Towards an Understanding of Atmospheric Balance

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Why is the extra-tropical troposphere approx. quasi-geostrophic ?

The stability of quasi-geostrophic flow with respect to ageostrophic perturbations

Derivation of (2-layer, f_plane) PE in terms of Normal Modes

Errico JAS 1981

$$\frac{d}{dt} b_{\mathbf{K}} = \sum_{\mathbf{L},\mathbf{M}} [C_1 b^*_{\mathbf{L}} b^*_{\mathbf{M}} + C_2 g^*_{\mathbf{L}} g^*_{\mathbf{M}} + C_3 g^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_3^* g^*_{\mathbf{L}} d^*_{\mathbf{M}} + C_4 a^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_4^* d^*_{\mathbf{L}} d^*_{\mathbf{M}} + C_5 a^*_{\mathbf{L}} d^*_{\mathbf{M}}],$$

$$\frac{d}{dt} g_{\mathbf{K}} = \sum_{\mathbf{L},\mathbf{M}} [C_6 b^*_{\mathbf{L}} g^*_{\mathbf{M}} + C_7 b^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_7^* b^*_{\mathbf{L}} d^*_{\mathbf{M}}],$$

$$\frac{d}{dt} a_{\mathbf{K}} = i\omega_K a_{\mathbf{K}} + \sum_{\mathbf{L},\mathbf{M}} [C_8 b^*_{\mathbf{L}} g^*_{\mathbf{M}} + C_9 b^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_{10} b^*_{\mathbf{L}} d^*_{\mathbf{M}}],$$

$$d_{\mathbf{K}} = a^*_{-\mathbf{K}}.$$

$$C_1 = -\tfrac{1}{2}\mathbf{L} \times \mathbf{M}(L^{-2} - M^{-2}),$$

$$C_2 = -\tfrac{1}{2}\mathbf{L} \times \mathbf{M}(\omega_L^{-2} - \omega_M^{-2}),$$

$$C_3 = -(M^2\omega_M^2\omega_L^2)^{-1}[\mathbf{L} \times \mathbf{M}(M^2 - L^2) - i\omega_M(L^2\mathbf{M}\cdot\mathbf{K} + M^2\mathbf{L}\cdot\mathbf{K})],$$

$$C_4 = \tfrac{1}{2}\omega_M^{-2}\omega_L^{-2}[\mathbf{L} \times \mathbf{M}(M^{-2} - L^{-2})(1 + \omega_M\omega + i(\omega_L + \omega_M)(M^{-2}\mathbf{M}\cdot\mathbf{K} + L^{-2}\mathbf{L}\cdot\mathbf{K})],$$

$$C_5 = \omega_M^{-2}\omega_L^{-2}[\mathbf{L} \times \mathbf{M}(M^{-2} - L^{-2})(1 - \omega_M\omega_L) + i(\omega_L - \omega_M)(M^{-2}\mathbf{M}\cdot\mathbf{K} + L^{-2}\mathbf{L}\cdot\mathbf{K})],$$

$$C_6 = -\mathbf{L} \times \mathbf{M}L^{-2}\omega_M^{-2}(\omega_M^2 - \omega_L^2 + 1),$$

$$C_7 = \omega_M^{-2}M^{-2}(\mathbf{L} \times \mathbf{M} + i\omega_M\mathbf{K}\cdot\mathbf{M}),$$

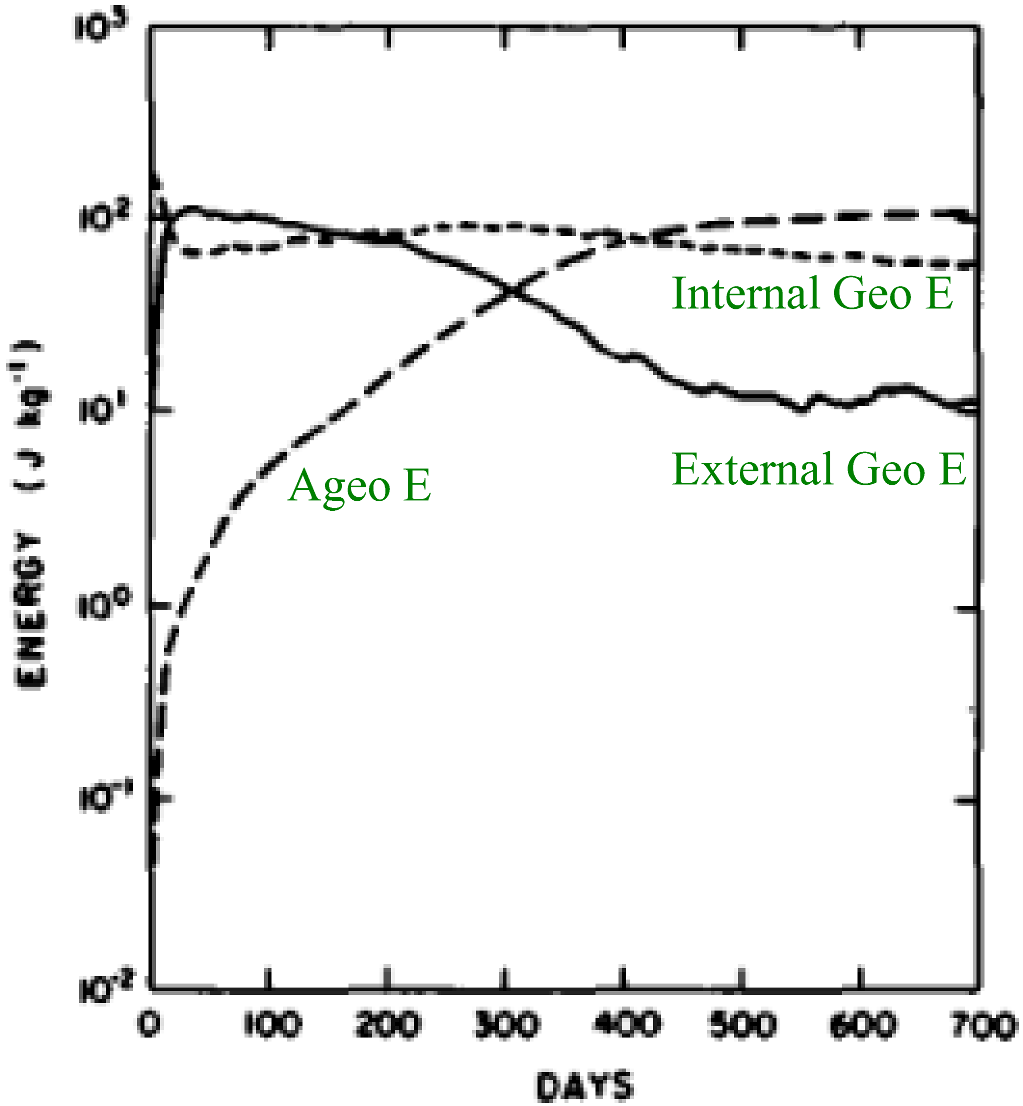
$$C_8 = -\omega_M^{-2}L^{-2}\mathbf{L} \times \mathbf{M}(\mathbf{K}\cdot\mathbf{L} - i\omega_K\mathbf{L} \times \mathbf{M}),$$

$$C_9 = (2\omega_M^2L^2M^2)^{-1}\{\mathbf{L} \times \mathbf{M}[L^2(1 - \omega_K\omega_M) - \omega_K(\omega_K M^2 - \omega_M K^2)] + i[2\omega_K(\mathbf{L} \times \mathbf{M})^2 + \omega_M L^2\mathbf{K}\cdot\mathbf{M}]\},$$

$$C_{10} = (2\omega_M^2L^2M^2)^{-1}\{\mathbf{L} \times \mathbf{M}[L^2(1 + \omega_K\omega_M) - \omega_K(\omega_K M^2 + \omega_M K^2)] + i[2\omega_K(\mathbf{L} \times \mathbf{M})^2 - \omega_M L^2\mathbf{K}\cdot\mathbf{M}]\}.$$

Demonstration of equipartition in an adiabatic model

Errico *Tellus* 1984



What are the characteristics of atmospheric balance?

