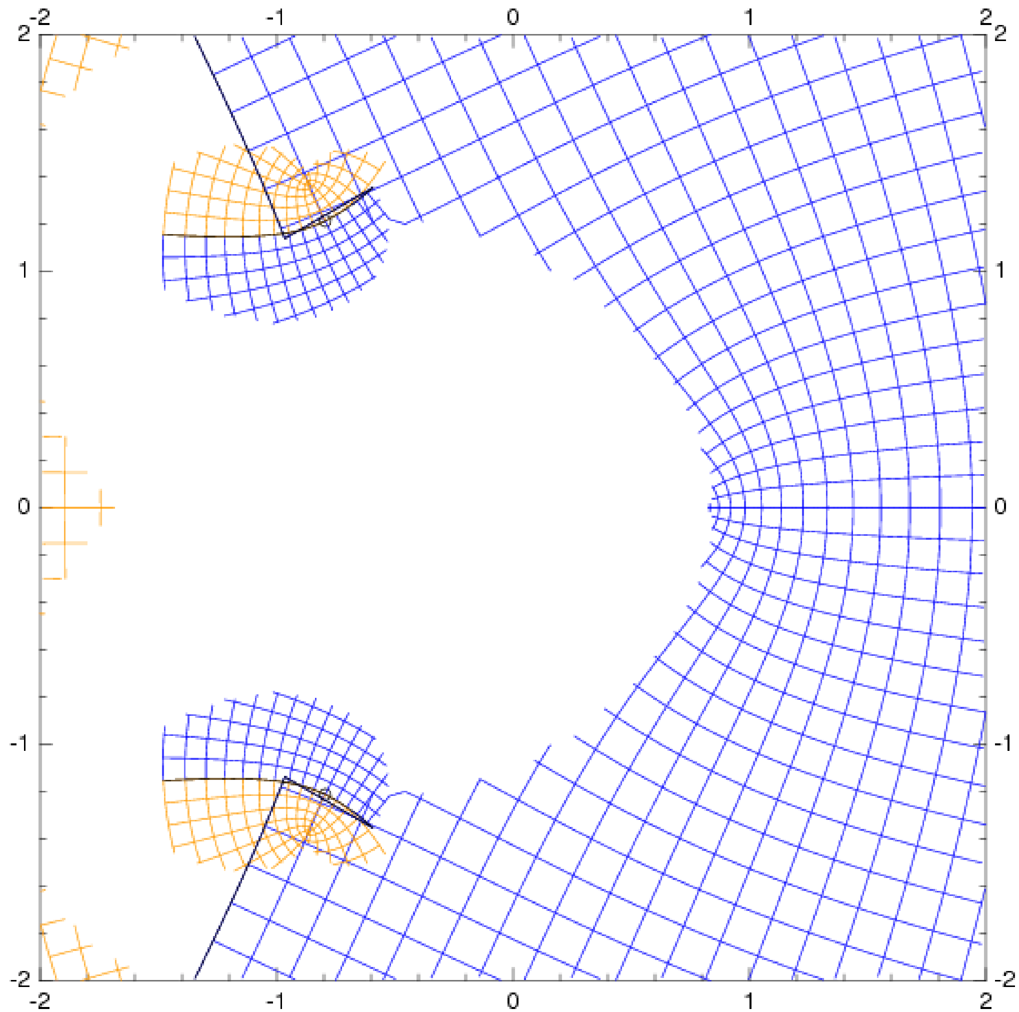
The solutions are iterated towards a state of self-consistency.


