Multiscale Theories and Models for the Madden–Julian Oscillation

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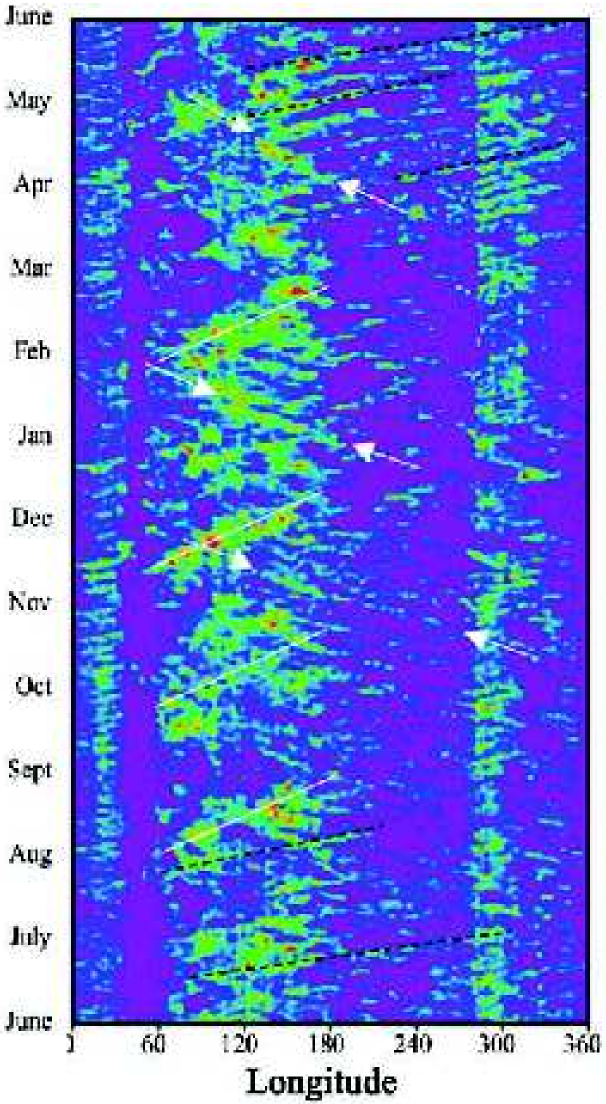
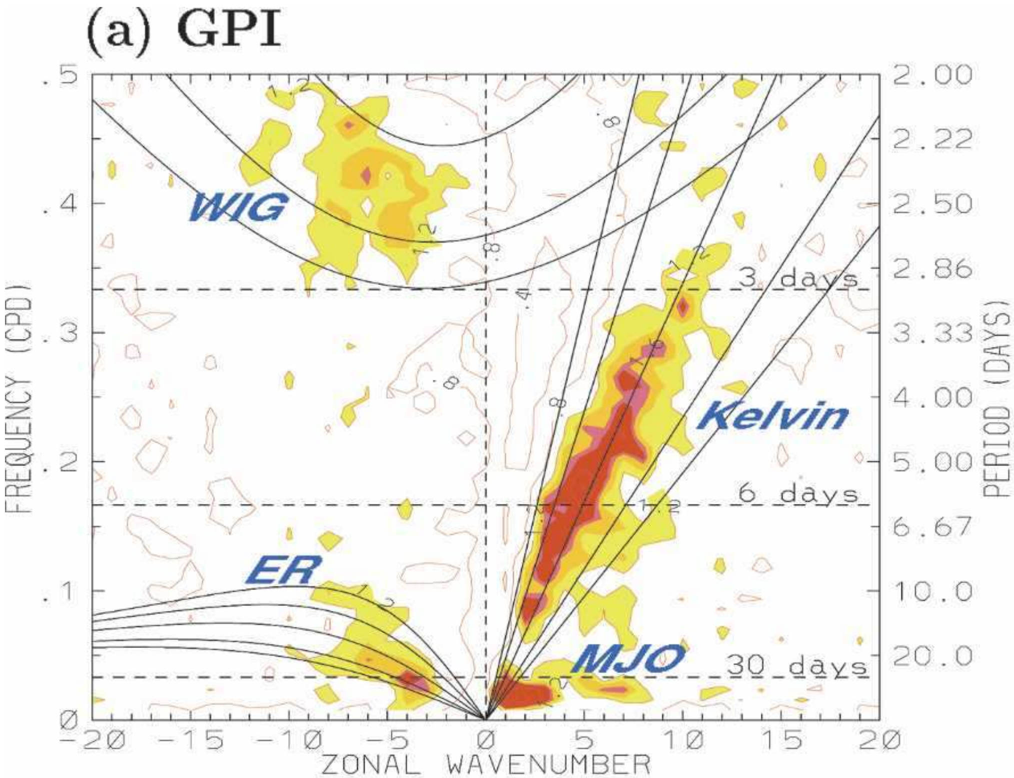
Multi-scale clouds and waves in the tropics

Precipitation



2000–2001 (from Zhang 2005)

