Sparse Grids for Spectral Elements using L-Galerkin Operators on Polygonal Grids

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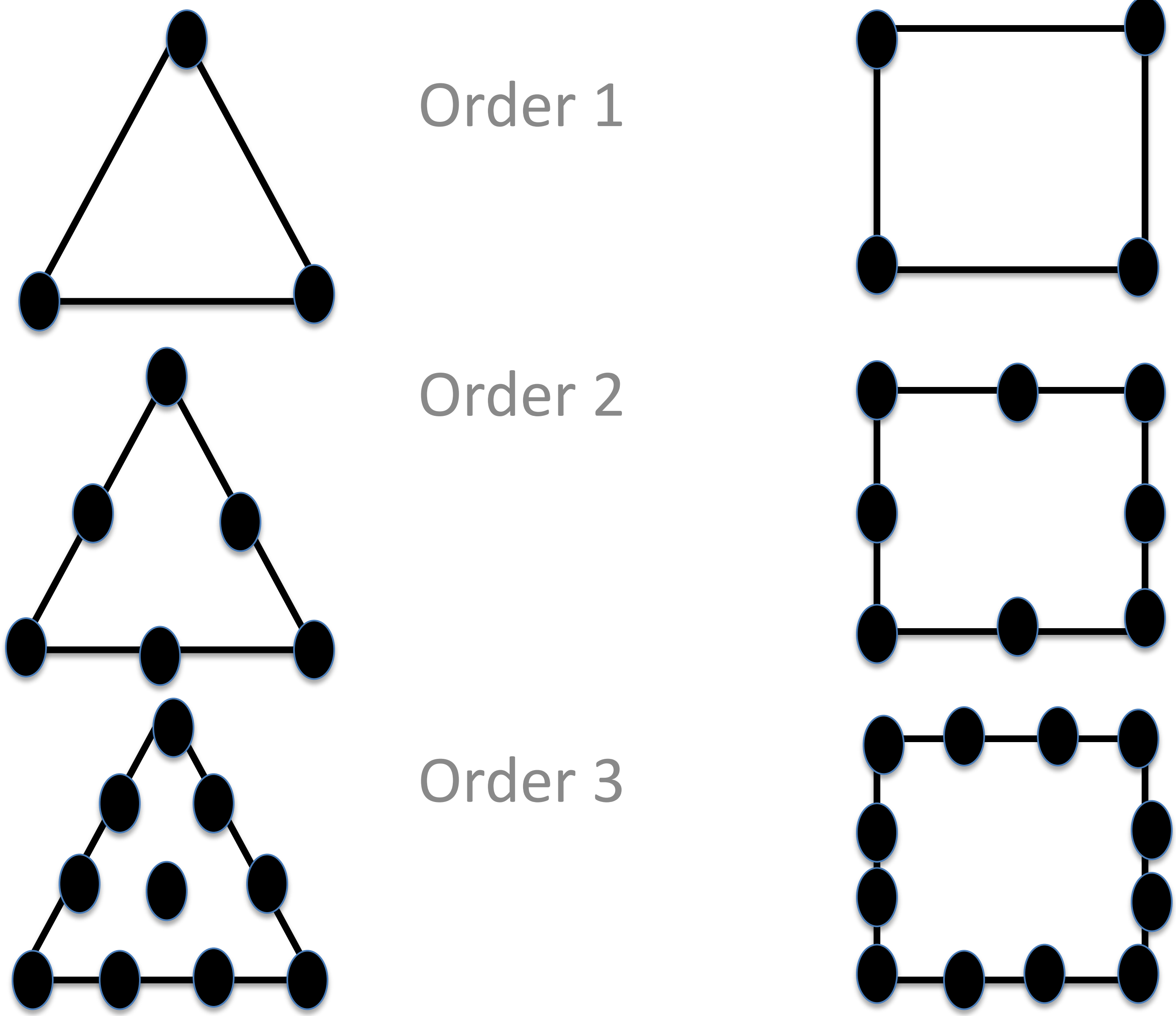
Plan of Lecture:

