MODES WORKSHOP

Observations of Free, or Normal Mode Rossby-Haurwitz Waves in the Atmosphere

By Roland A. Madden

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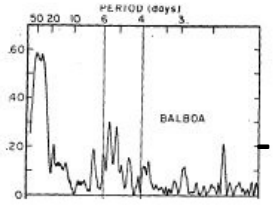
With NCAR Command Language (NCL) help from

Dennis Shea, Mary Haley, Adam Phillips, and Maria Gehne

What I will Talk About

- How I became interested in Free, or Normal Mode, Rossby-Haurwitz Waves
- Leading to Theoretical Predictions of the Structures and Periods as given by Kasahara (1976, 1980), and Kasahara and Puri (1981)
- Using NCAR Command Language (NCL) to Look for Observational Evidence Confirming These Predictions

Coherence Squared between Kanton (172W) and Balboa (80W) Station Pressures



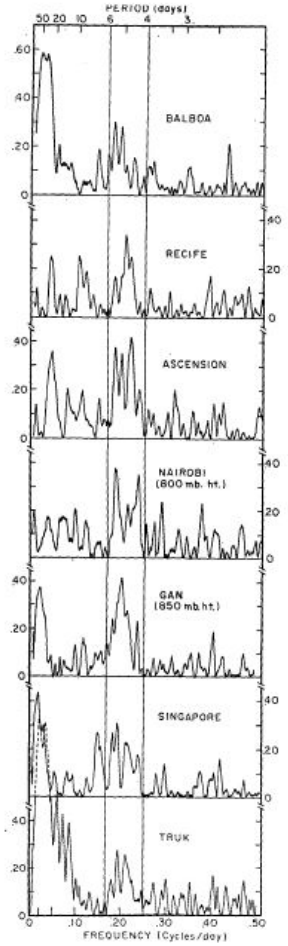
0.20

50 6 4 2

day periods

95% significance <0.19

Coherence Squared between Kanton and other Equatorial Stations' Station Pressures



Balboa

Recife

Ascension

Nairobi

Gan

Singapore

Truk

From MJ 1972:JAS

Phase at 4-5 Day Periods between Kanton and other Equatorial Stations' Station Pressures

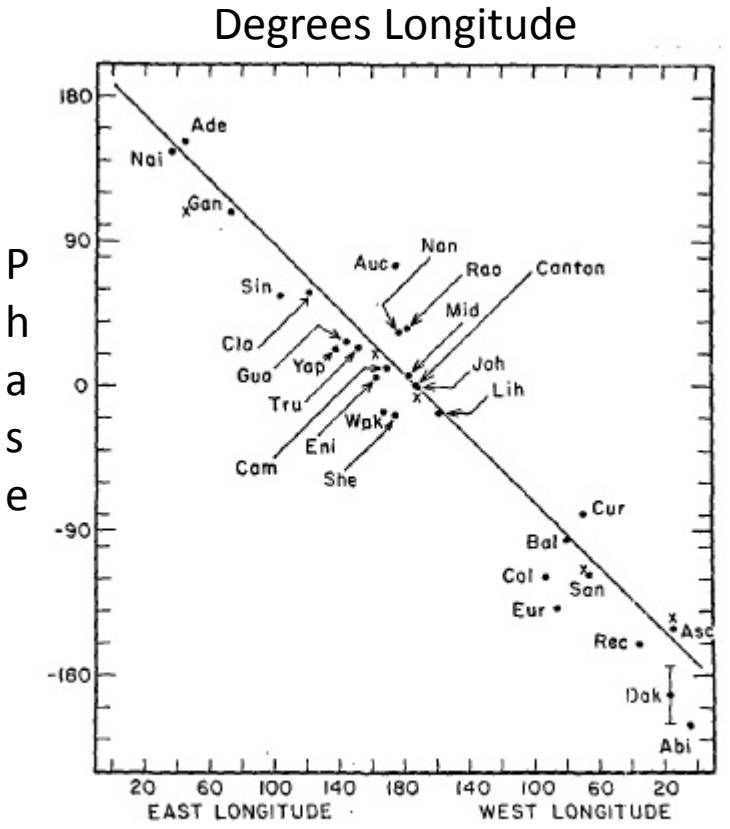
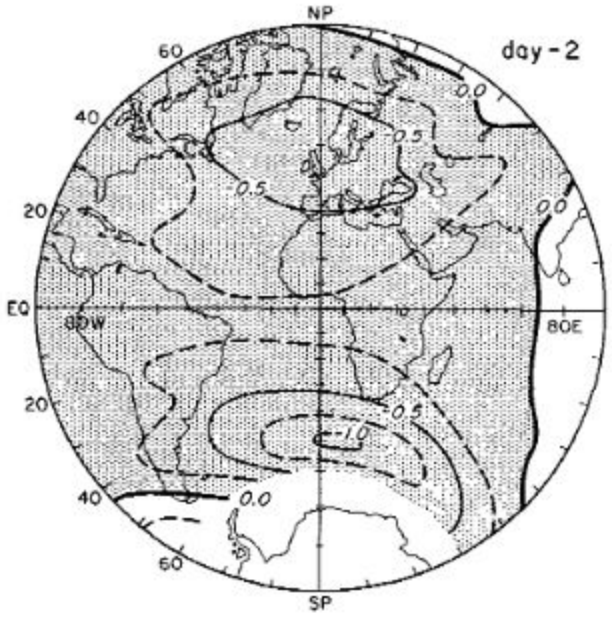


FIG. 2. Phase angles between station pressure at indicated stations and that at Canton Island. Phase angles between 500-mb heights at stations listed in Table 2 and the Canton station pressure are indicated by X's.

From MJ 1972:JAS

SLP Composite for 5-day Wave

Based on 18-Month IGY (1957-1958) SLP Data Set



Units hPa

From MJ 1973:JAS

About 1971

- I said, “Bob, what do you think of this?”

- Bob said, “Read Eliasen and Machenhauer, 1965, 1969:Tellus”

Haurwitz, B., 1937:The Oscillations of the Atmosphere. Gerl. Beitr. Geophys. 51. p195

- In this paper, Bernhard Haurwitz had offered the LaPlace Tidal Equation and the work of Margules (1893) and Hough (1898) as relevant to atmospheric motions. “Hough Functions” describe the latitudinal structures.