To which normal modes should balance be applied?

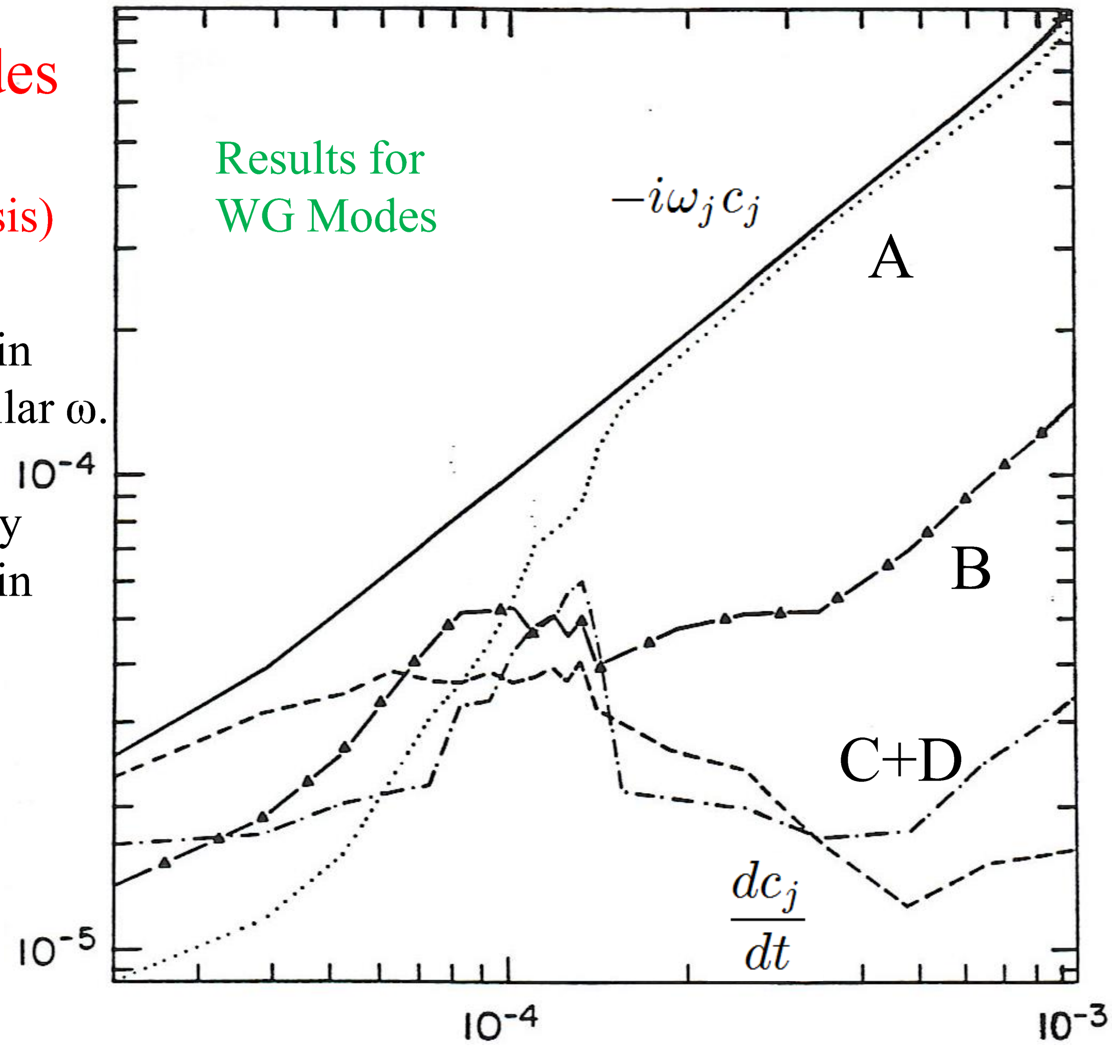
$$\frac{dc_j}{dt} = -i\omega_j c_j + A(r,r) + B(r,g) + C(g,g) + D$$

Balance of WG modes in a climate model

(a sophisticated scale analysis)

Term magnitudes averaged in time and within bins of similar ω.

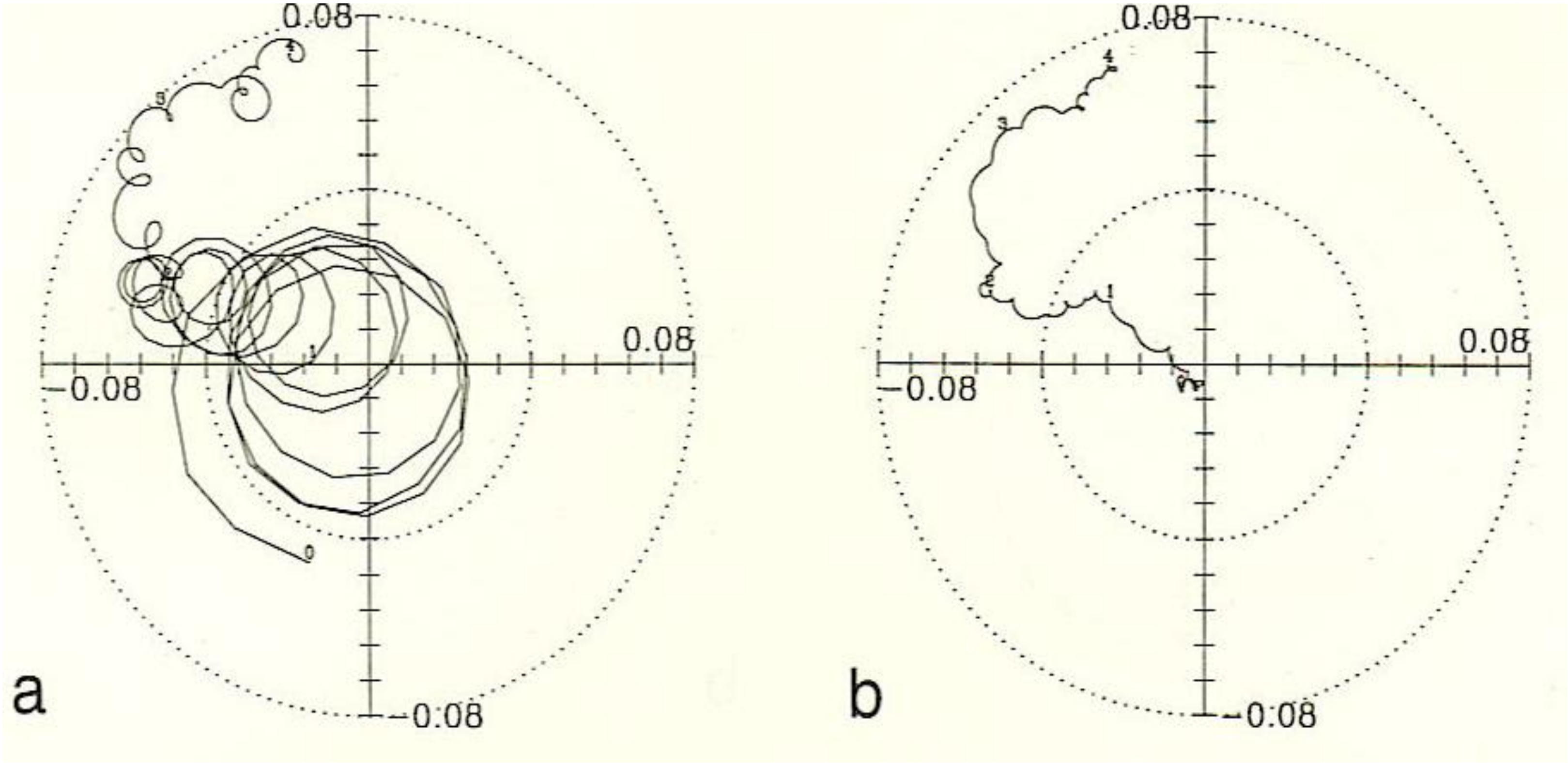
Plotted values normalized by mean energy of modes within the ω bin, so units are 1/s