Spectral Power



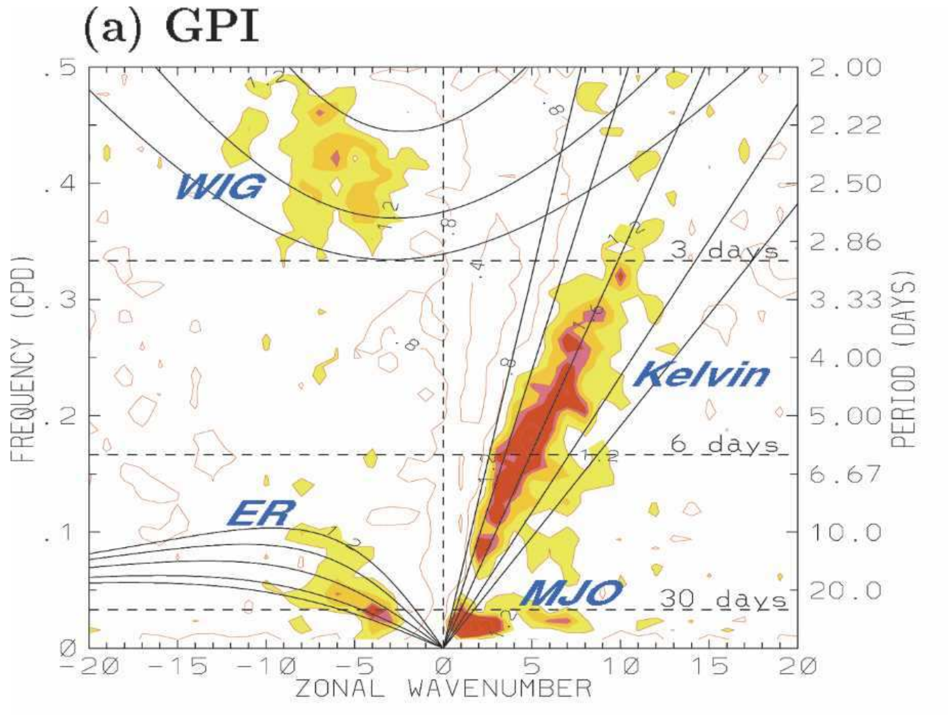
from Lin et al. 2006

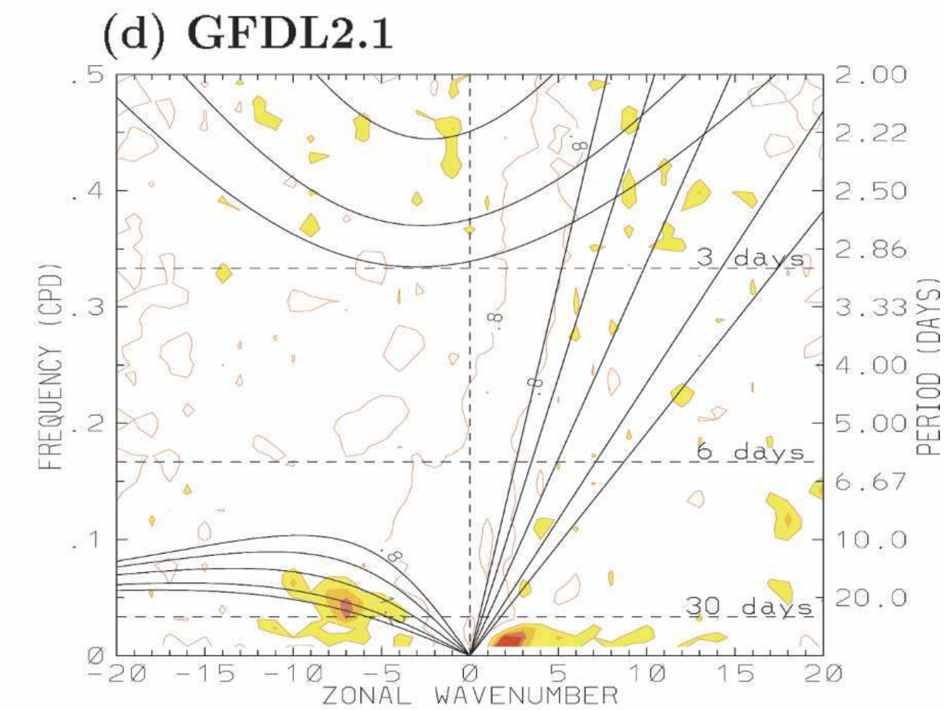
MJO is an envelope of smaller-scale convection/waves

Multi-scale clouds and waves in the tropics

Observations

General Circulation Model (GCM)





from Lin et al. (2006)

GCMs typically don't adequately represent convectively coupled equatorial waves and the MJO

What are the physical mechanisms of the MJO?

Outline

1. A model for the MJO "skeleton"

   - A minimal, nonlinear oscillator model

   - Based on multiscale ideas

   - Planetary/intraseasonal-scale features of MJO (not smaller-scale details)

2. A model for part of the MJO's "muscle"

   - Multiscale model for convectively coupled wave–mean flow interactions

   - Acceleration of lower tropospheric jet during westerly wind burst (due to convective momentum transports)

Dry fluid dynamics of the tropical atmosphere

Dry fluid dynamics of the tropical atmosphere

$$\frac{Du}{Dt} - \beta y v = -\frac{\partial p}{\partial x}$$

$$\frac{Dv}{Dt} + \beta y u = -\frac{\partial p}{\partial y}$$

$$0 = -\frac{\partial p}{\partial z} + g\frac{\theta}{\theta_{ref}}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{D\theta}{Dt} + w\frac{d\bar{\Theta}}{dz} = 0$$

$(u, v) =$ horizontal velocity

$w =$ vertical velocity

$p =$ pressure

$\theta =$ potential temp.

Vertical modes: Equatorial shallow water equations

Linear waves

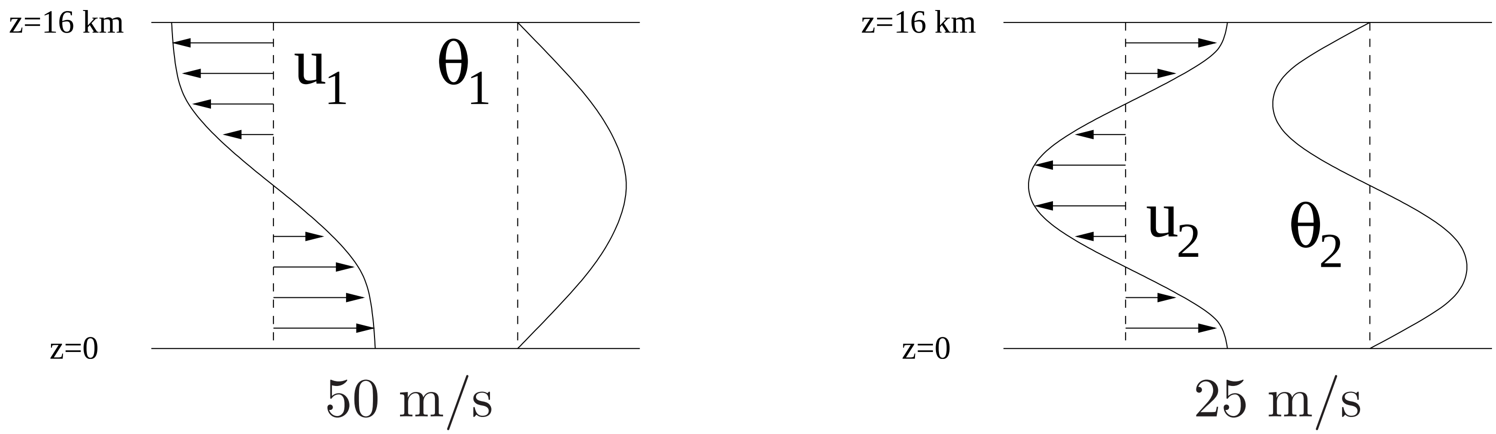
- Expand in vertical modes: $u(x,y,z,t) = \sum_j u_j(x,y,t)\cos jz$, etc.
- Equatorial shallow water system for each vertical mode j:

$$\frac{\partial u_j}{\partial t} - y v_j - \frac{\partial \theta_j}{\partial x} = 0$$

$$\frac{\partial v_j}{\partial t} + y u_j - \frac{\partial \theta_j}{\partial y} = 0$$

$$\frac{\partial \theta_j}{\partial t} - \frac{1}{j^2}\left(\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y}\right) = 0$$

- Gravity wave speed $\propto 1/j$



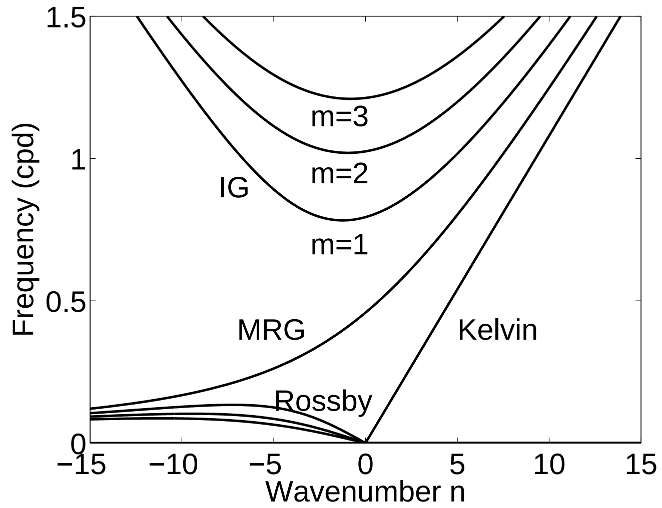
Meridional modes: Equatorially trapped waves