Hough Function Depictions of Z, u, v

Solutions to the Horizontal Structure Equation for Equivalent Depth of 10km

S=zonal wave number
l=meridional index

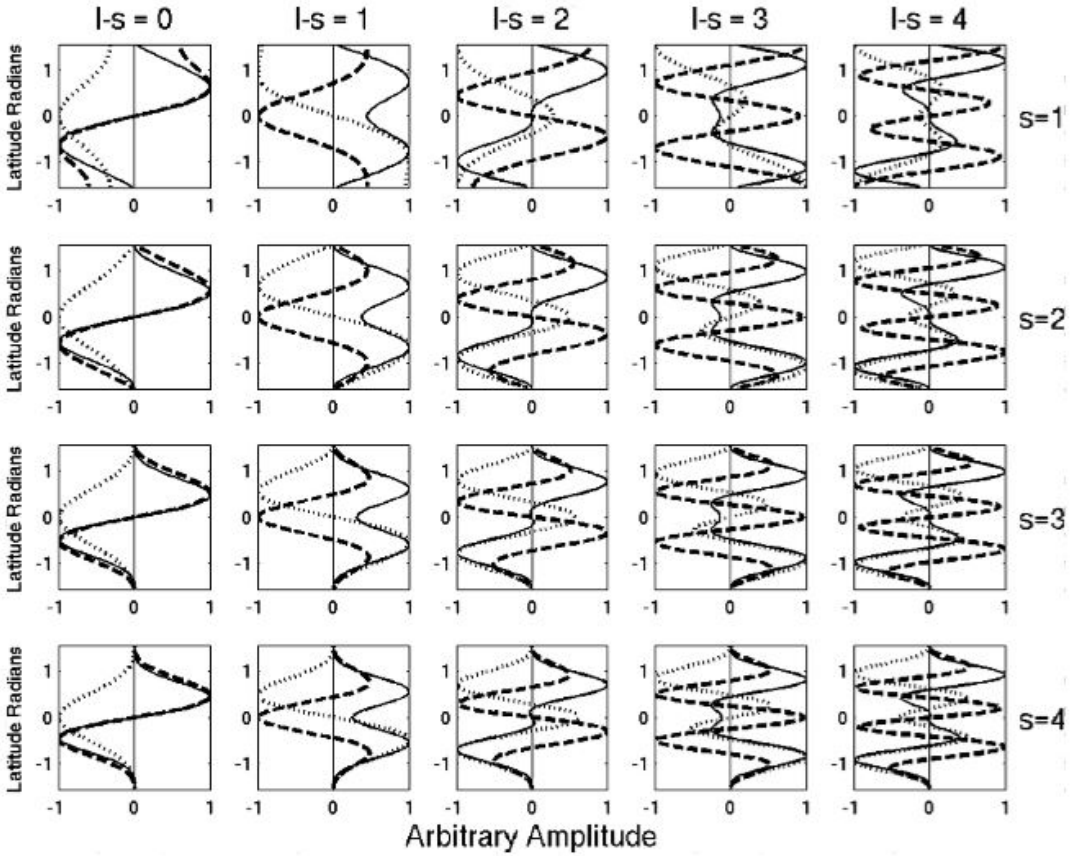
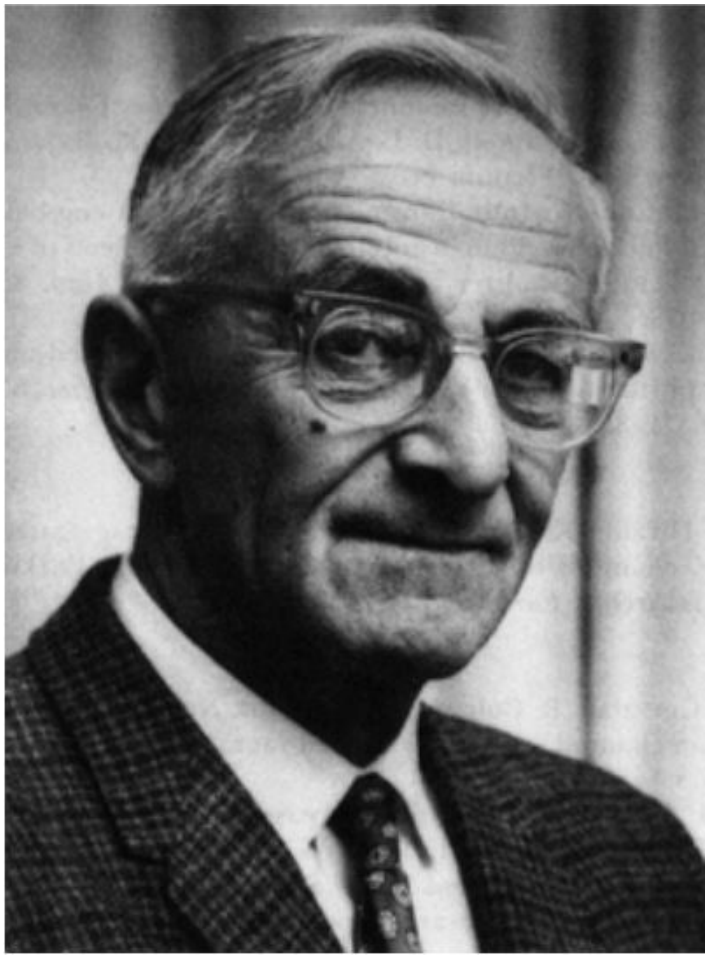


Fig. 2. Hough function depictions of the latitudinal structure of u (dashed), v (dotted) and Z (solid) for zonal wavenumber s (rows), and meridional index $l - s$ (columns). Hough functions are after Kasahara (1976). Each variable is normalized to a maximum amplitude of one.

After Kasahara, 1976

Professor Bernhard Haurwitz
Colorado State University mid-1970ties



- Bob Dickinson pointed out to me that since, in an isothermal atmosphere at rest, the vertical structure equation gives for free waves:

$$\ln(a(p)) = K\ln(p_0/p) + \ln(a(p_0))$$

I should plot amplitude estimates for various pressure levels on a log-log scale.

Estimates of the Vertical Structure of the 16-Day Wave

(Second Symmetric Mode; **s**=1, **l**-**s**=3)

