A consistent framework for discrete integrations of soundproof and compressible PDEs of all-scale atmospheric dynamics

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A numerical framework is developed for consistent integrations of soundproof and the fully compressible nonhydrostatic equations of motion for all-scale atmospheric flows. The reduced anelastic and pseudo-incompressible soundproof equations and the fully compressible Euler equations are combined into a common form of conservation laws for mass, momentum and entropy that facilitates the design of a sole principal algorithm for its integration, with minimal alterations for accommodating each special case. The development extends a proven numerical framework for integrating the soundproof equations. It relies on non-oscillatory forward-in-time transport methods, applied consistently to all dependent variables of the system at hand, and with buoyant and rotational modes of motion treated implicitly in the integration. When the fully compressible equations are solved, the framework admits congruent schemes with explicit or implicit representation of acoustic modes, so the former can provide a reference for the latter. The consistency of the framework minimises the numerical differences between the soundproof and compressible integrators, thus admitting conclusive comparisons between compressible and soundproof solutions, unobscured by algorithmic disparities. For the large-time-step implicit schemes, technical differences between the soundproof and compressible integrators reduce to the selection of either a prescribed or a numerically prognosed density, and extension of the generalised Poisson solver to a bespoke Helmholtz solver. The numerical advancements and merits of the approach are illustrated with canonical simulations of planetary baroclinic instability, an archetype of global weather, and the breaking of a deep stratospheric gravity waves, an example of nonhydrostatic mesoscale dynamics.