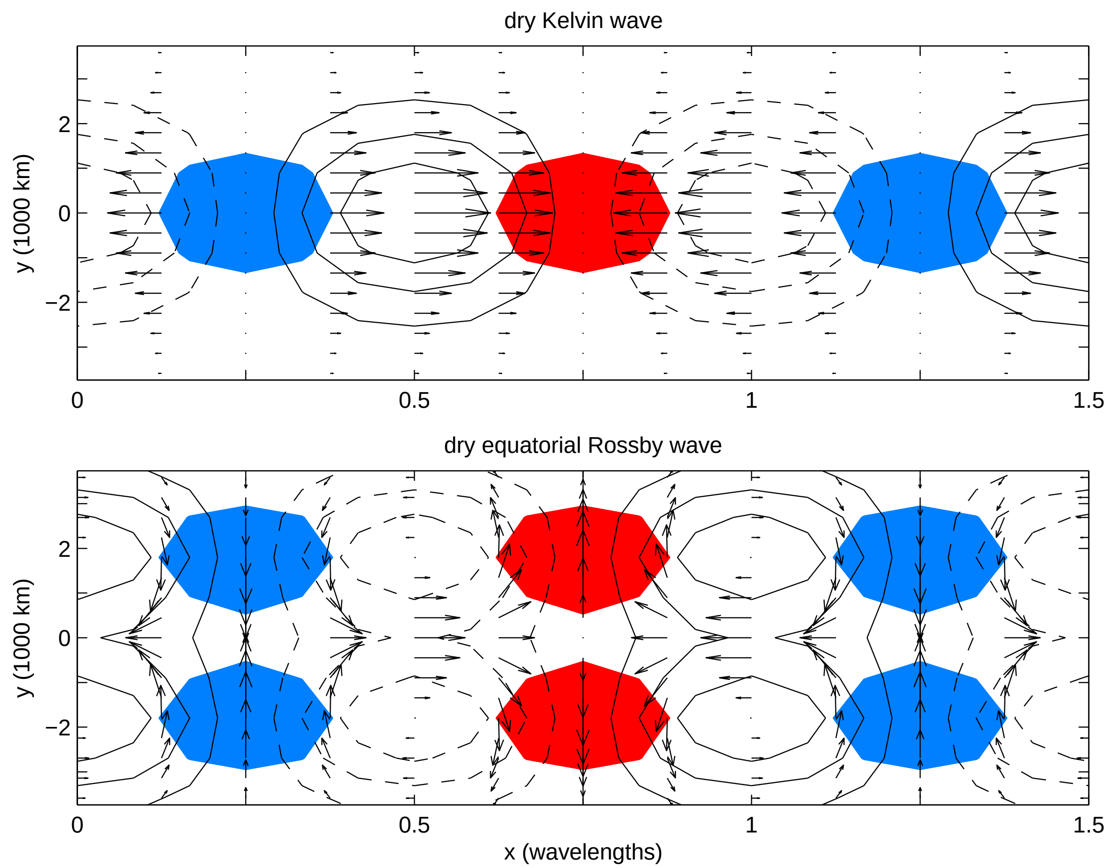
Expand in meridional modes: $u(x,y,t) = \sum_m u_m(x,t)\phi_m(y)$, etc.

$$\phi_0(y) \propto \exp\left(-\frac{y^2}{2}\right), \quad \phi_1(y) \propto y\exp\left(-\frac{y^2}{2}\right), \quad \phi_2(y) \propto (2y^2-1)\exp\left(-\frac{y^2}{2}\right)$$

Result: Zonally propagating waves $K(x,t)$, $R_m(x,t)$, etc.

Dispersion curves for equatorial shallow water eqns.





Dry fluid dynamics of the tropical atmosphere

Summary

- Primitive equations $(x, y, z) \longrightarrow$ equatorial shallow water equations (x, y)
  - Expand in vertical modes $\cos(jz)$
- Equatorial shallow water equations $(x, y) \longrightarrow$ zonally propagating waves (x)
  - Expand in meridional modes $\phi_m(y)$

Result: Zonally propagating waves (Kelvin, Rossby, etc.)

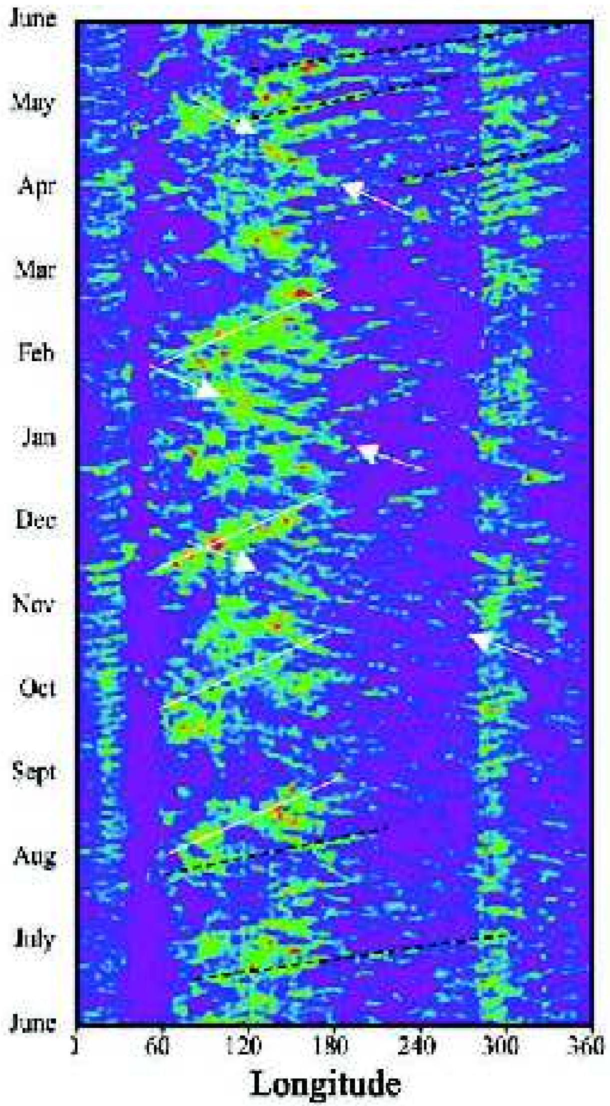
The Madden–Julian Oscillation (MJO)