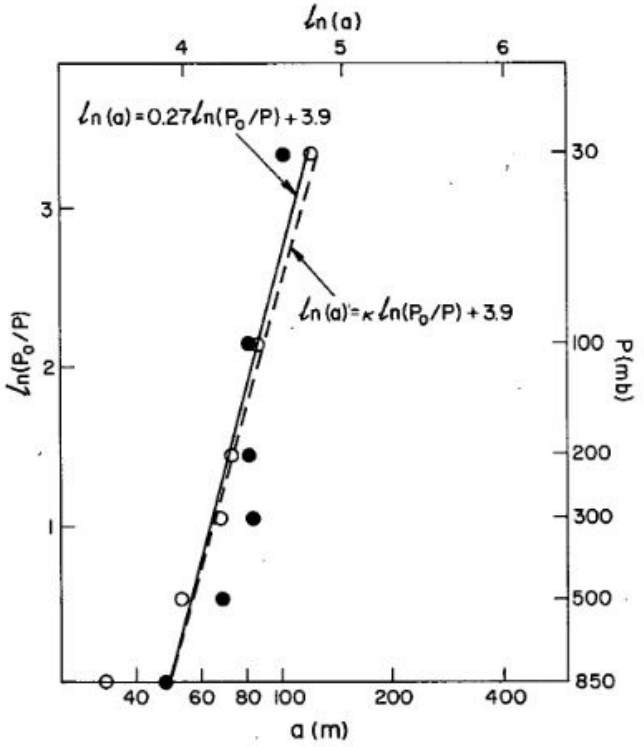
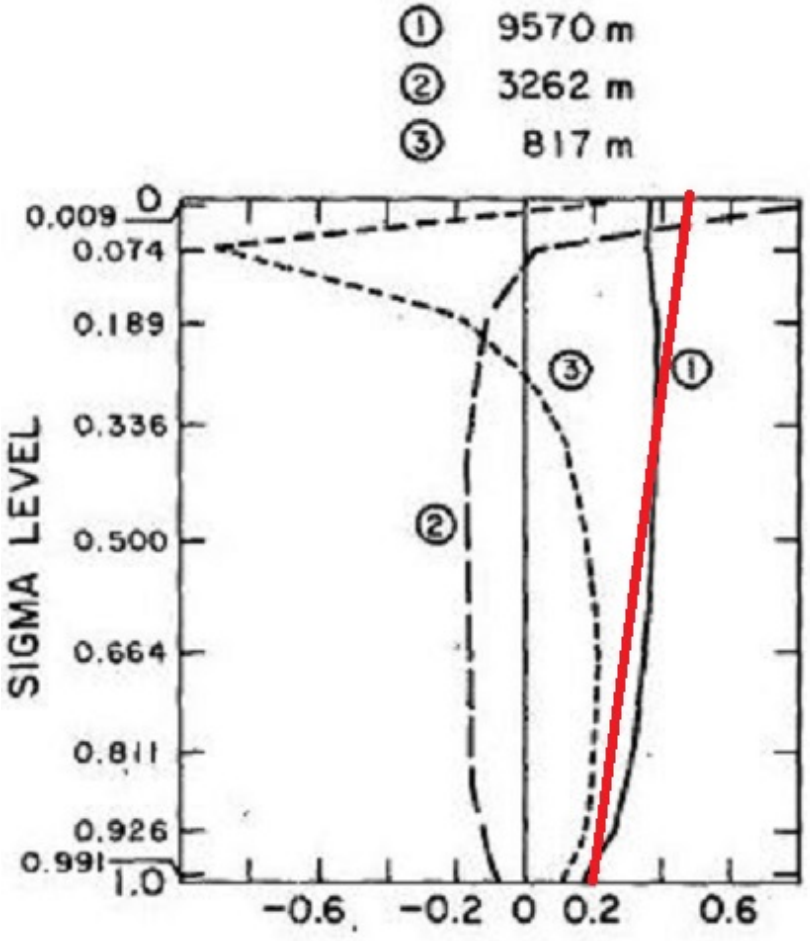


FIG. 7. Amplitudes a of 16-day composite winter season wave at 60°N based on 100 mb category dates (open circles determined from Fig. 6) and 850 mb category dates (dots determined from a composite wave not shown). Regression line determined from a's is the solid line. Dashed line is the theoretical expectation derived from (5.2) (constant is arbitrary).

From Madden,1978

Vertical Structures for Various Equivalent Depths from Kasahara and Puri, 1981: JAS

Solutions to the Vertical Structure Equation with Static Stability from January 1977



Red Line: $K\ln(P_0/P) + c$

Predicted Periods after Kasahara, 1980
(DJF Winds)

	l-s = 0	l-s = 1	l-s = 2	l-s = 3	l-s = 4	
s=1	1.20	4.85	9.91	18.39	28.08	s=1
s=2	1.71	3.84	7.27	14.23	21.47	s=2
s=3	2.30	4.28	7.40	13.65	xxx	s=3
s=4	2.90	5.21	8.20	13.55	xxx	s=4

Predicted Period in Days after Kasahara, 1980

Using NCL (NCAR Command Language) to Look for Evidence of Rossby-Haurwitz Normal Modes

ERA Interim Data Vorticity Fields 2010-2012 at 00, 06, 12, 18GMT

Using NCL to Look for Evidence of Rossby-Haurwitz Normal Modes

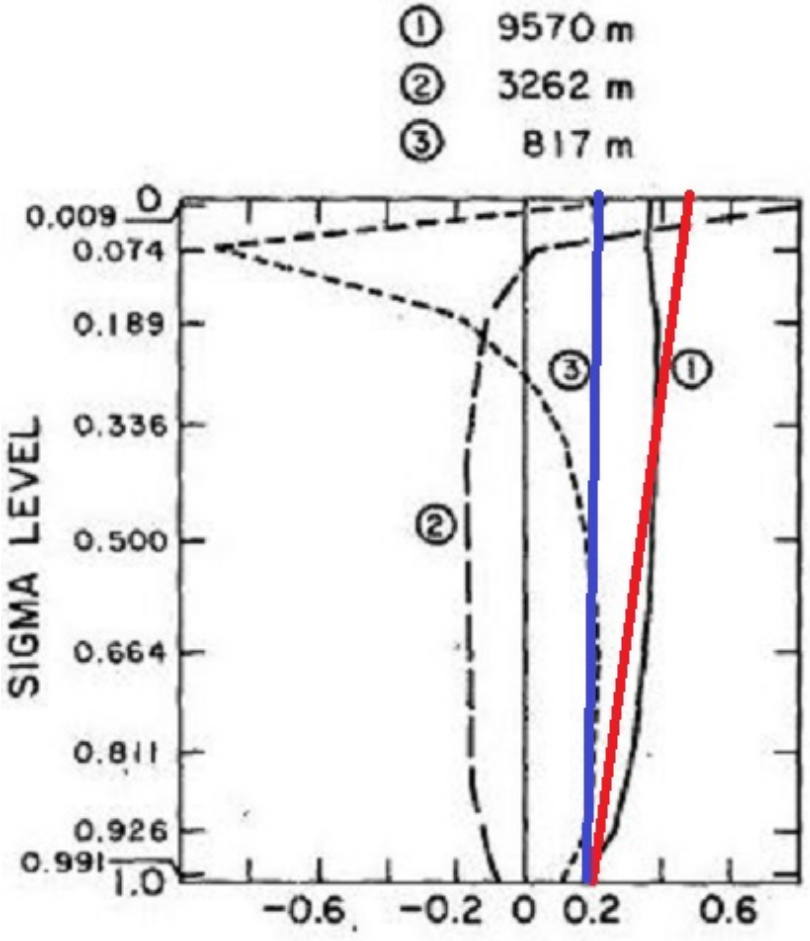
ERA Interim Data Vorticity Fields 2010-2012 at 00, 06, 12, 18GMT

Steps:

1) Average Six Levels (850,700,500,300,150, 100hPa) Vertically at Full Horizontal Resolution

Vertical Structures for Various Equivalent Depths from Kasahara and Puri, 1981: JAS

Solutions to the Vertical Structure Equation with Static Stability from January 1977



Red Line: $K\ln(P_0/P) + c$

Blue Line: Constant Whts.