-1.5
-1.0
-0.5
0
0.5
1.0
1.5
2
1
0
-1
-2
Riemann sheet 1
-2.0
-1.5
-1.0
-0.5
0
0.5
2
1
0
-1
-2
Riemann sheet 2

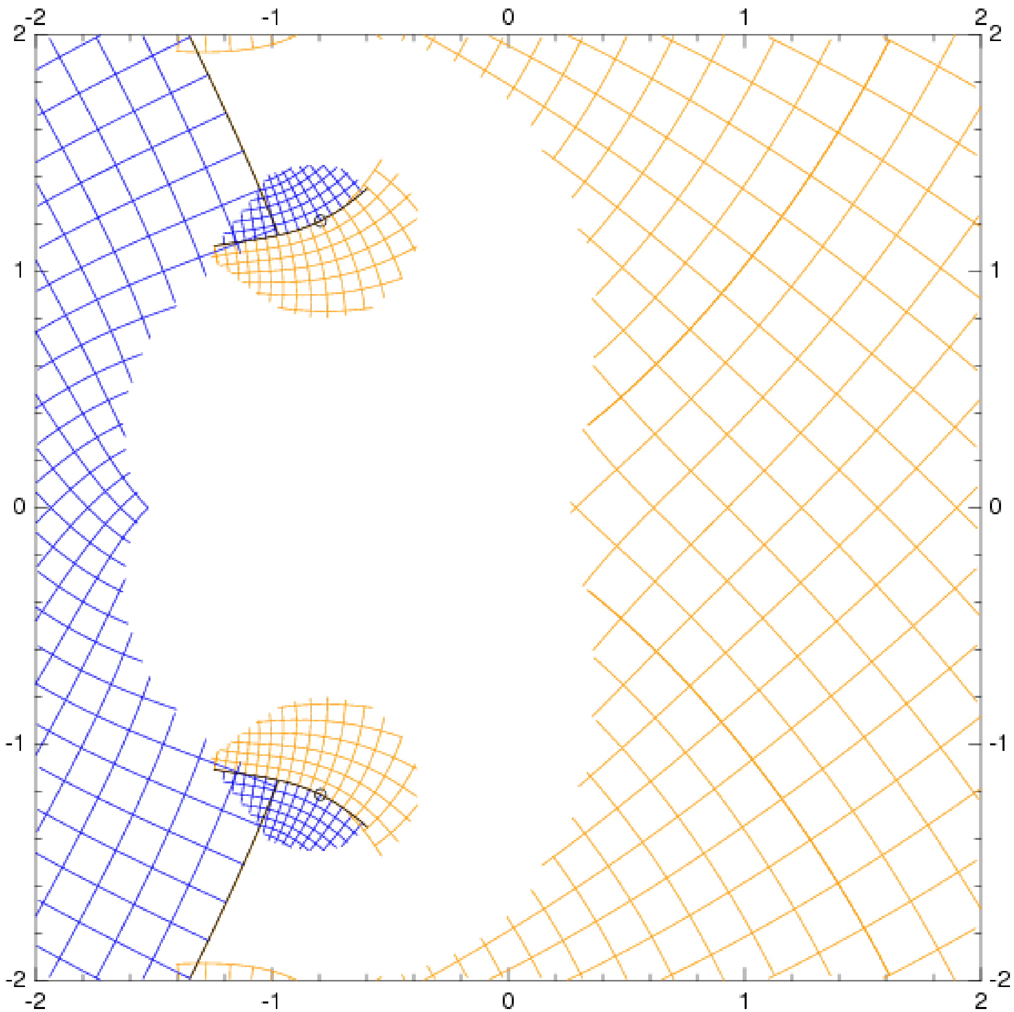
We construct what (for obvious reasons) we refer to as a “boomerang” function, mapping the exterior of an angled symmetric pair of unit segments to the interior of a true circle, CONFORMALLY.



The following animations show vividly how the solution “comes together” as the iterations proceed.



-2
-1
0
1
2
2
1
0
-1
-2



-2
-1
0
1
2

We have not had enough time to develop the numerical methods for integrating a model and reconciling the overlap by interpolation and merging, but we intend to work on this soon.

We are especially interested in the issues of conservation and potential numerical noise that might occur within the overlap areas.

But the singularities that remain are of a sufficiently attenuated form that they, at least, are unlikely to present us with serious numerical difficulties.

We thank you for your attention.