in the Observational Record

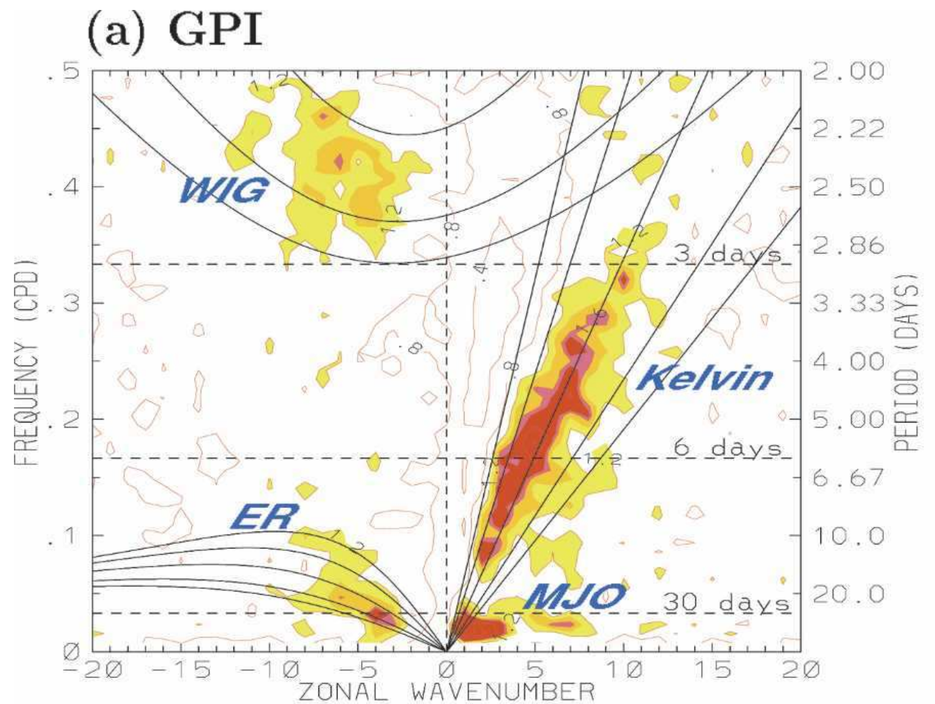
Observations of the MJO

Precipitation

2000–2001 (from Zhang 2005)



Spectral Power



from Lin et al. 2006

MJO: slow eastward propagation ≈ 5 m/s

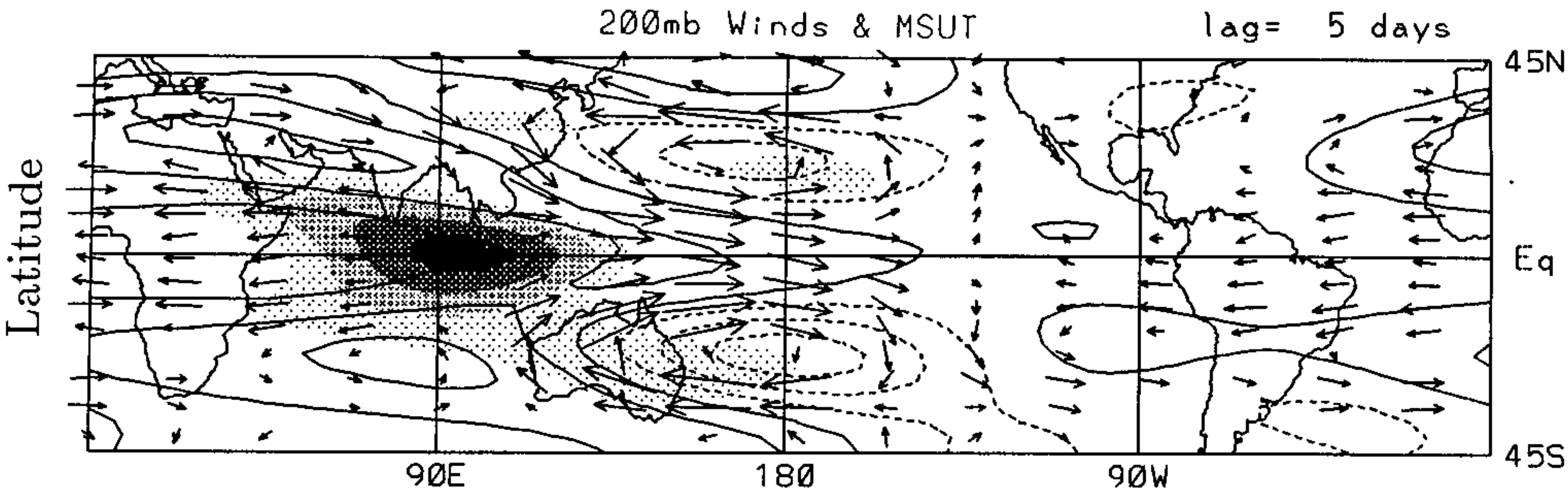
MJO: peculiar dispersion relation $\frac{d\omega}{dk} \approx 0$

MJO is an envelope of smaller-scale convection/waves

Horizontal structure of MJO

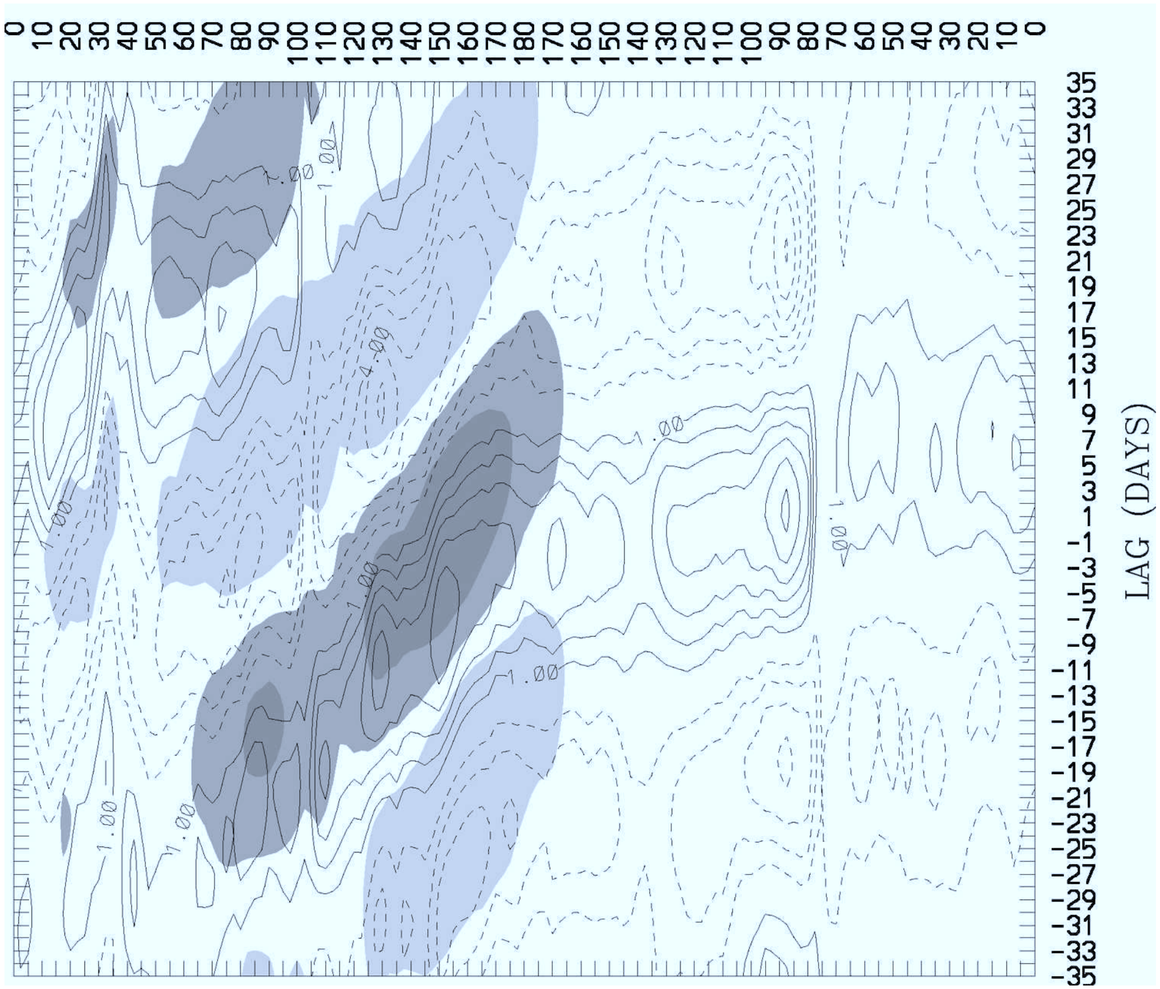
Quadrupole vortices:

Hendon and Salby (1994)



Moisture preconditioning in the MJO

Kiladis, Straub, & Haertel (2005)



Lower tropospheric moisture (contours) *leads* enhanced convection (dark shading)

A new model for the MJO

Majda and Stechmann (2009) PNAS

The Skeleton of Tropical Intraseasonal Oscillations

Majda and Stechmann (2011) JAS, submitted

Nonlinear Dynamics and Regional Variations in the MJO Skeleton

Fundamental mechanism proposed for MJO skeleton

Minimal, nonlinear oscillator model

Neutrally stable interactions between

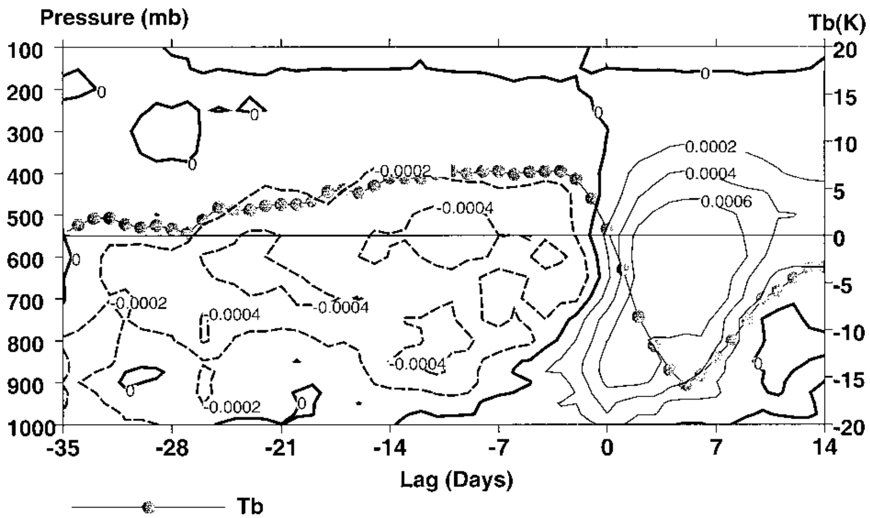
1. planetary-scale, lower-tropospheric moisture
2. sub-planetary-scale, convection/wave activity

- Tacit assumption: primary instabilities/damping occur on synoptic scales

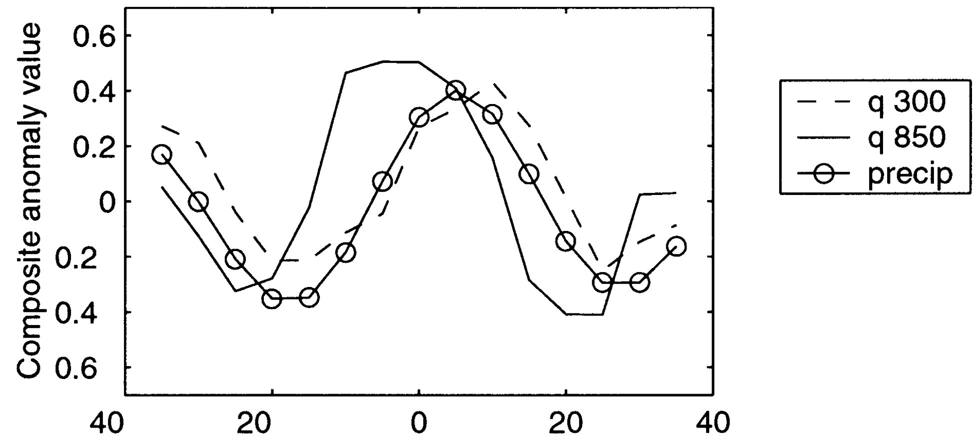
- MJO "muscle" from other potential upscale transport effects from synoptic scales
  - convective momentum transports from synoptic-scale waves
  - variations in land–sea contrasts and surface fluxes

Convective Activity on Planetary/Intraseasonal Scales

Kemball–Cook and Weare (2001)



Myers and Waliser (2003)



Lower-tropospheric moisture *leads* convective activity, suggesting ...

q : lower-tropospheric moisture

a : amplitude of convective activity envelope

Not mainly $a = a(q)$,

but

$$\partial_t a = \Gamma q a$$

Minimal nonlinear oscillator model

$$
\begin{aligned}
u_t - yv &= -p_x \\
yu &= -p_y \\
0 &= -p_z + \theta \\
u_x + v_y + w_z &= 0 \\
\theta_t + w &= \bar{H}a - s^\theta \\
q_t - \tilde{Q}w &= -\bar{H}a + s^q \\
\\
a_t &= \Gamma q a
\end{aligned}
$$

Linearized primitive equations

- Equatorial long-wave scaling
- Coriolis term: equatorial β-plane approx.

+

Dynamic equation for convective activity

- q: lower tropospheric moisture
- a: amplitude of convective activity envelope

Key mechanism: positive q creates a tendency to enhance convective activity a

Minimal number of parameters: $s^\theta, \tilde{Q}, \Gamma$

Minimal nonlinear oscillator model

(vertical truncation)

$$\boxed{\begin{aligned} u_t - yv - \theta_x &= 0 \\ yu - \theta_y &= 0 \\ \theta_t - u_x - v_y &= \bar{H}a - s^\theta \\ q_t + \tilde{Q}(u_x + v_y) &= -\bar{H}a + s^q \\ a_t &= \Gamma q a \end{aligned}}$$

- Truncate at first vertical baroclinic mode
- Matsuno–Gill-like model
  without dissipative mechanisms
  but with
  - lower tropospheric moisture, q
  - envelope of synoptic-scale wave activity, a, provides dynamic planetary-scale heating

Conserved energy:

$$\partial_t \left[\frac{1}{2}u^2 + \frac{1}{2}\theta^2 + \frac{1}{2}\frac{\tilde{Q}}{1-\tilde{Q}}\left(\theta + \frac{q}{\tilde{Q}}\right)^2 + \frac{\bar{H}}{\Gamma\tilde{Q}}a \right] - \partial_x(u\theta) - \partial_y(v\theta) = 0. \qquad (1)$$