Using NCL to Look for Evidence of Rossby-Haurwitz Normal Modes

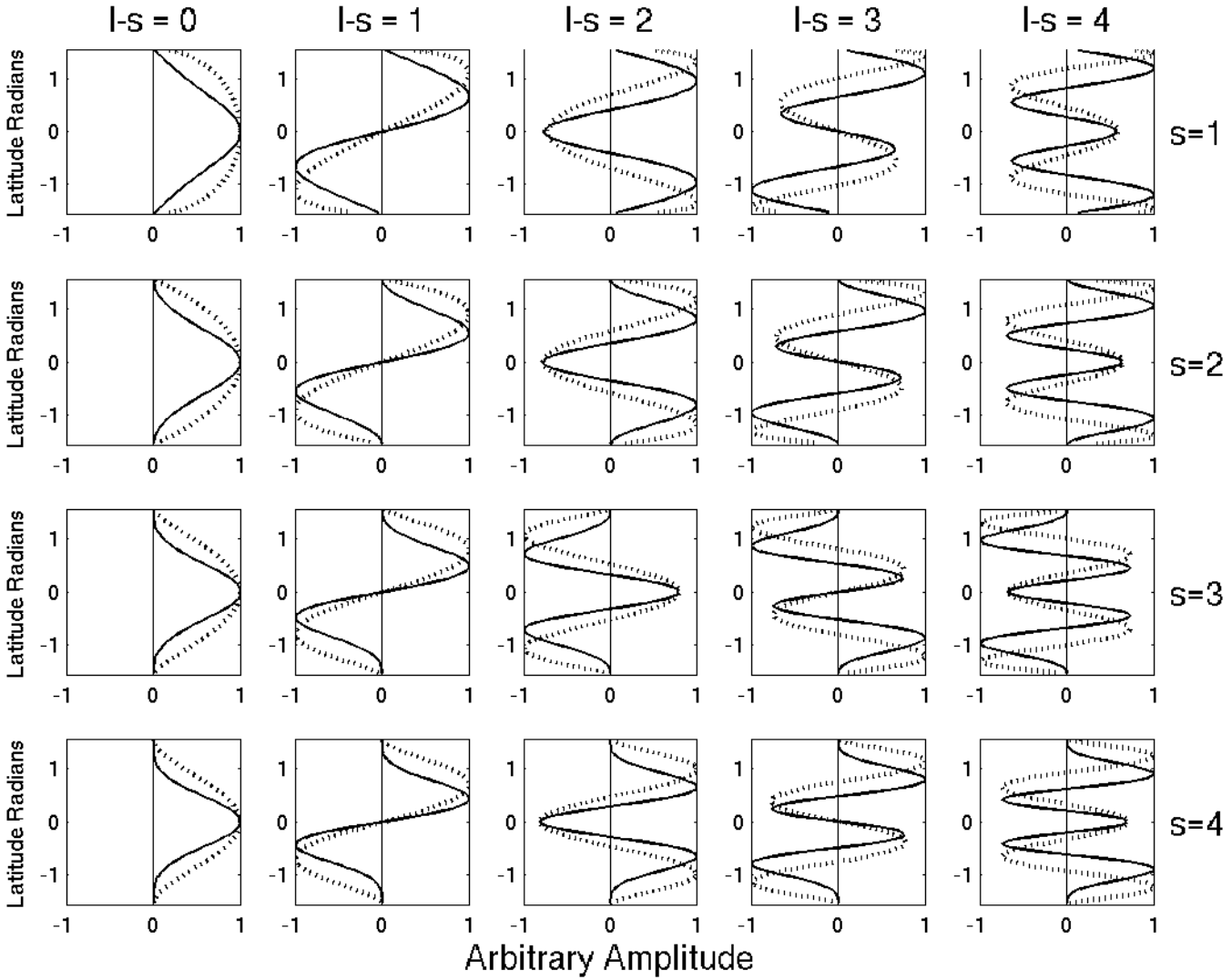
ERA Interim Data Vorticity Fields 2010-2012 at 00, 06, 12, 18GMT

Steps:

1) Average Vertically
2) Project onto Spherical Harmonics (**shagC**)

Latitudinal Profiles

Depictions of the Stream Function for Various Modes



Associated Legendre Functions ($\mathbf{P}^s_l$) - dotted; Hough Functions - solid

for example: $\Psi^1{}_2 = 0.993\mathbf{P}^1_2 + 0.110\mathbf{P}^1_4 + \ldots$ after Diky and Golitsyn,1968

Using NCL to Look for Evidence of Rossby-Haurwitz Normal Modes

ERA Interim Data Vorticity Fields 2010-2012 at 00, 06, 12, 18GMT

Steps:

1) Average Vertically
2) Project onto Spherical Harmonics (**shagC**)
3) Triangular Truncate ***l-s*** = 0:4; ***s***=0:5(**tri_trunC**)

Predicted Periods after Kasahara, 1980
(DJF Winds)

	l-s = 0	l-s = 1	l-s = 2	l-s = 3	l-s = 4	
s=1	1.20	4.85	9.91	18.39	28.08	s=1
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Predicted Period in Days after Kasahara, 1980

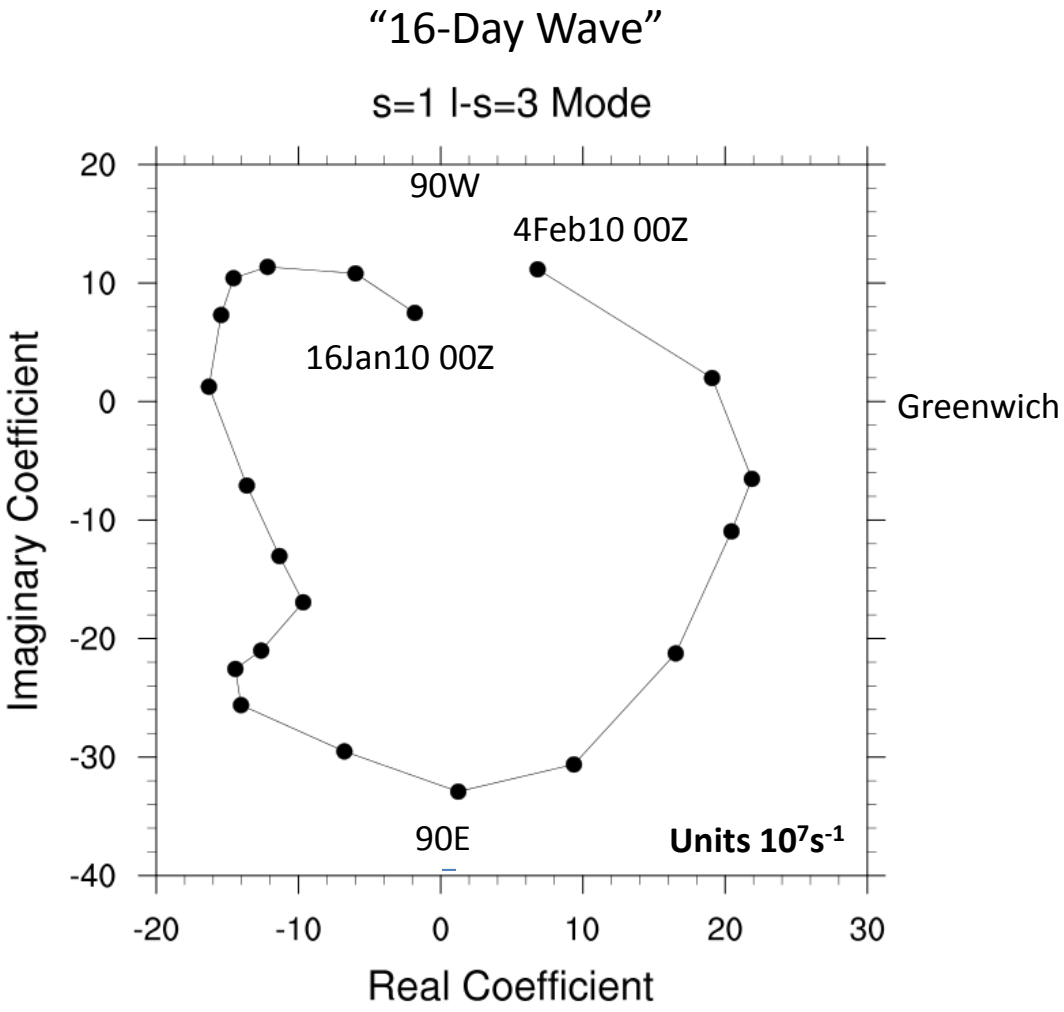
Using NCL to Look for Evidence of Rossby-Haurwitz Normal Modes