Errico 1984, 1990 *MWR*; Errico et al. *1988* MWR

Harmonic Dial for External m=4 Mode, Period=3.7h

4-day forecast without NNMI With NNMI



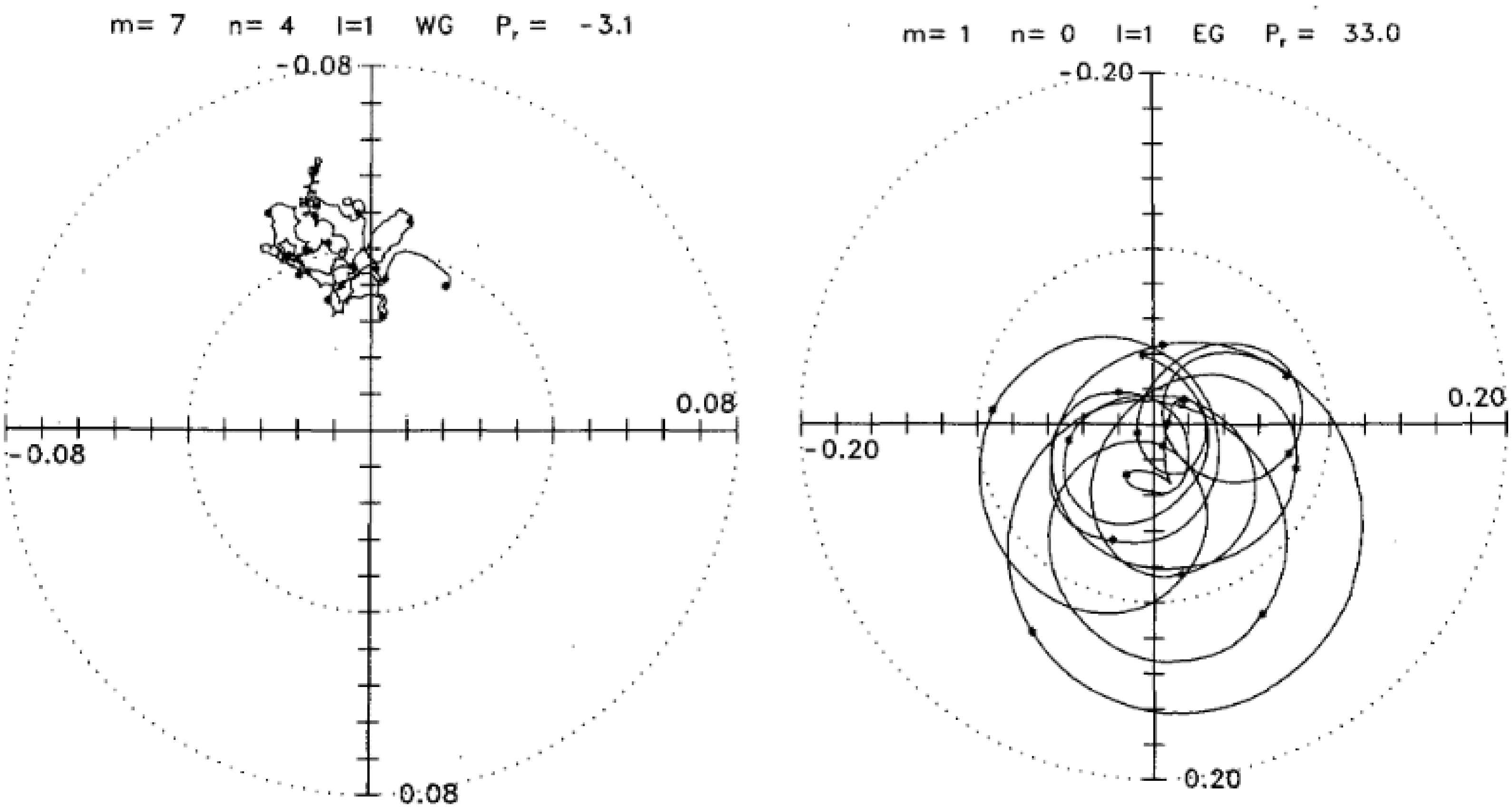
Errico and Williamson 1988 *MWR*
Errico 1997 *J Japan Met Soc*

How do moist diabatic processes affect balance ?

How should NMMI consider diabatic heating?

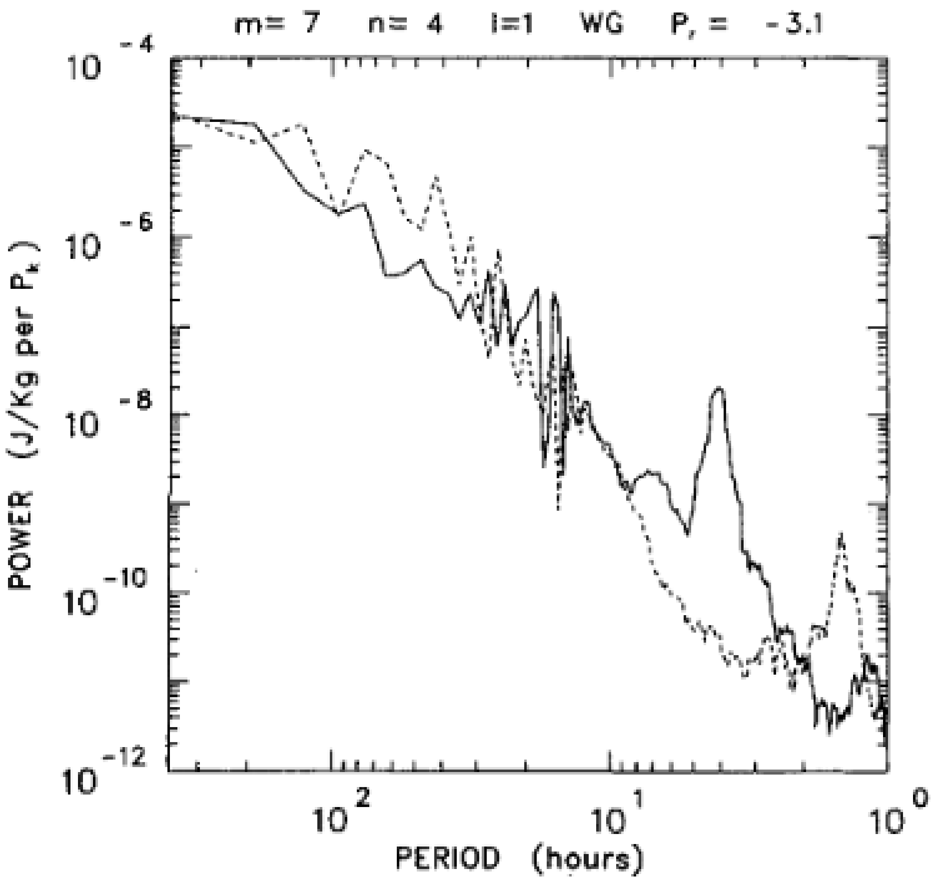
Behavior of gravitational modes in a climate model:
Time series (harmonic dials) of complex mode amplitudes
Errico MWR 1989