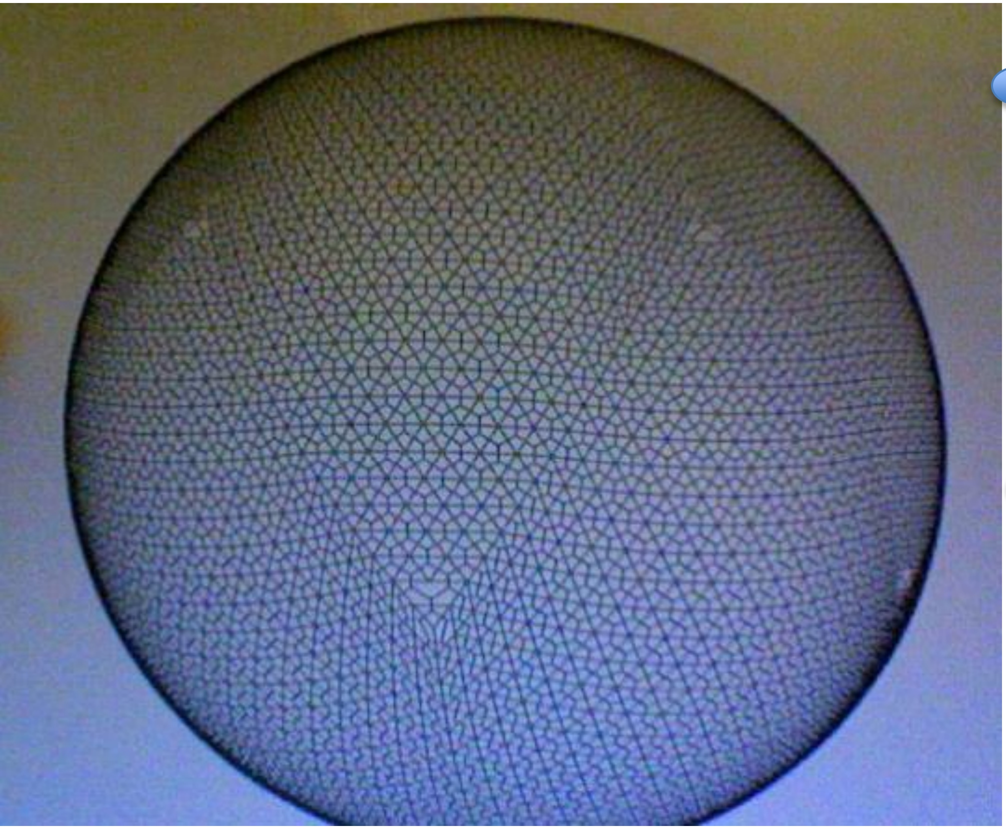
- SE schemes use a basis function representation
- Grid points < ==== > function amplitudes
- == > most schemes: FV, SE, FD, C-grid on icosahedon, Cubed sphere, ying yan etc. can be understood as SE schemes
- L-Galerkin on sparse grids are a generalization of point quadrature SE to achieve:
  – High order on very irregular grids and superconvergence
  – Efficiency by **sparseness**, **spectral procedure, better position of point cloud for high order differencing**
  – Conservation
  – Effective implementation of cut cells for any order

Sparse Grids / Serendipity

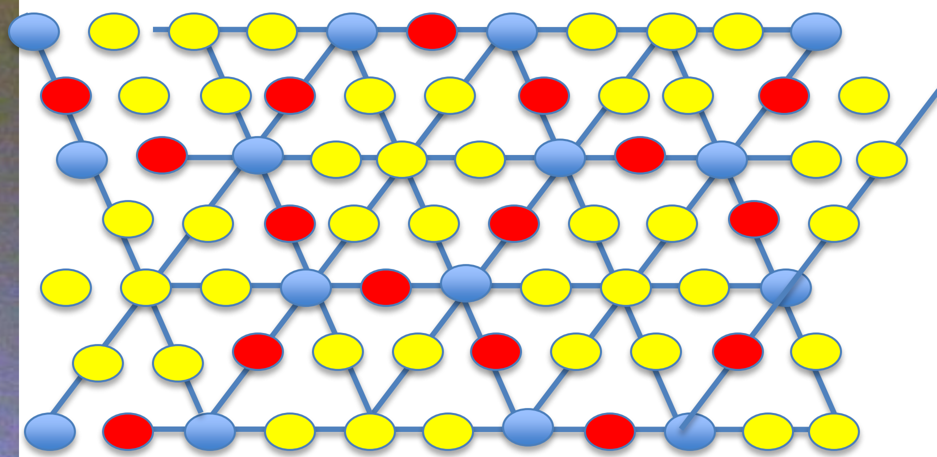


Sparse grids using diagnostic Points

Hexagonal Grids: o2



O2: full grid (yellow) **Corner points** (blue)
O2 points (red)



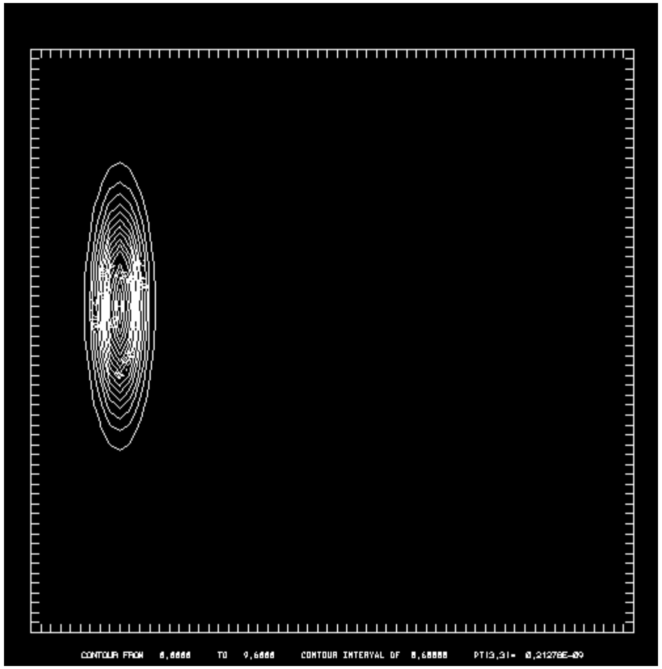
3-d, o2:

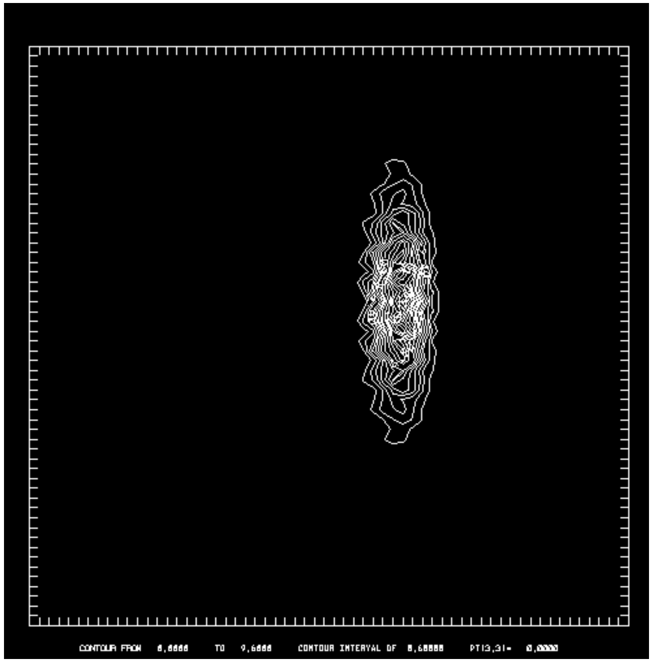
12 : 48

2-d

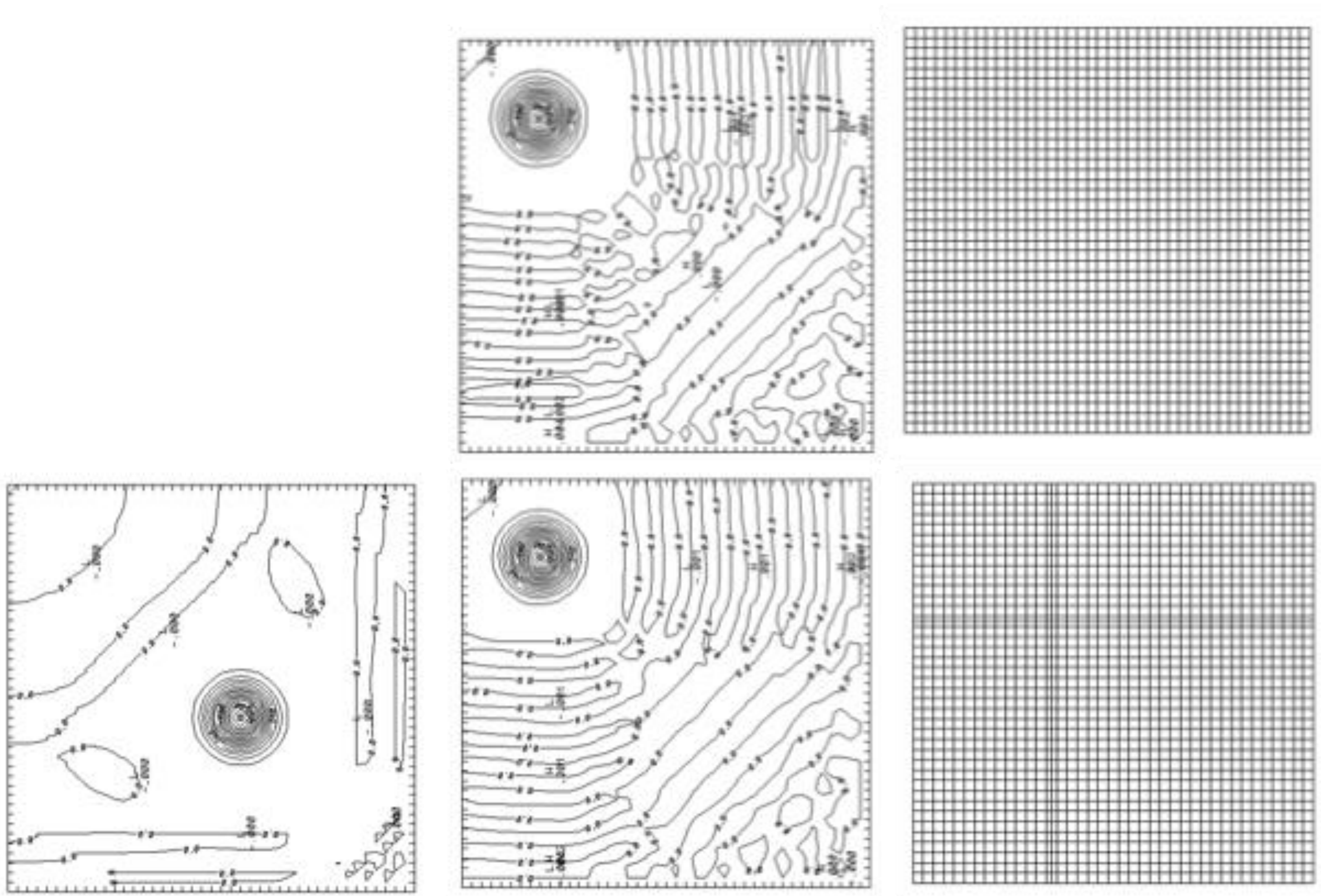
8 : 24

Hexagon sparse grid O2 / o3 advection in x-direction





Standard O3 L-Galerkin (equivalent to point quadrature) is suitable for irregular resolution



The L-Galerkin spectral elements

- SE: Standard spectral elements (Quadrature G-Lobatto points or averaging at discontinuities)
- L-Galerkin schemes are local and use basis functions: Pre-regularisation (simple o3), Mass conserving interpolation and more
- Example (pre-regularization): $\partial/\partial t\,\rho = \partial/\partial x\, RC\rho u$
- C: Collocation; R: Regularization O2-O3 superconvergence is possible

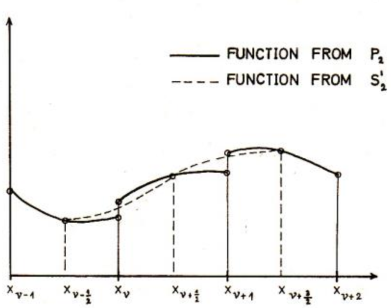


FIG. 2. Approximating functions from P_2 by functions from S_2.