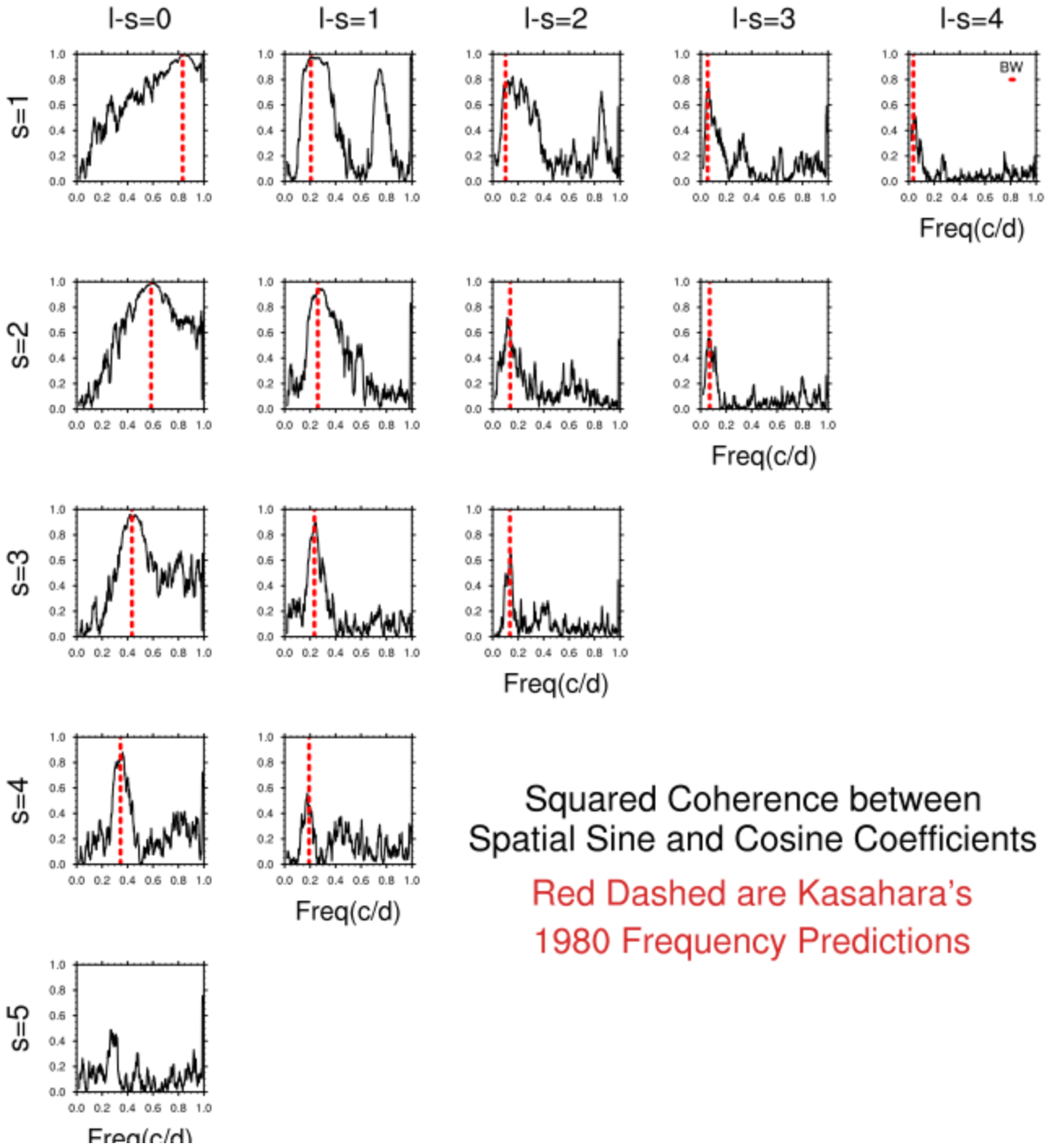
ERA Interim Data Vorticity Fields 2010-2012 at 00, 06, 12, 18GMT

Steps:

1) Average Vertically
2) Project onto Spherical Harmonics (**shagC**)
3) Triangular Truncate ***l-s*** = 0:4; ***s***=0:5 (**tri_trunC**)
4) Compute Coherence and Phase between Real and Imaginary Spatial Coefficients (**specxy_anal)**