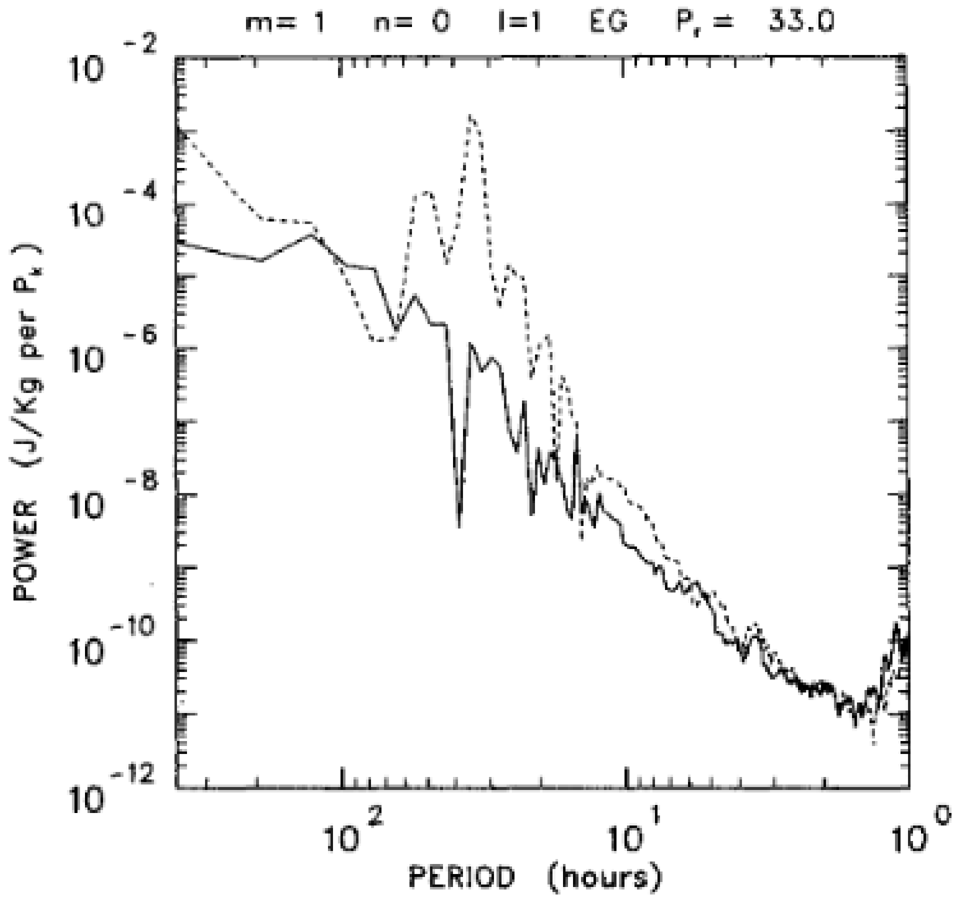
16-day simulation shown



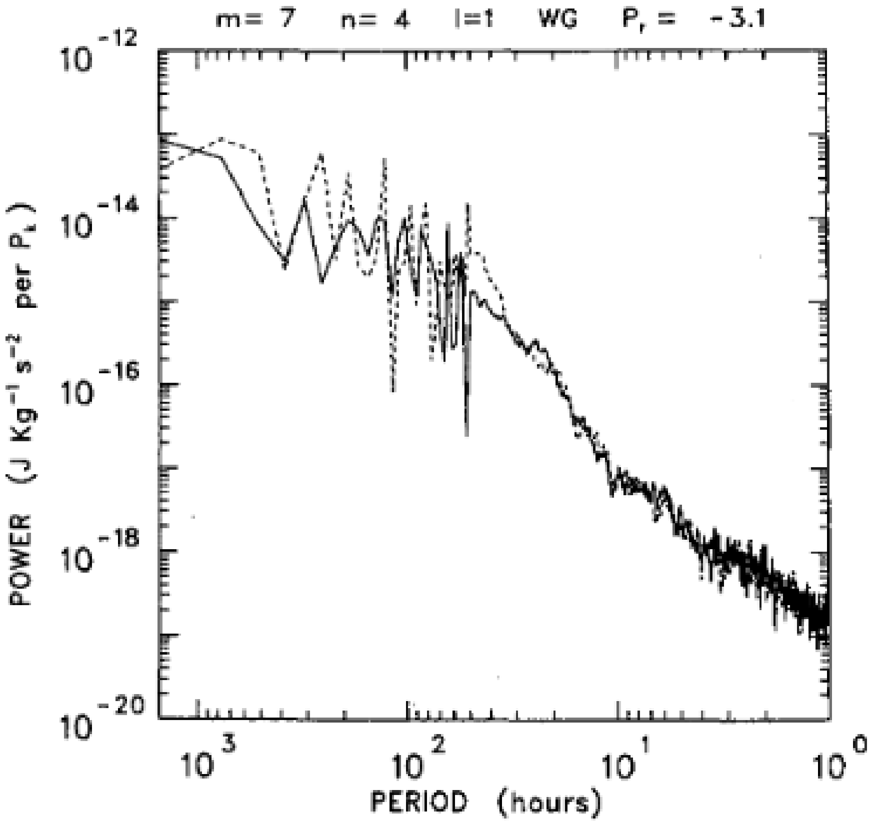
Behavior of gravitational modes in a climate model:
Power spectra of complex mode amplitudes
Errico MWR 1989

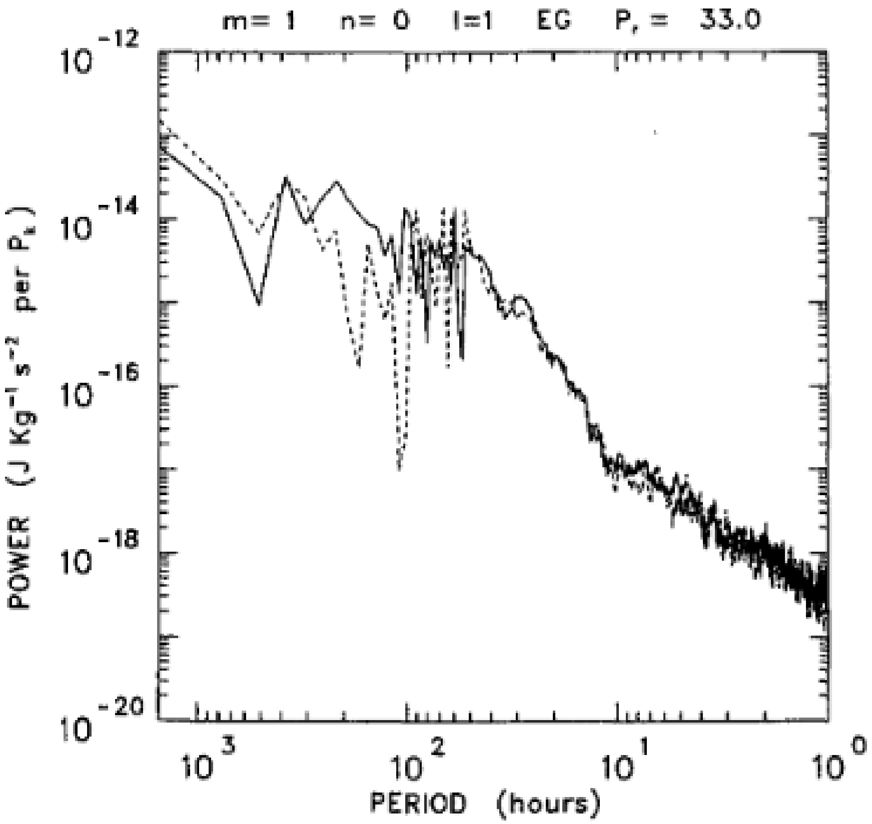
Solid: Westward propagating   Dashed: Eastward propagating