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On a High Accuracy Finite Difference Method

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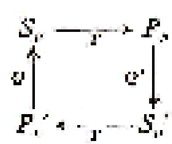
Numerical Procedure

A time step is just a time translation mapping T fol
shiftings Q or Q'. For example, one obtains in this wa
from a function in S_ν. In the next time step one agai



L-Galerkin procedure

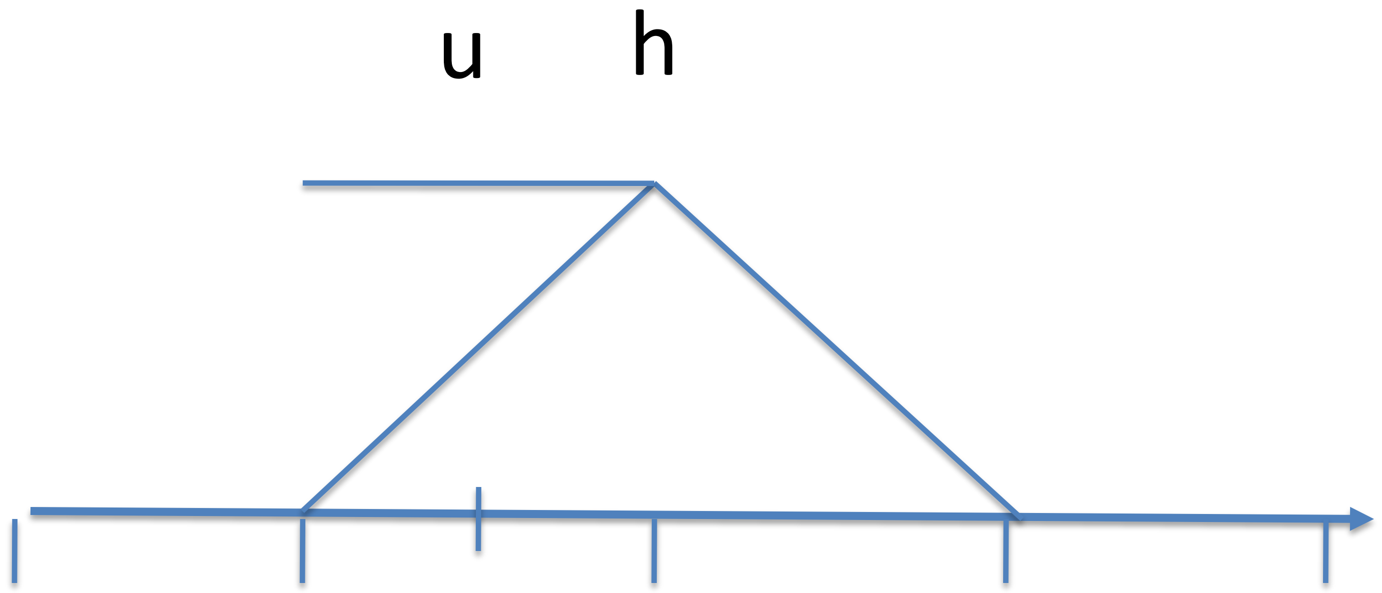
Numerical properties of different Serendipity Schemes

Table 1:

CFL numbers for different schemes

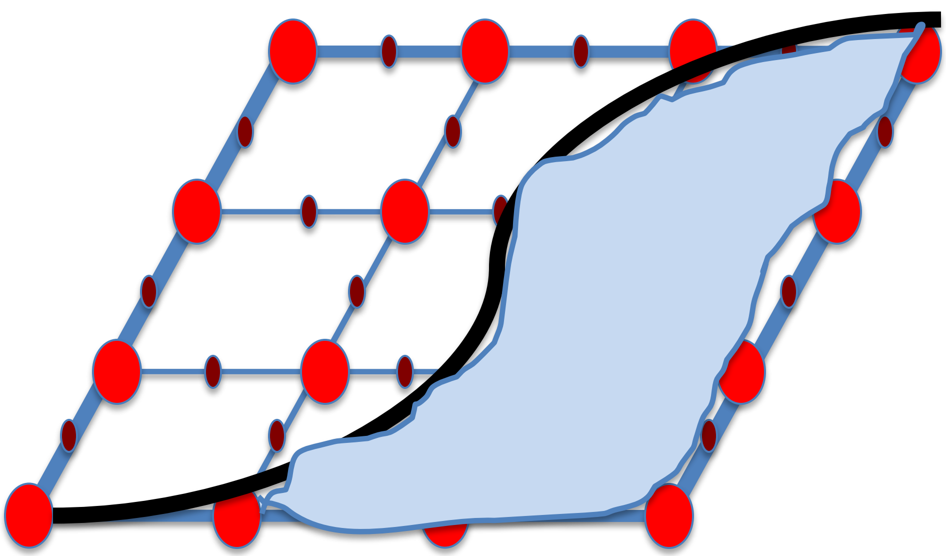
Scheme	CFL	conservation	suitable for irregular resolution
O1O1	2.8	yes	no
Classic O4:	2.1	not known	in 1-d
O2 Standard:	1.9	yes	yes
O2 with O4 diff:	4.	Yes	limited
O2 with O2/O4 diff:	1.4	yes	yes
O3 with O4 diff:	3.8	yes	limited
O3 standard	1.6	yes	yes