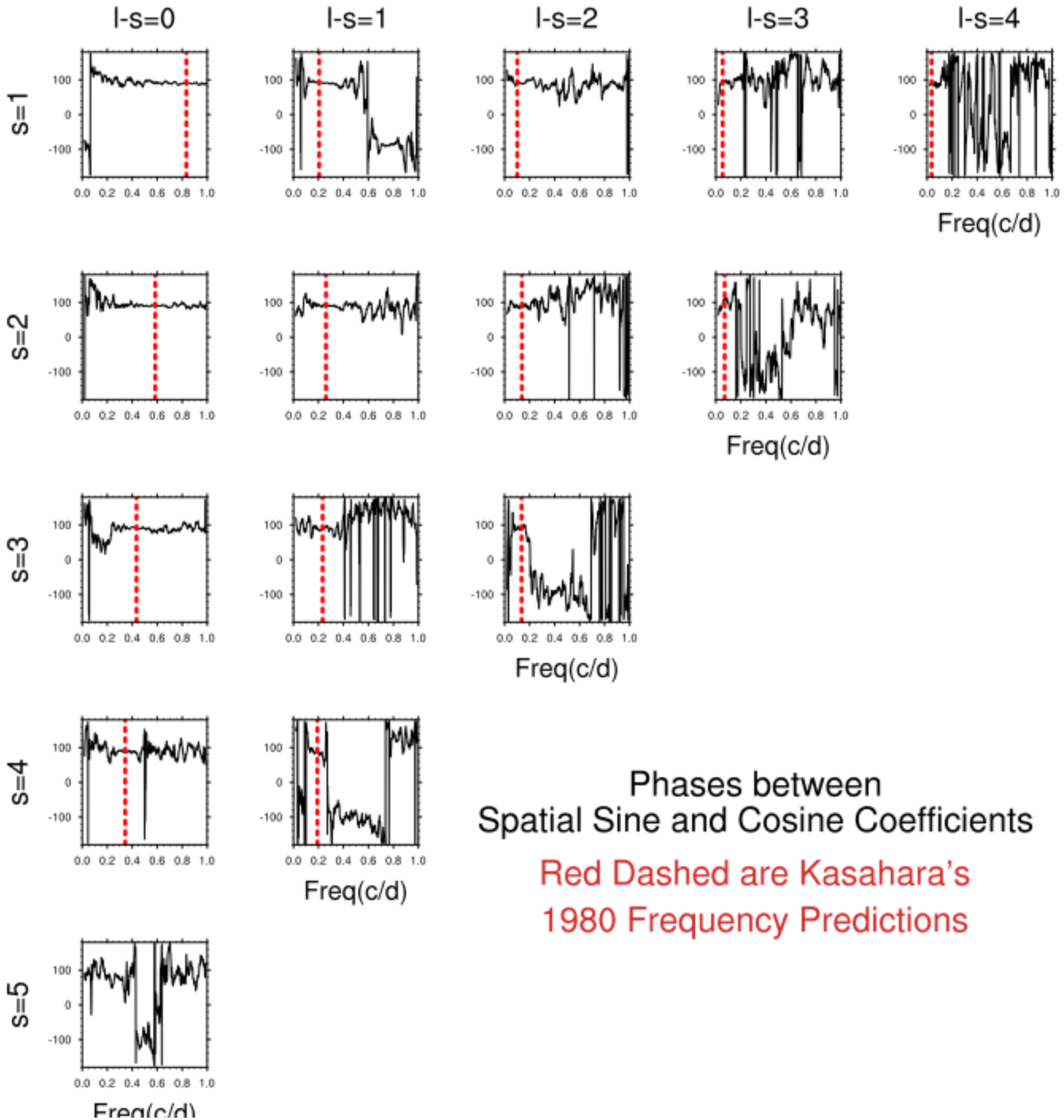
Hodograph of Second Symmetric Mode





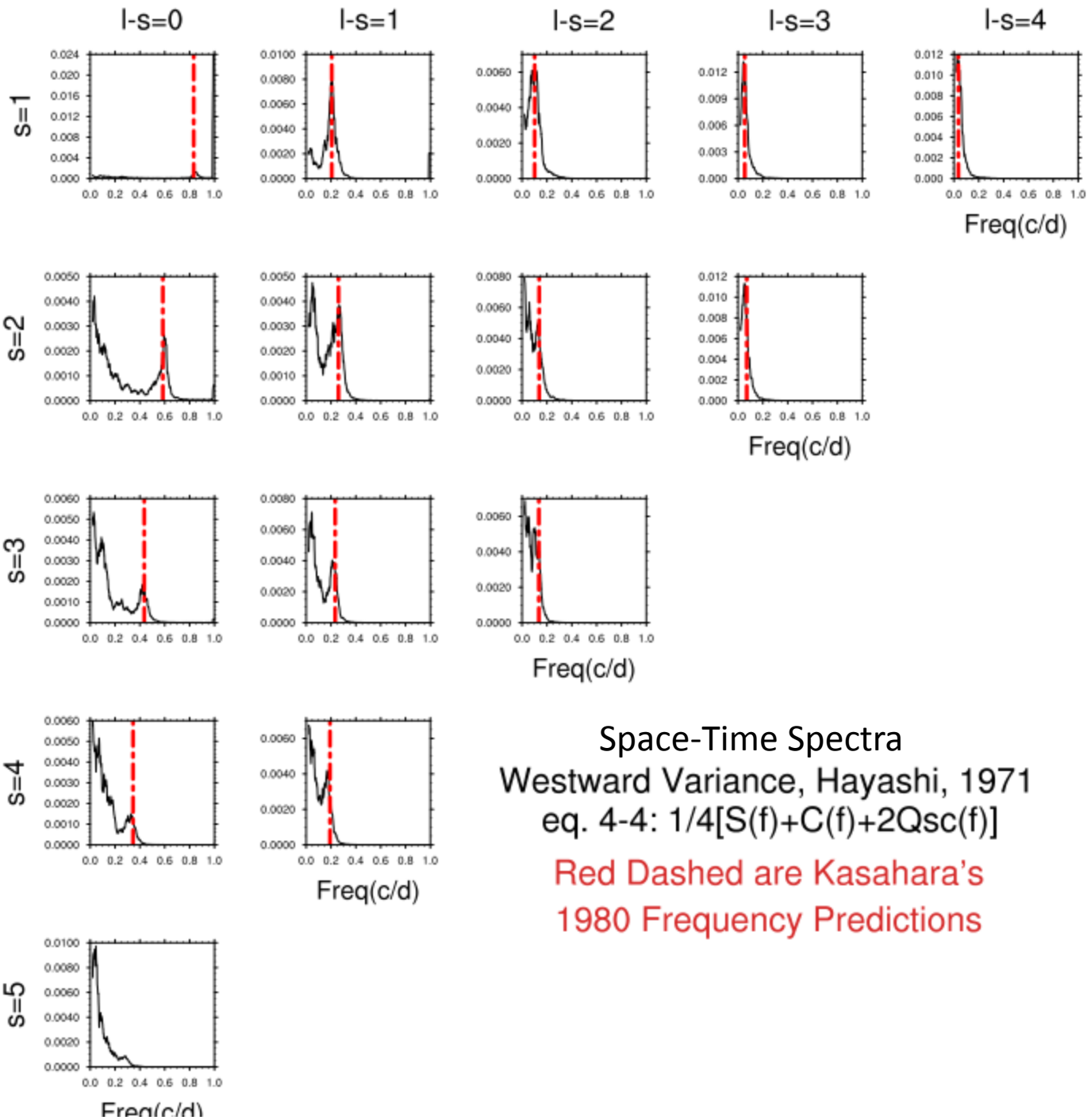
Squared Coherence between
Spatial Sine and Cosine Coefficients

Red Dashed are Kasahara's
1980 Frequency Predictions



Phases between
Spatial Sine and Cosine Coefficients

Red Dashed are Kasahara's
1980 Frequency Predictions



Space-Time Spectra
Westward Variance, Hayashi, 1971
eq. 4-4: 1/4[S(f)+C(f)+2Qsc(f)]