Minimal nonlinear oscillator model

(vertical and meridional truncation)

$$K_t + K_x = -\frac{1}{\sqrt{2}}\bar{H}A$$

$$R_t - \frac{1}{3}R_x = -\frac{2\sqrt{2}}{3}\bar{H}A$$

$$Q_t + \frac{1}{\sqrt{2}}\tilde{Q}K_x - \frac{1}{6\sqrt{2}}\tilde{Q}R_x = \left(-1 + \frac{1}{6}\tilde{Q}\right)\bar{H}A$$

$$A_t = \Gamma Q(\bar{A} + A)$$

Meridional structures:

K: Kelvin wave

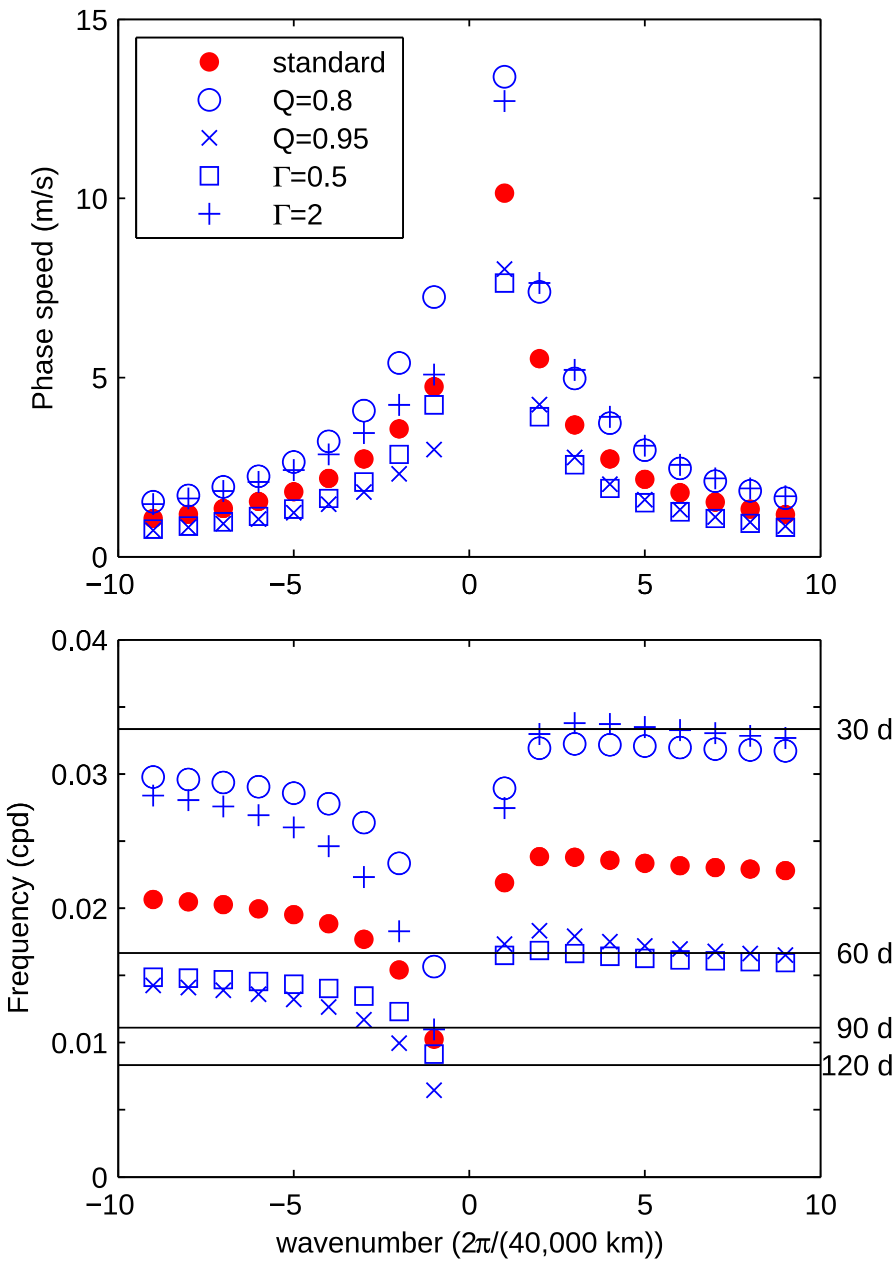
R: first symmetric equatorial Rossby wave

Q: $\exp(-y^2/2)$

A: $\exp(-y^2/2)$

Linear Theory

Phase speed and oscillation frequency



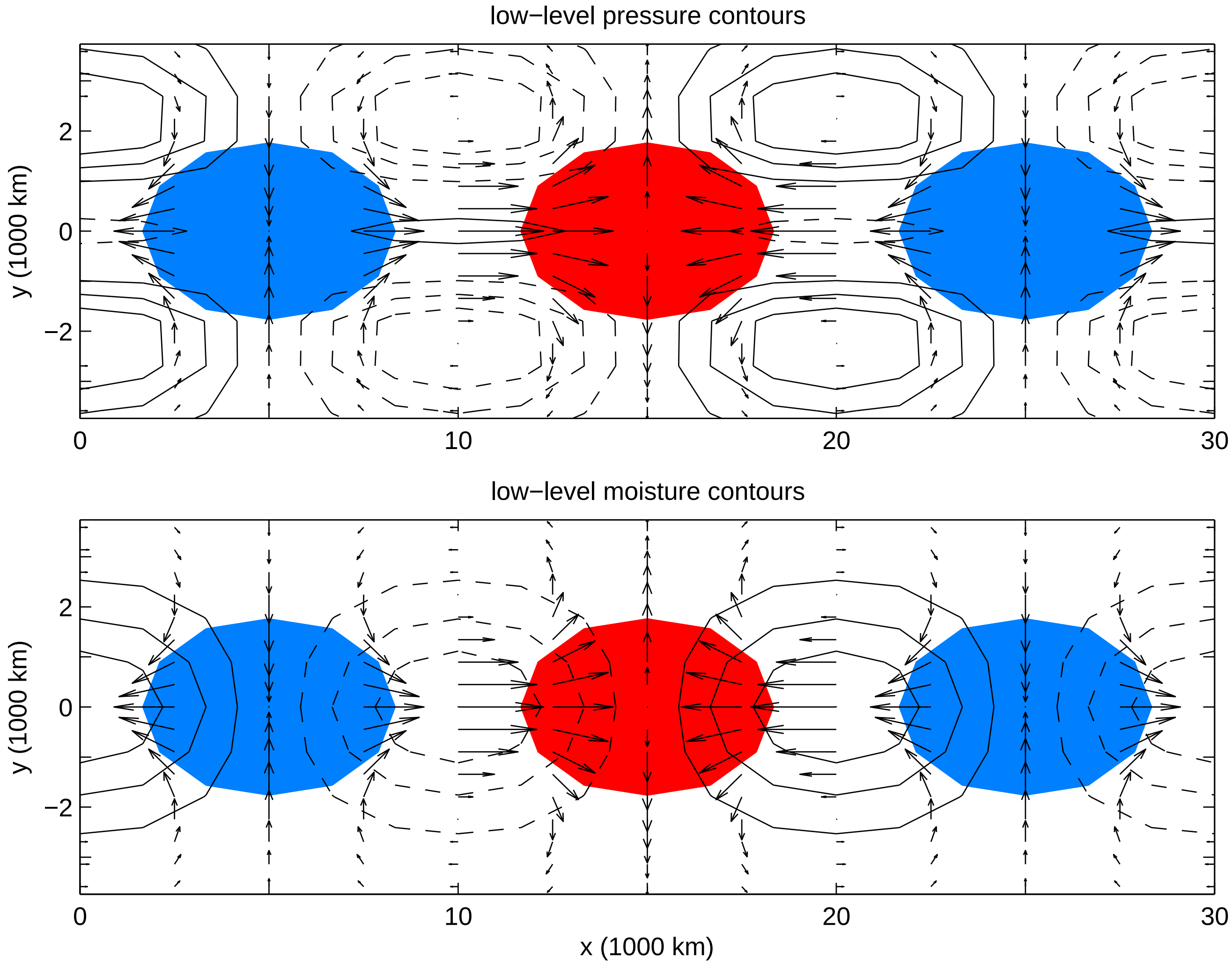
- Phase speeds of ≈ 5 m/s
- Results robust over parameter space