Basis functions for C-grid Scheme



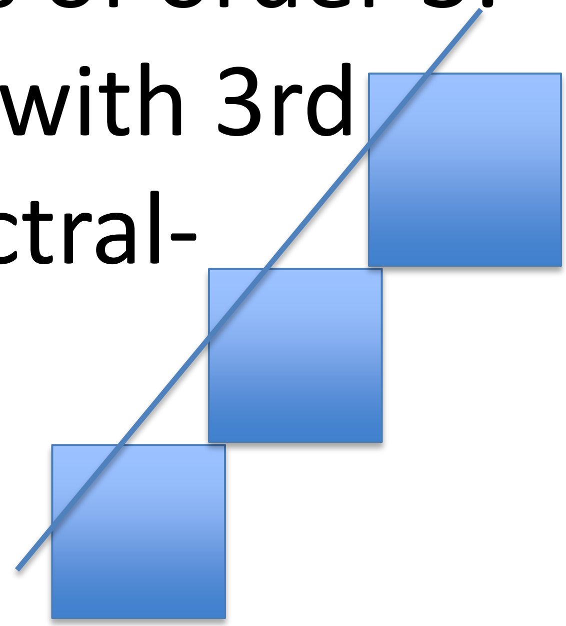
Cut cells in a high (2 nd) order grid

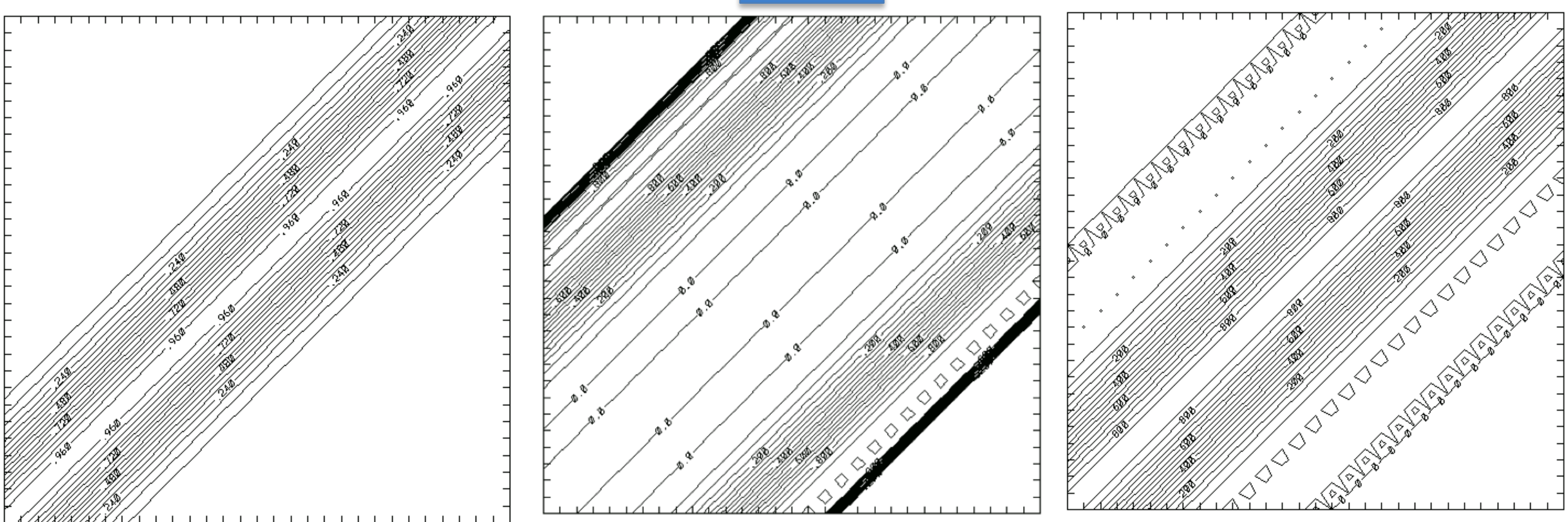
- Only a part of the cell is taken by the atmosphere



3rd spectral elements with cut cells

spectral elements of order 3:
=> cut cells work with 3rd order sparse Spectral-elements



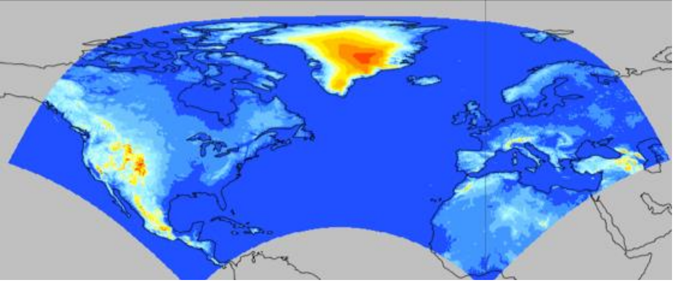


Why sparse L-Galerkin SE

- Construct high order conserving schemes, Numerical efficiency, cut cells of any order and for any polygonal cell (➔improved forecasts)
- For (continuous) **SE**: efficiency by sparseness, cut cells
- For **FD**: high order + conservation, cut cells in high order, irregular cell structure and small cells as occur in cut cells or ying yang