Red Dashed are Kasahara's
1980 Frequency Predictions

Identified Modes 1982-2007

	l-s = 0	l-s = 1	l-s = 2	l-s = 3	l-s = 4	
s=1	W93 M	A82 L84 V89 H89 W93 E93 M	A82 L84 V89 H89 W93 E93 M	A82 L84 V89 H89 W93 E93 M	V89 W93 E93 H89 M	s=1
s=2	W93 M	A82 L84 V89 H89 W93 E93 M	A82 L84 V89 H89 W93 E93 M	A82 L84 V89 H89 W93 E93 M	A82 E93 H89 M	s=2
s=3	A82 W93 E93 M	A82 L84 V89 W93 E93 M	A82 L84 V89 W93 E93 M	L84 V89 W93 E93		s=3
s=4	A82 W93 E93 M	A82 V89 W93 E93 M	A82 W93 E93		E93	s=4

TABLE 5

A-Ahlquist,1982; L-Lindzen et.al.,1984; V-Venne,1989; H-Hirooka and Hirota,1989; W-Weber and Madden,1993; E-Elbern and Speth,1993;,M-Madden,2007

Summary

Since 1954 (Kubota and Iida: *Pap. Meteorol. Geophys.*) Numerous Studies Using Different Data and Different Analysis Techniques Have Shown Evidence for Free Rossby-Haurwitz Modes

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The Overall Correspondence Between Observations and Theory is, in My Opinion, the Best of Any in Meteorology

Further Questions:

1) What are the Details of Mode Forcing?
2) How Well do Forecast Models Handle Free, Rossby-Haurwitz Waves?

Thank you

EOFs of 300 hPa (40 year record) Stream Function for s=1

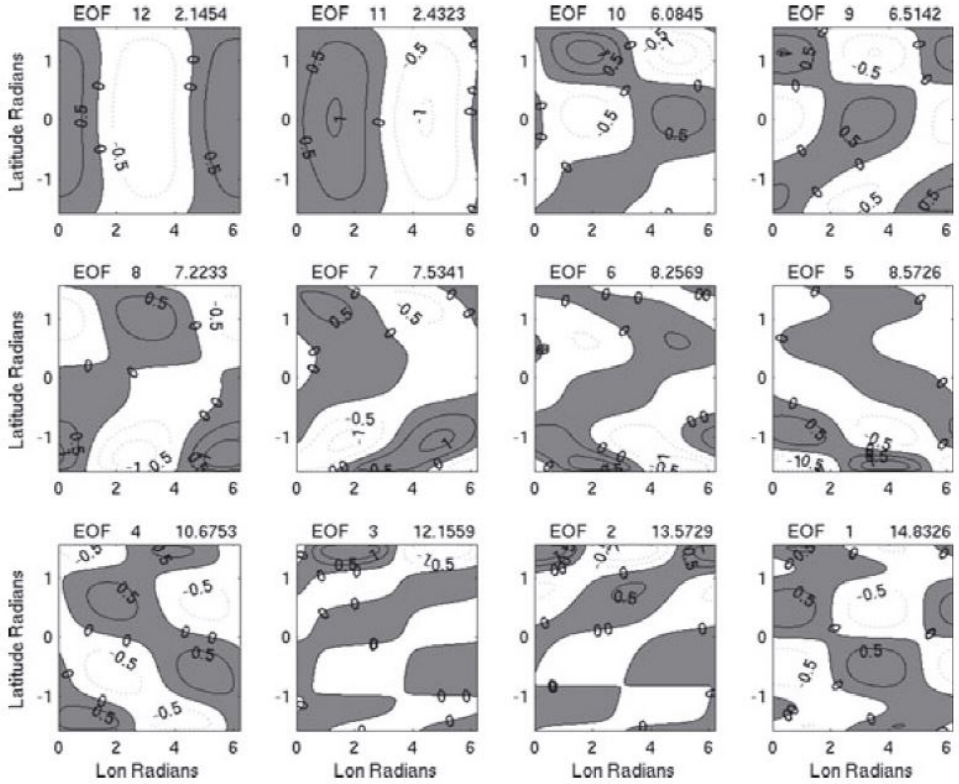


Fig. 3. EOFs of the basic ψ (stream function) data for zonal wavenumber 1 ($s = 1$). Each panel is a map going eastward from the Greenwich Meridian for 2π radians (horizontal) and from the South Pole ($-\pi/2$ radians) to to the North Pole ($+\pi/2$radians). Shaded values are positive. EOF weightings for these maps are shown in Table 3. Amplitudes are arbitrary. The EOF number and percent variance explained by each EOF are indicated above the upper left and upper right of each panel, respectively. EOF1 (lower right) explains the most variance and EOF12 (upper left) the least.

Stream Function courtesy Branstator and Mai

Latitudinal Structures of EOF Pairs Determined from a 40-Year Record of 300hPa Stream Function Data