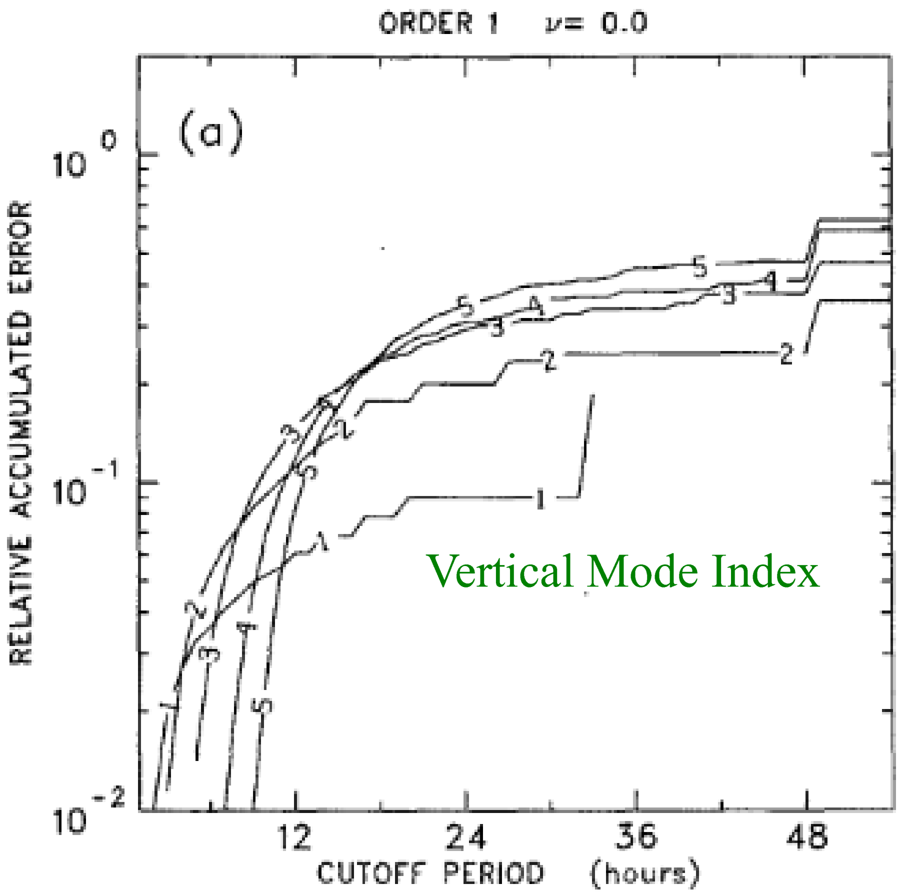
Behavior of gravitational modes in a climate model:
Power spectra of convective heating
Errico MWR 1989

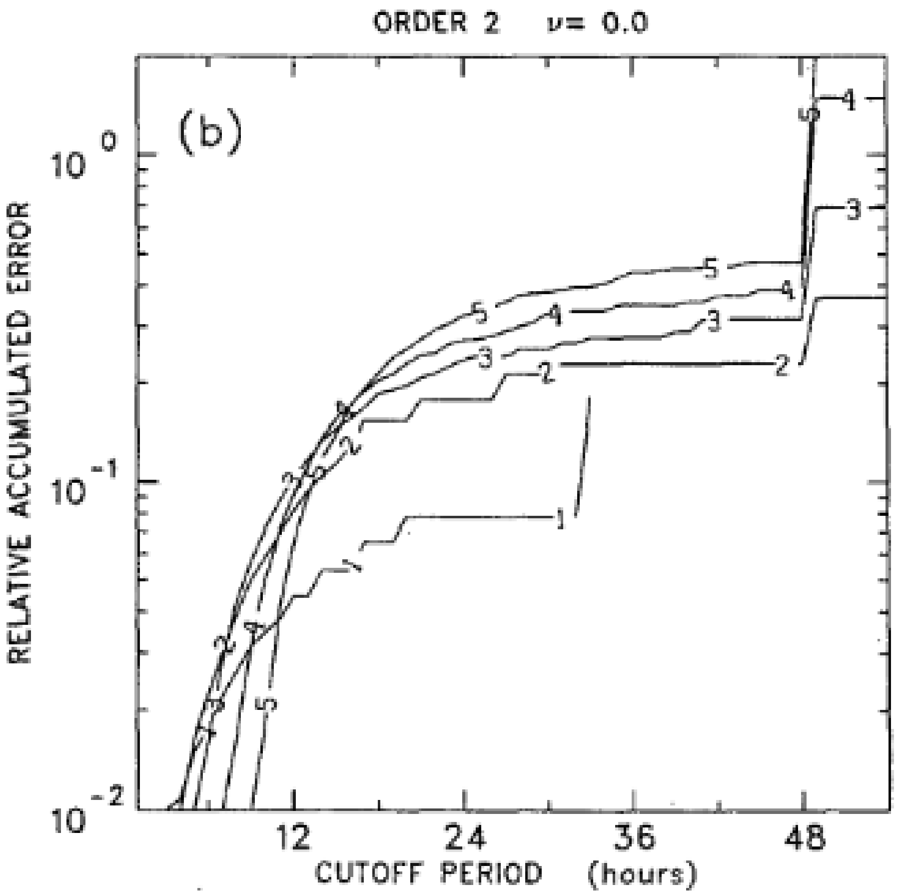




Higher-order Machenhauer schemes

Errico *MWR* 1989



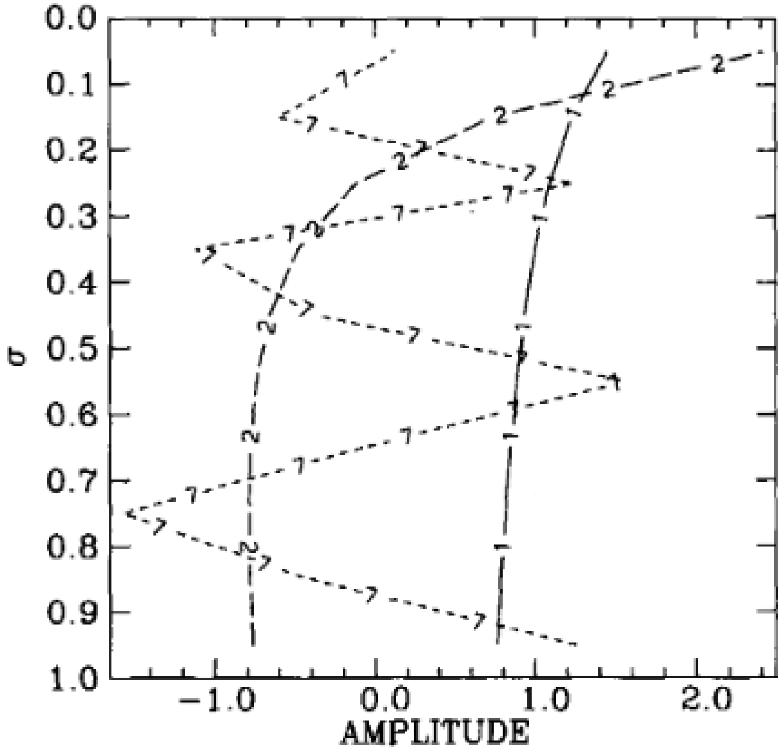


What are some aspects of normal modes that make their interpretations awkward?