State of development

- Terrain following SE on Rhomboids: play models OK
- FD-hexagons: ongoing
- Cut cells: non conserving on C grid 3-d with real data OK



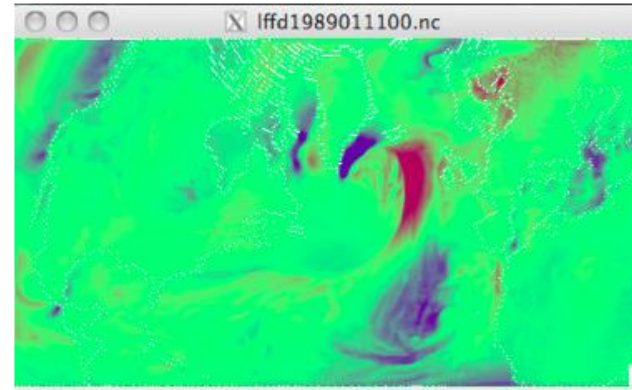
Impact of cut cells on cyclonic forecasts

For tropical forecasts see:

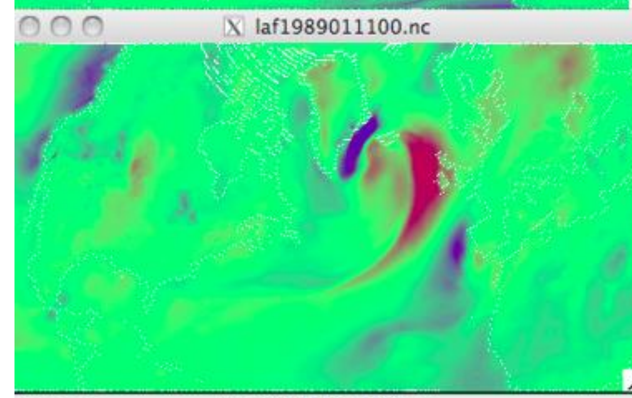
Atmos. Sci. lett. (2011), **<u>12</u>** *320pp;*