- Eastward MJO branch: $\frac{d\omega}{dk} \approx 0$ on *intraseasonal* time scales

$$\omega \approx \sqrt{\Gamma \bar{R}(1 - \tilde{Q})}$$

- Westward branch: *seasonal* time scales for wavenumbers 1 and 2

Physical structure of MJO skeleton



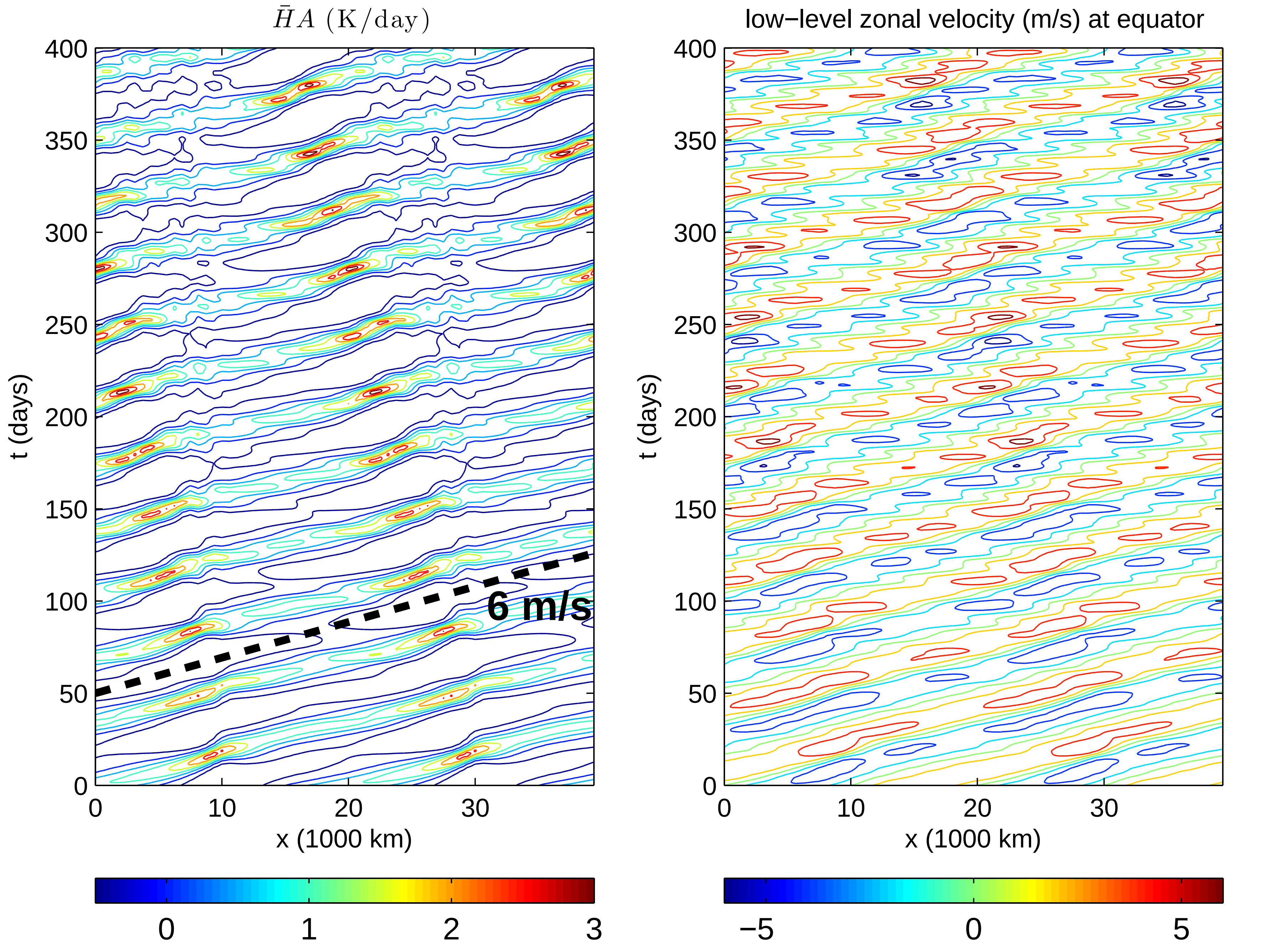
suppressed convection ($A < 0$)

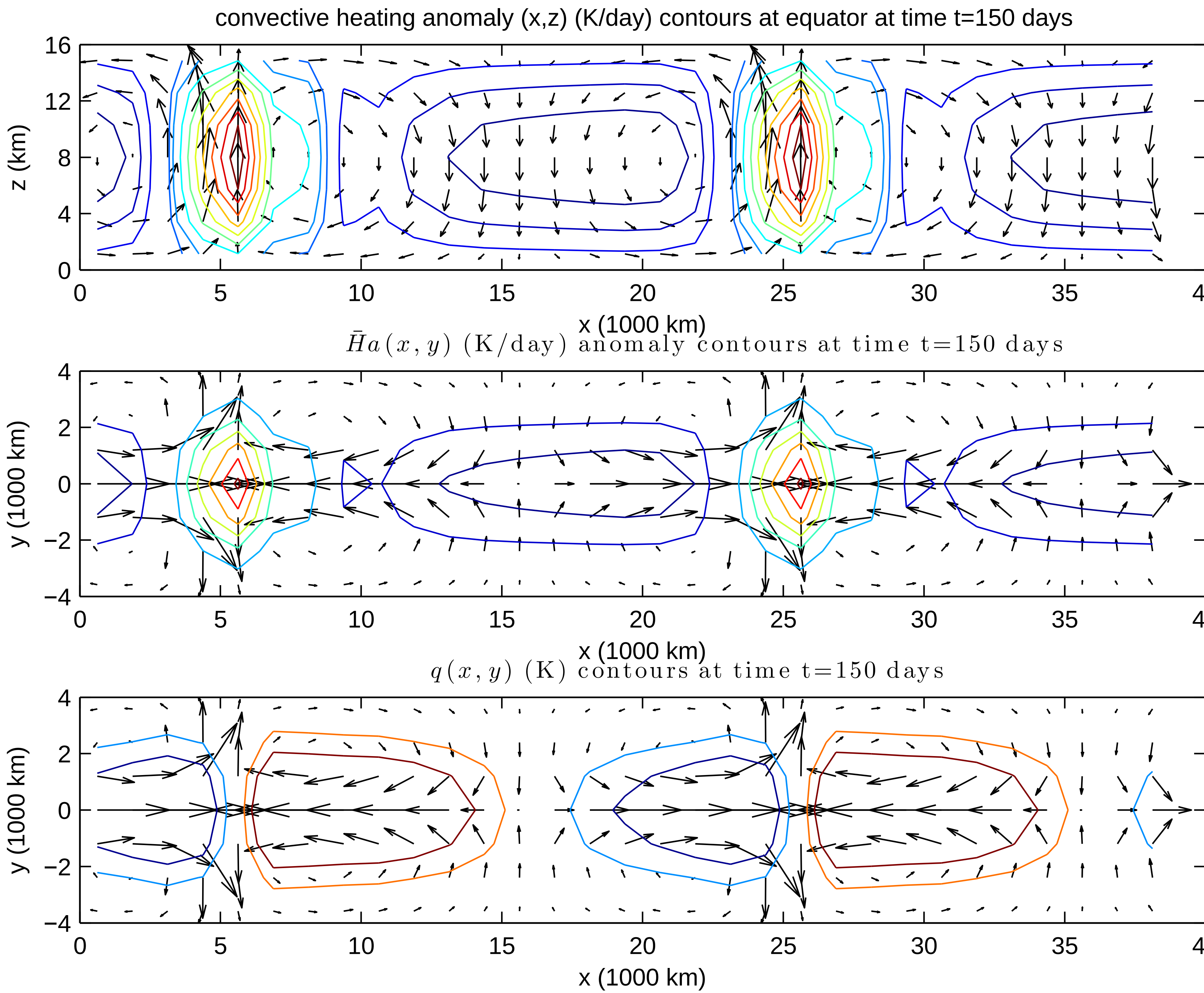
enhanced convection ($A > 0$)