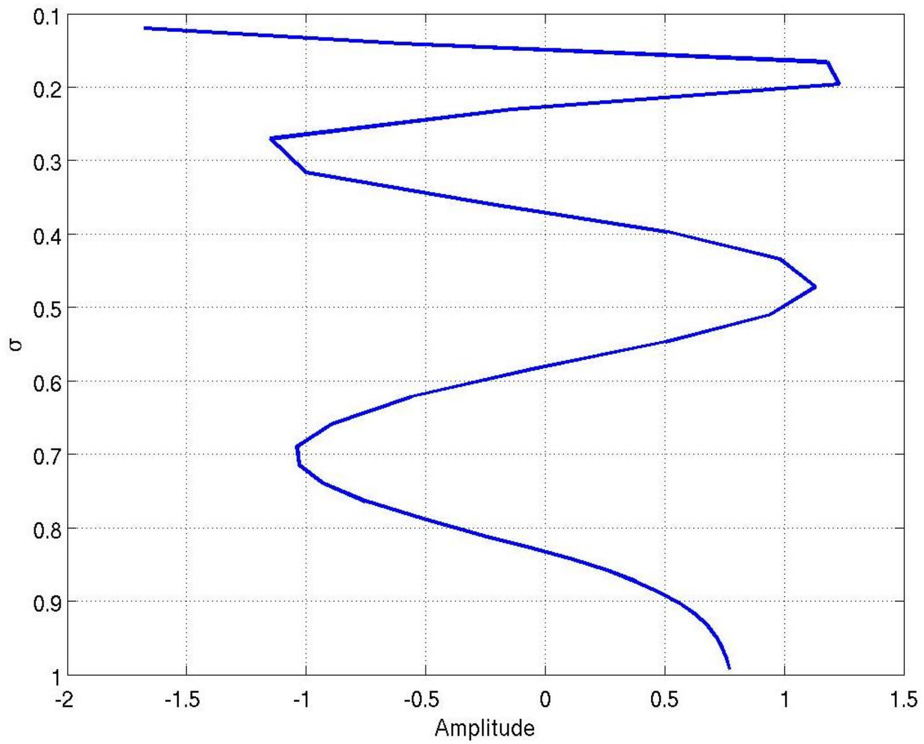
The nature of vertical modes and comparison with singular vectors.

Vertical modes in discrete models

10 level MAMS
Modes 1, 2, 7 (H=10,000, 2050, 13 m)



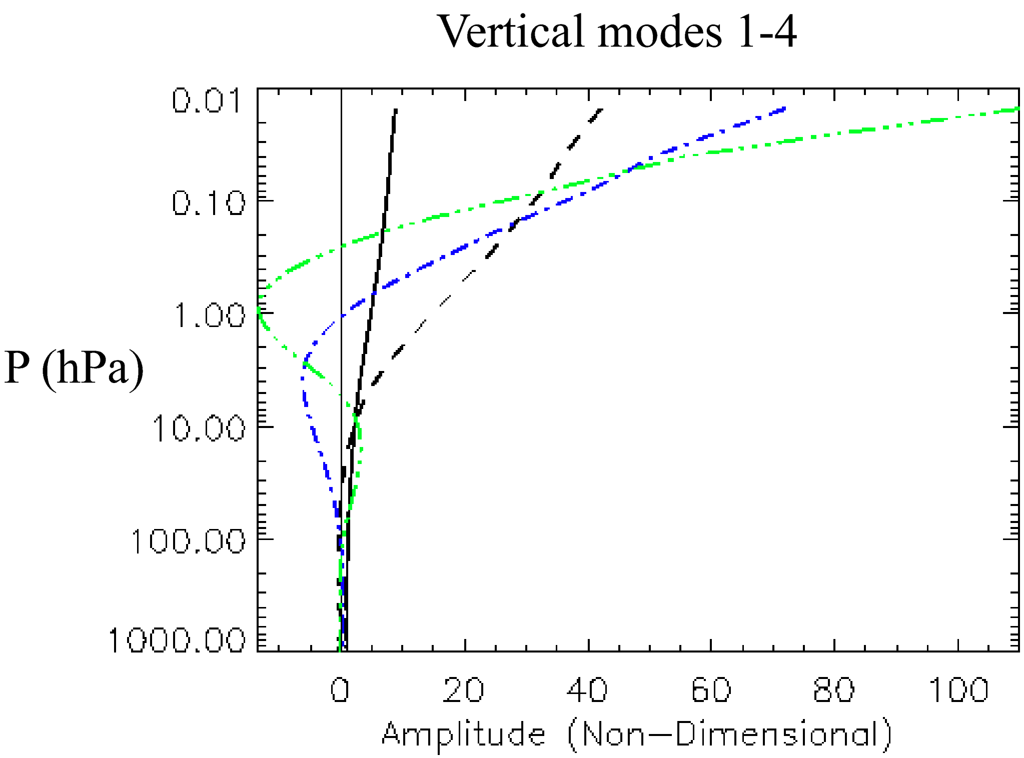
72 level GEOS-5
Mode 29 (H=13m)
(Notice only plotted up to σ=0.1



23 zero crossings above for σ<0.1

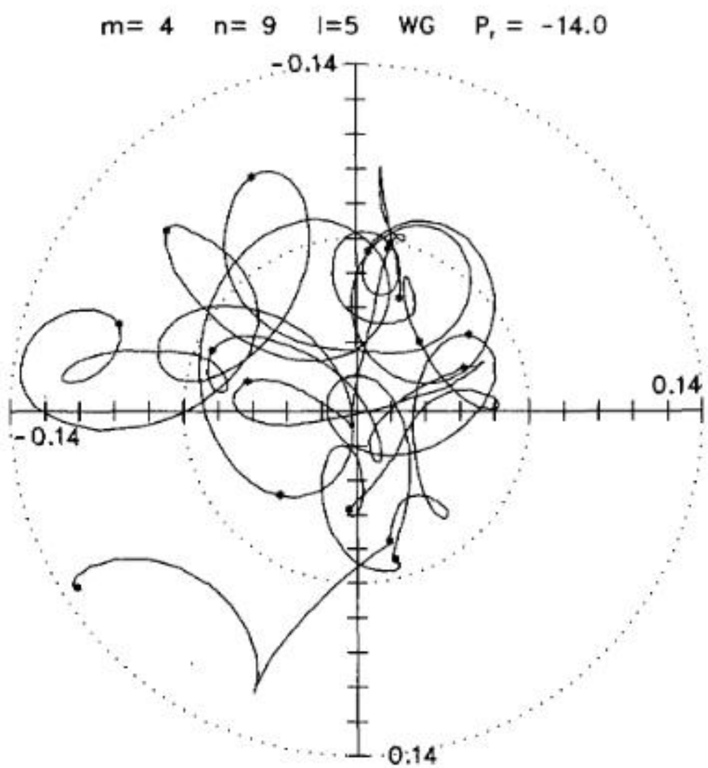
High amplitude modes in the upper atmosphere

