Riemann solver for the adjoint shallow water equations with discontinuous coefficients

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The application of the dual weighted residual method for adaptive refinement requires the evaluation of residuals of the primal and the adjoint equations of the underlying problem, see [1]. We consider the 2D shallow water equations with a discontinuous Galerkin method, as done in [2], which requires to solve a Riemann problem across cell boundaries. This approach can lead to discontinuous coefficients in the adjoint equation. Though the adjoint equations are linear, the time and space dependency, as well as the discontinuities keep the problem from being trivial. For this reason, we tested two different approaches:
First, we applied the Rusanov numerical flux [3], which requires the eigenvalues of the system. Since the eigenvalues of the adjoint equations are the velocity solutions of the primal problem, we can obtain them easily. Here, the discontinuities of the eigenvalues are avoided by choosing the largest eigenvalue on each element edge.
Another approach is the solution for discontinuous coefficients as presented in [4], as a mixture of upwind, downwind and weighted ansatz, depending on the positive or negative value of the two coefficients on the element edge. We transfered this method from 1D to 2D and applied it to our test case, a simplified cyclone track benchmark problem, which is highly sensitive to the grid refinement.

References

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