

Abstract

Two strategies for the mitigation of coordinate singularities of a spherical polyhedral grid.

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The symmetric conformal mapping of the gridded surface of a polyhedron, such as the cube or the icosahedron, to the sphere entails a moderately strong coordinate singularity at each vertex. The problem can be considered to have two distinct aspects: the grid area (or Jacobian) becomes singular, with progressively smaller geographical areas associated with each grid cell approaching the vertex; the grid curvature becomes singular towards the vertex.

We have pursued two strategies to alleviate the numerical difficulties. On the one hand, we have developed a new scheme guaranteed to produce, in a unique way, a perfectly equal-area mapping for any sufficiently symmetric polyhedron or dihedron. This is done by following the symmetry-preserving conformal mapping with a symmetry-preserving “map migration” process. Here, the mapped grid on the sphere evolves according to an irrotational flow for a unit period of a time-like parameter, with the geographical distribution of divergence of this flow, apart from the spatially independent part needed to maintain a zero global integral, being the negative logarithm of the Jacobian of the conformal solution at each advecting grid point. The map migration flow is itself evolving according to this definition during the unit period, since the divergence pattern is subjected to the flow that it is forcing. The result is a polyhedral mapping for which the Jacobian singularity is entirely removed (the Jacobian becomes uniform) but the singularity in the curvature of the grid remains. On the other hand, we have developed an alternative resolution of the mapping difficulty through an application of the method of complex analytic functions, in which conformality is preserved, but the vertex singularities are avoided by the judicious placement of a pair of weak branch-point singularities approximately straddling the position of the former vertex, and having the modified mapping function become self-overlapping on a so-called “Riemann surface”. The result in this case is a mapping configuration that naturally lends itself to an “overset grid” technique, including a configuration for the cubic case resembling the “Yin-Yang” overset, except that the grid between adjacent square panels join in a perfectly smooth and continuous way except within a compact region covering the former vertex positions where solution blending (of the kind familiar in the conventional Yin-Yang configuration) is still needed to reconcile the overlaps. This second strategy avoids both the Jacobian and curvature singularities and furthermore preserves the advantageous feature of perfect conformality. Moreover, it can be adapted to complicated (nonconvex) polyhedral configurations, in such a way as to enable a single smooth conformal global grid to exhibit multiple compact regions of enhanced resolution at chosen locations of particular interest or dynamical significance. The only coordinate singularities of

such a configuration are the branch-points, at which several (although finitely many) derivatives still remain continuous, and which are therefore almost imperceptibly weak as far as model numerical processes are concerned.

We plan to present the grid construction methods for both mitigation strategies, and some preliminary results using an atmospheric model.