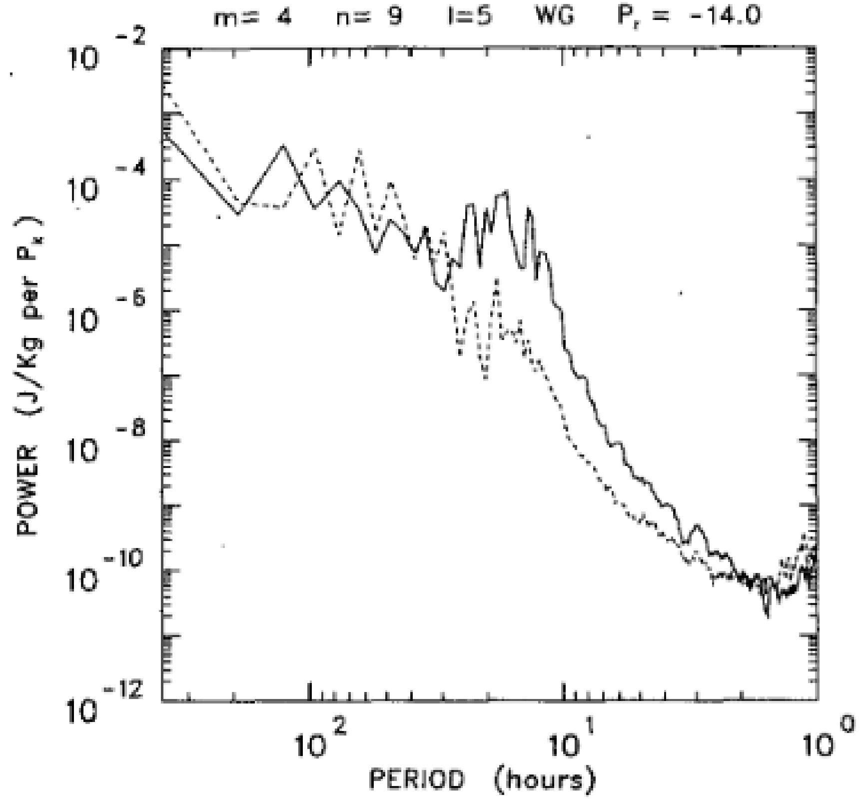
72 level GEOS-5 model with top at 0.01 hPa



Modal interaction

Errico and Williamson *MWR* 1988, Errico *MWR* 1989





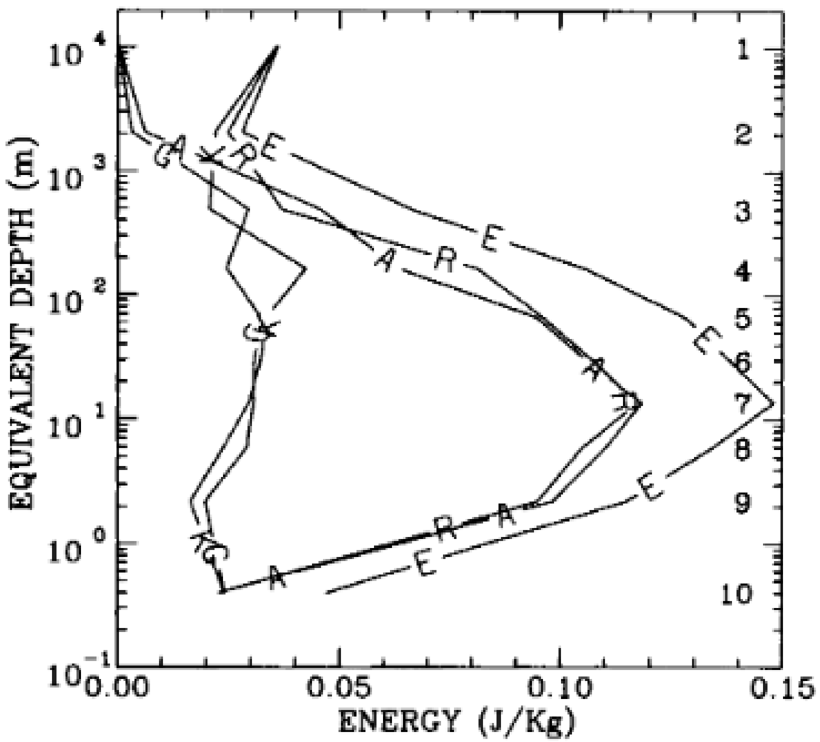
The balance of a singular vector

Errico *QJRMS* 2000

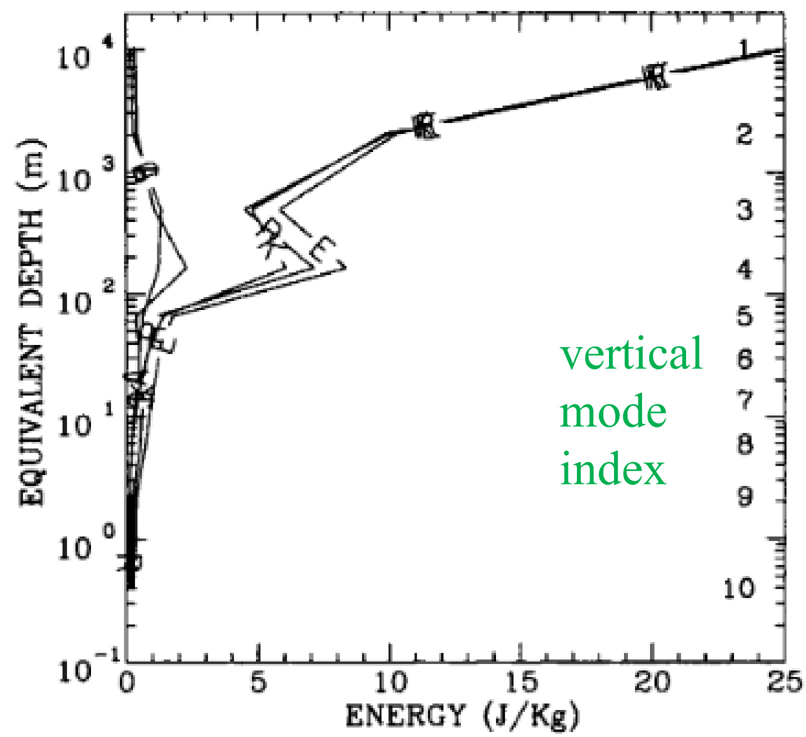
Perturbation "energy" contributed by each vertical mode