Stream Function Data Courtesy of Branstator and Mai

Solid Lines are Hough Functions

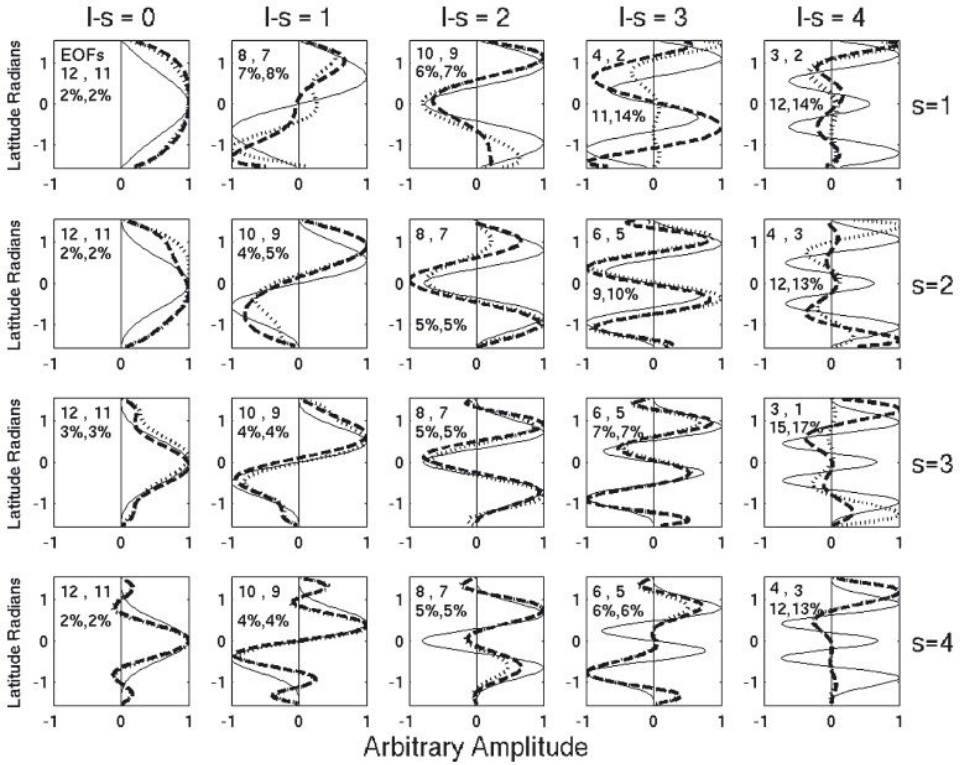


Fig. 4. Solid curved lines are the Hough function depictions of the latitudinal structure of ψ (stream function) for the $s, l-s$ mode in an atmosphere with an equivalent depth of 10 km like those of Fig. 1 (after Kasahara, 1976). Each row is for a different zonal wavenumber s, and each column is for a different meridional index, $l-s$. Selected EOF pairs and the percent variance that they explain are indicated in the upper left of each panel. Dashed line represents the first, or left-hand side, indicated EOF, and dotted line the second, or right-hand side indicated EOF. Profiles in the top row correspond to the EOFs shown in Fig. 3.

After Madden, 2007: Tellus

Rough Estimates of % Variance Explained based on EOF Pairs and Coherence between them

Table 6. Approximate percent of the variance of the basic data explained by a free Rossby mode. Each row is for a different zonal wavenumber, s, and each column is for a different meridional index, $l - s$

	$l-s=0$	$l-s=1$	$l-s=2$	$l-s=3$	$l-s=4$	**Sum**
$s=1$	2%	9%	6%	2%	10%	29%
$s=2$	3%	5%	2%	2%	3%	15%
$s=3$	4%	5%	3%	XXX	XXX	12%
$s=4$	2%	<1%	XXX	XXX	XXX	3%

From Madden,2007: Tellus

Summary

Since 1954 (Kobota and Iida: *Pap. Meteorol. Geophys.*) Numerous Studies Using Different Data and Different Analysis Techniques Have Shown Evidence for Free Rossby-Haurwitz Modes

The Overall Correspondence Between Observations and Theory is, in My Opinion, the Best of Any in Meteorology

In a Separate EOF Analysis of a 40-Year Record of 300hPa Stream Function Data, only outlined here, Leading Eigen Vector Pairs Reflect Latitudinal Structures and Westward Propagation of the Theoretically Predicted Modes. Rough Estimates of Variance explained range from 2% for ***s=4*** to as much as 30% for ***s=1***

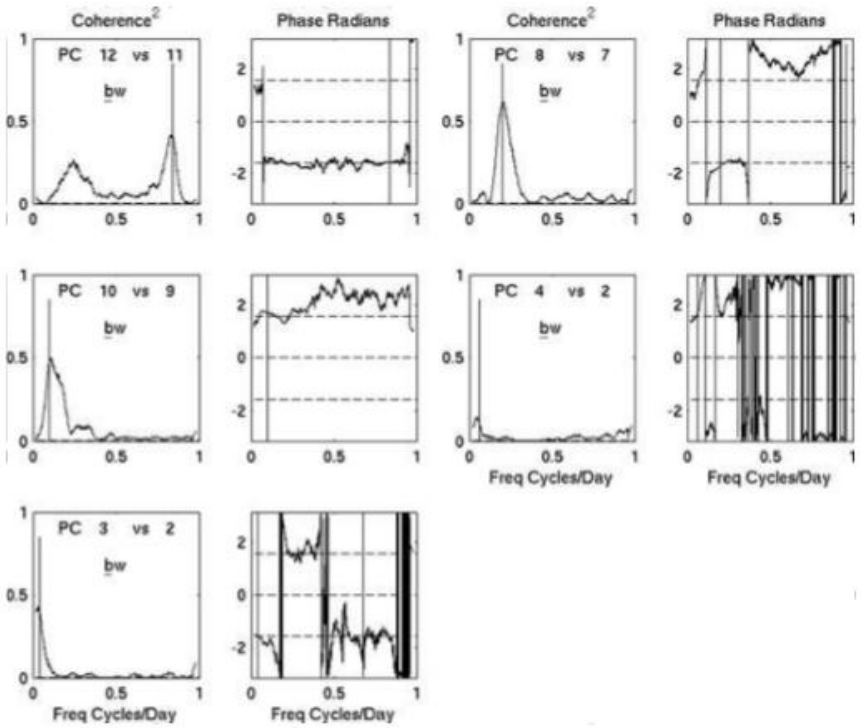
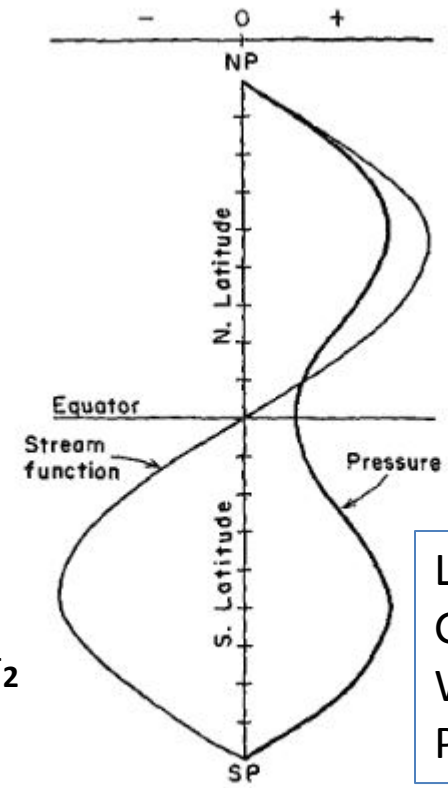


Fig. 5. Squared coherence (left of each pair of panels) and phase (right of each pair of panels) between the PC time series of selected pair of EOFs from Fig. 3 for $s = 1$. Phases are in radians and frequency in cycles d^{-1}. Negative phase angles means the first mentioned EOF (e.g. EOF12 in upper left panels) trails the second mentioned EOF (e.g. EOF11 in upper left panels). Vertical line in each panel marks the 1.2-, 5-, 10-, 18- and 28-d period predicted by Kasahara's (1980) analysis for $s = 1$. The 95% significance level assuming zero coherence and 1168*d.f.* of 0.0051 (dashed line) is barely distinguishable from the zero line. In the phase diagrams, $-\pi/2$, 0 and $+\pi$ are indicated by the horizontal dashed lines.

Non-divergent Approximation to the "5-Day Normal Mode" from Haurwitz,1940

Latitudinal Dependence of the Streamfunction $\mathbf{P^1_2}$



Latitudinal Dependence of Pressure
Goes like $3/2*\mathbf{P^1_2} - 1/3*\mathbf{P^1_3}$
Where $\mathbf{P^s_l}$ are Associated Legendre Polynomials

FIG. 5. Latitudinal dependence of pressure and streamfunction for the wave mode $m=1$, $n=2$. The amplitude is arbitrary.