Geo. Mod. dev. (2013) **6** *875 pp*

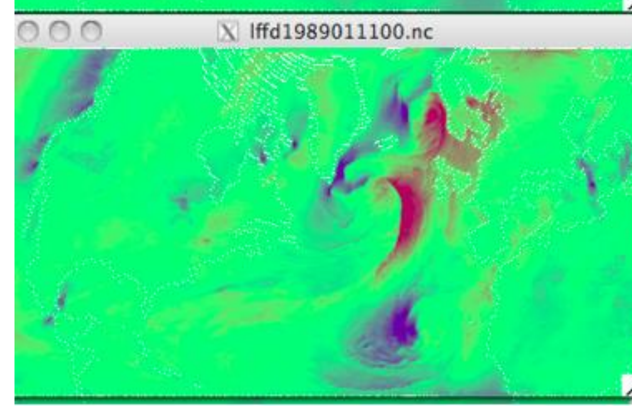
Cut cells 10 days, u10m



Observation

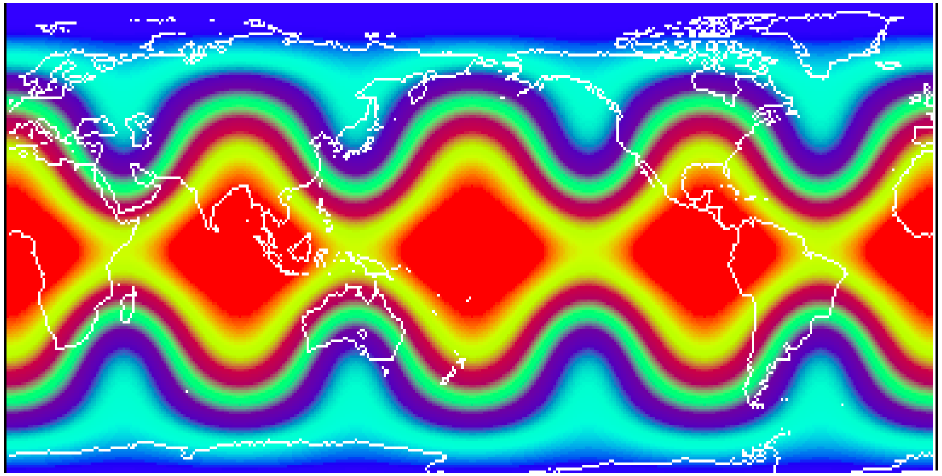


Terrain following, 10 days, u10m

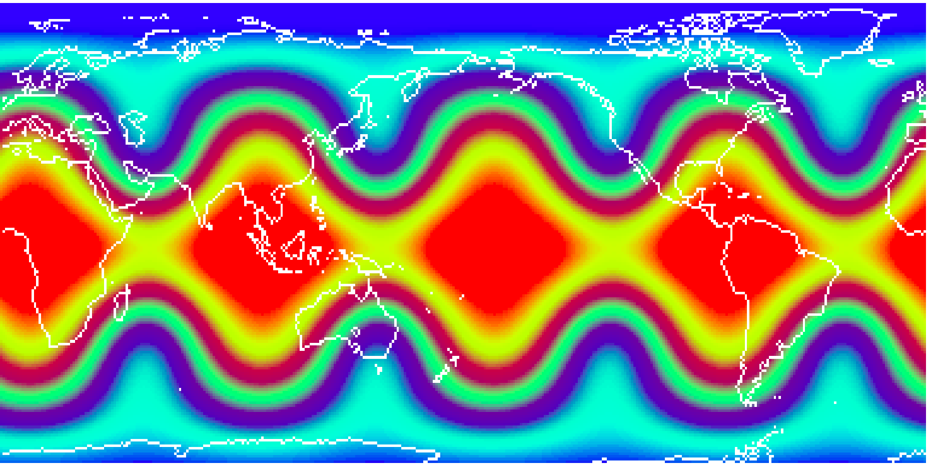


Sparse rhomboidal/conserving serendipity elements Test case 6

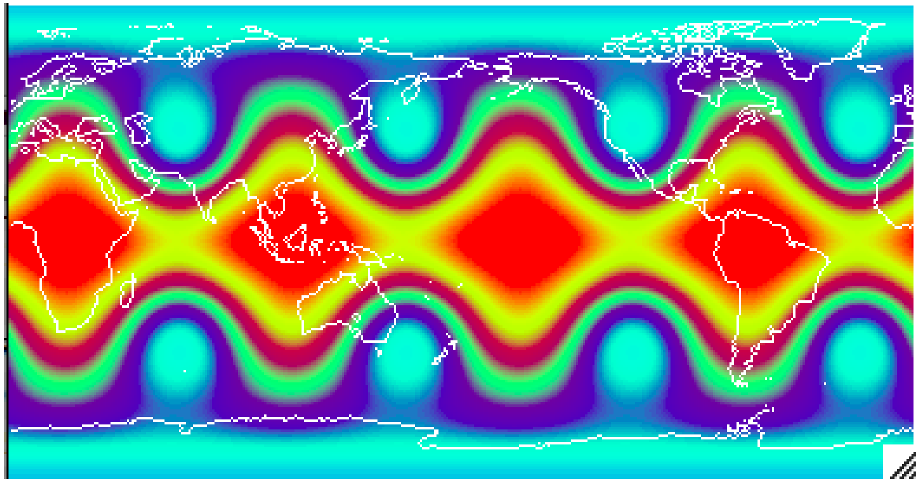
Initial



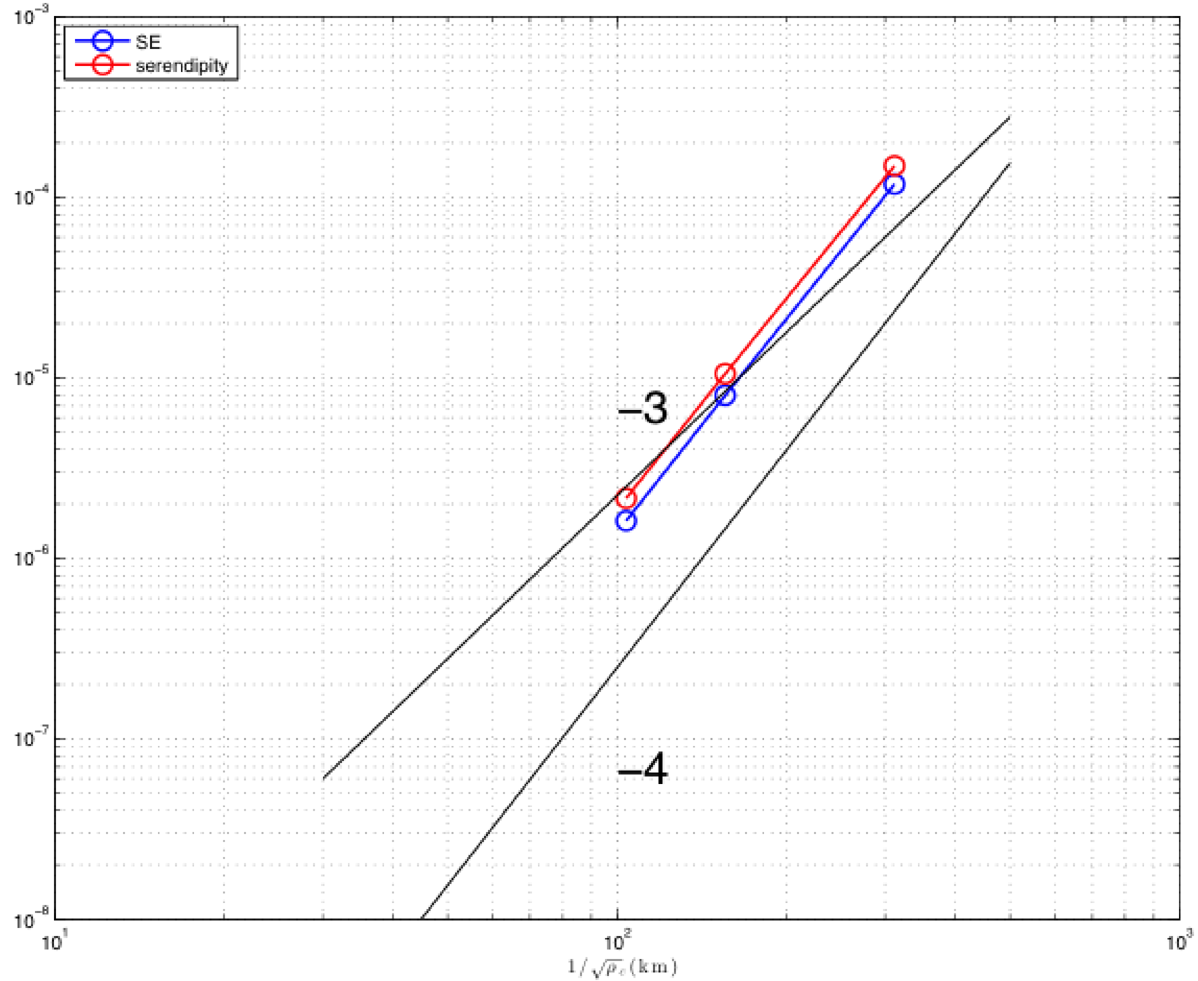
Day1



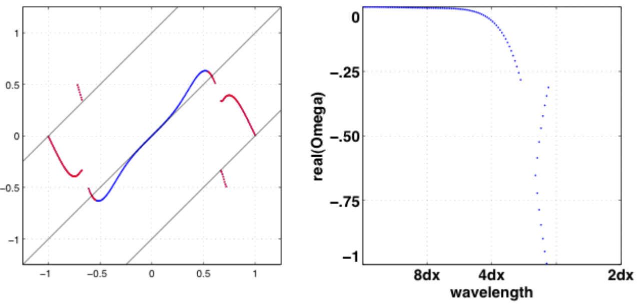
Day 10



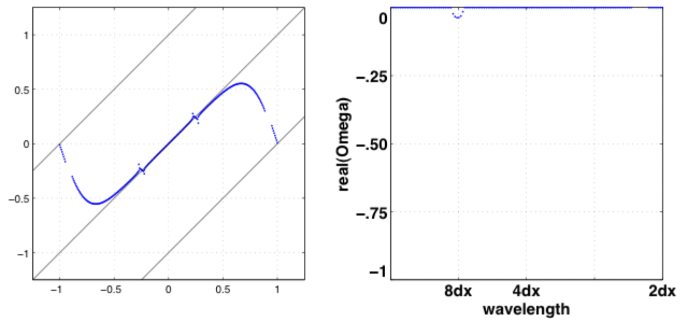
Third Order Convergence of Shallow Water Model at Day 3