R=rotational, G=gravitational, A=APE, K=KE, E=A+K=R+G

Time=0



Time=24 hr



The balance of a singular vector

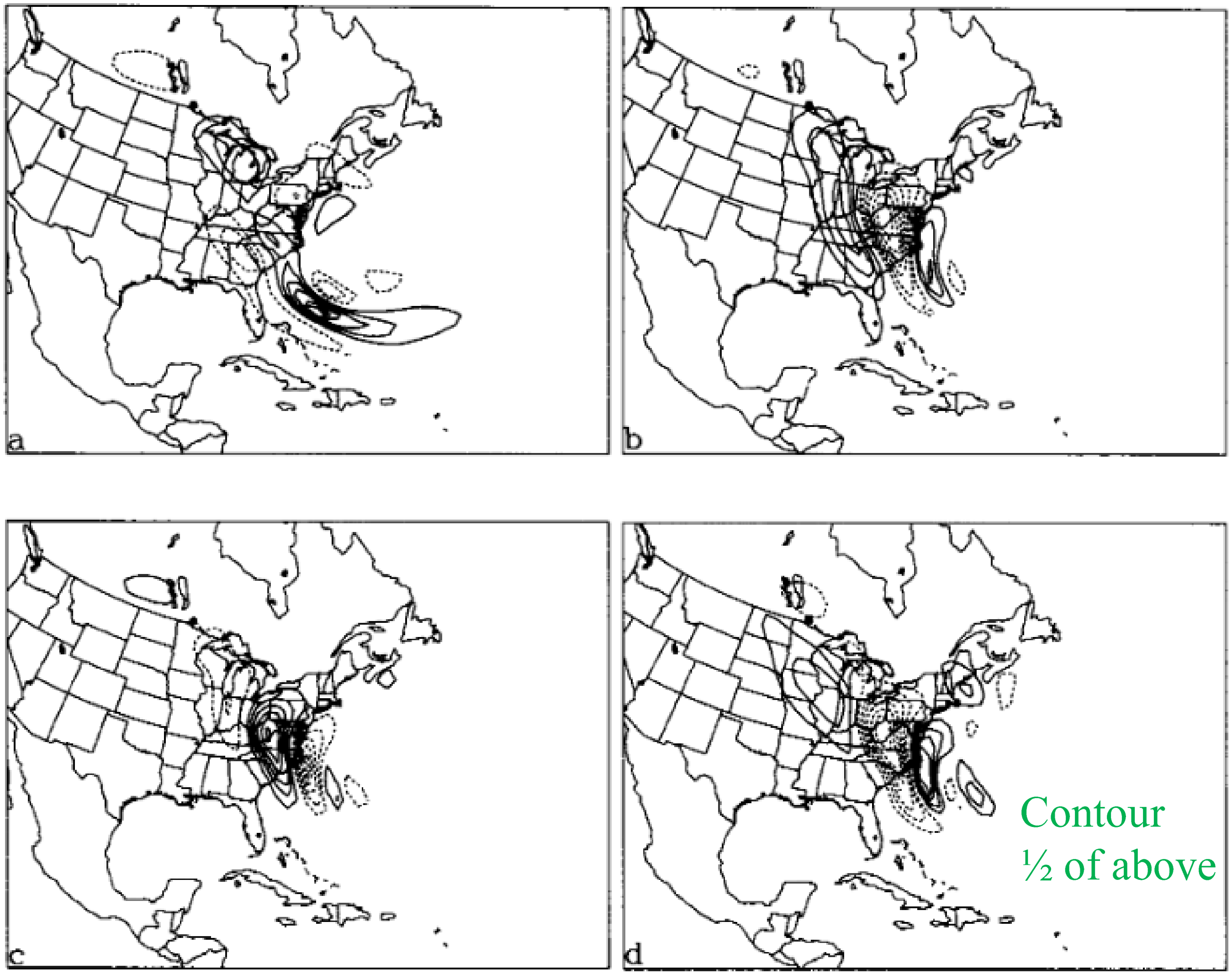
Errico *QJRMS* 2000

Leading SV at t=0 hr, p=500 hPa, $\lambda^2=55$

V field | T field

Rotational (geostrophic) component

Gravitational (ageostrophic) component



Contour ½ of above

The balance of a singular vector

Errico *QJRMS* 2000

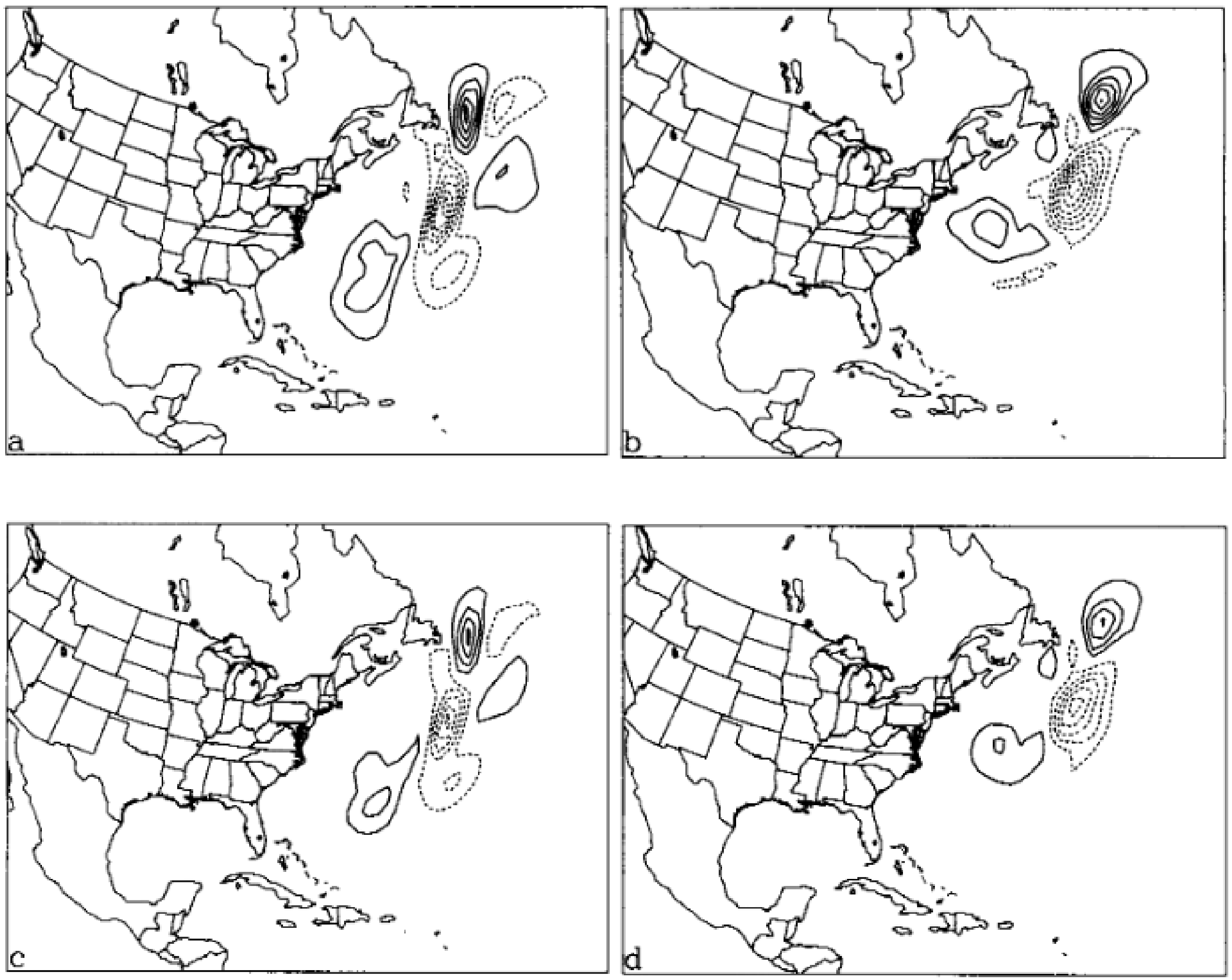
Leading SV at t=24 hr, p=500 hPa, λ^2=55

v field   T field

Rotational (geostrophic) component

Gravitational (ageostrophic) Component

(contours ½ those above)



Summary

1. Much can be learned from some old works
2. The standard Normal Modes provide useful concepts and tools
3. The standard Normal Modes have limitations
   a. the universality of vertical modes
   b. internal modes (when C not >> U)
   c. more realistic basic states (e.g. as for SVs)
4. Is Initialization still an issue ?
5. There is more to understand
   a. time scales of moist diabatic processes
   b. effects of top boundary conditions, non-hydrostatic behavior
   c. SV behavior