An accurate and efficient numerical framework for adaptive numerical weather prediction

We have introduced an accurate and efficient discretization approach for the shallow water equations on the sphere (extending to spherical geometry the technique proposed in Tumolo et al. JCP 2013), as well as for the non-hydrostatic Euler equations on a vertical slice, that can be effectively applied to all geophysical scale flows.

We combine a semi-Lagrangian approach with a novel TR-BDF2 based semi-implicit time integrator and with a spatial discretization based on adaptive discontinuous finite elements on hierarchical bases.

The resulting method is unconditionally stable and has full second order accuracy in time, thus improving standard off-centered trapezoidal rule discretizations without any major increase of the computational cost nor loss in stability, while allowing the use of time steps up to 50 times larger than those required by stability for explicit methods applied to corresponding DG discretizations.

The method also has arbitrarily high order accuracy in space and can effectively adapt the number of degrees of freedom employed in each element in order to balance accuracy and computational cost. The p-adaptivity approach employed does not require remeshing and is especially suitable for applications, such as numerical weather prediction, in which a large number of physical quantities has to be reconstructed on the mesh from the available data.

Furthermore, although the proposed method can be implemented on arbitrary unstructured and nonconforming meshes, like reduced Gaussian grids employed by spectral transform models, even in applications on simple Cartesian meshes in spherical coordinates the p-adaptivity approach can cure effectively the pole problem by reducing the polynomial degree in the polar elements, yielding a reduction in the computational cost that is comparable to that achieved with reduced grids.

Numerical simulations of classical shallow water and non-hydrostatic benchmarks have been employed to validate the method and to demonstrate its capability to achieve accurate results even at large Courant numbers, while reducing the computational cost thanks to the adaptivity approach. Preliminary results for the non-hydrostatic equations are also available, making the proposed method a good candidate for providing the basis of an accurate and efficient adaptive weather prediction system.

Work done in collaboration with L. Bonaventura (MOX-Politecnico di Milano).