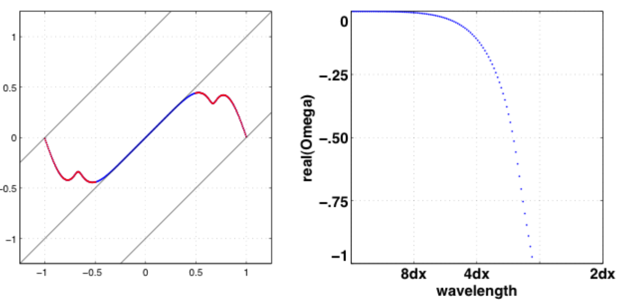
Simple O3: linear analysis, L-Galerkin



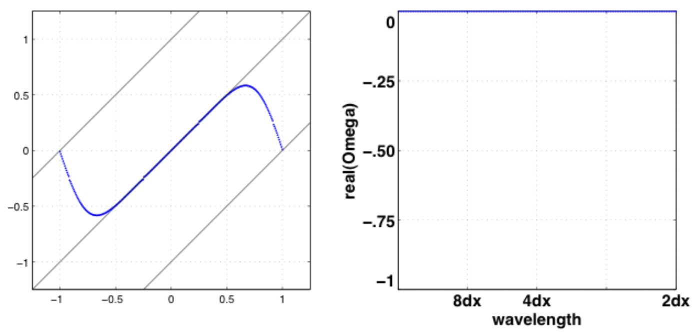
Eq spaced L-Gal, Diffusion o4



Eq spaced SE, no diffusion



Eq spaced SE, Diffusion o4



GL spaced SE no diffusion

Pre-Regularization (o3-grid)

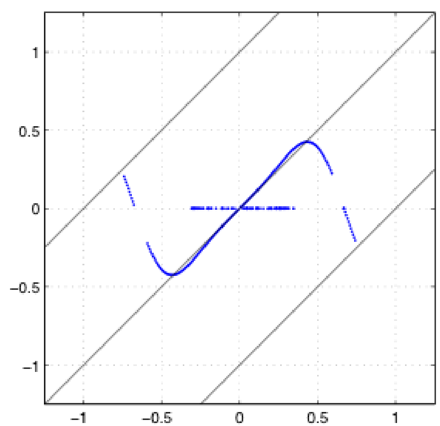
equally spaced method stable, CFL with RK4: 3.9

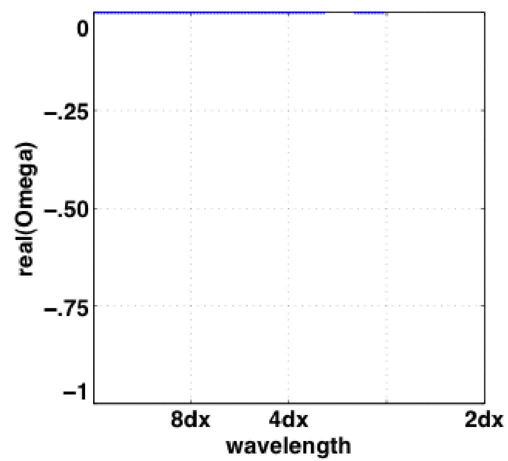
Comparison:

Standard o1 GL/SE: LA=2.7;normal Galerkin, O3 based: LA=2.

conservative eq spaced:LA=2.97;

eq. spaced/quadrature: unstable





Thank you for your attention