- horizontal quadrupole vortices
- moisture leads convection
- Kelvin wave structure on equator
- off-equatorial quadrupole Rossby gyres

Nonlinear Simulations

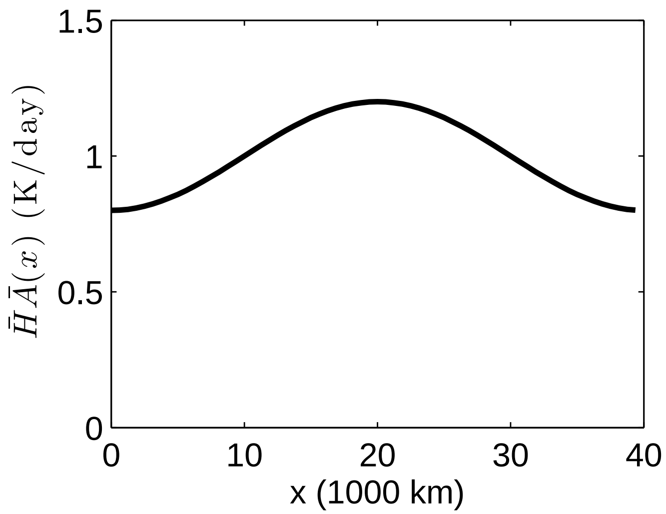
Constant SST

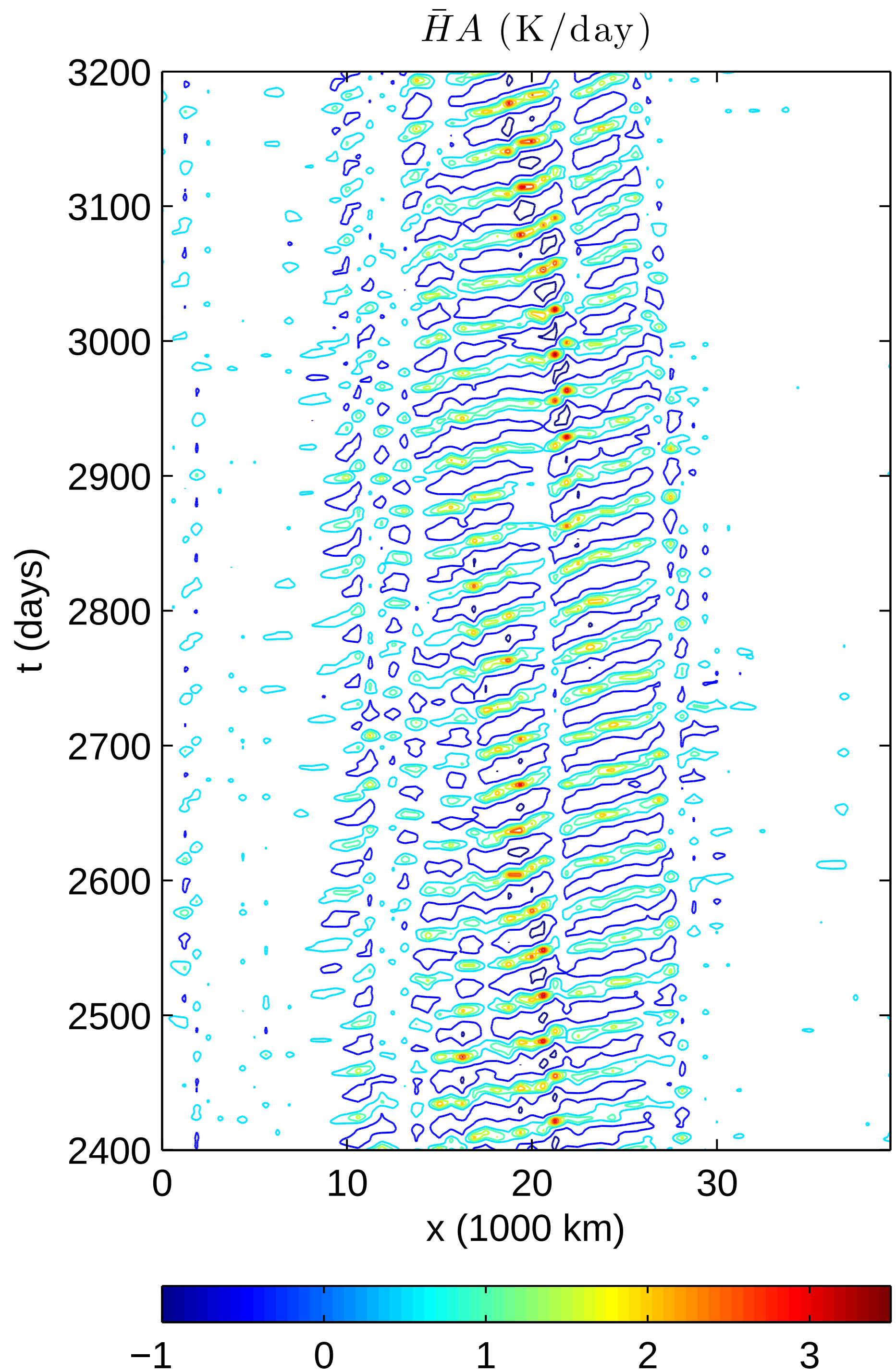

$\bar{H}A$ (K/day)
low−level zonal velocity (m/s) at equator
t (days)
x (1000 km)
6 m/s
400
350
300
250
200
150
100
50
0
10
20
30
1
2
3
−5
5

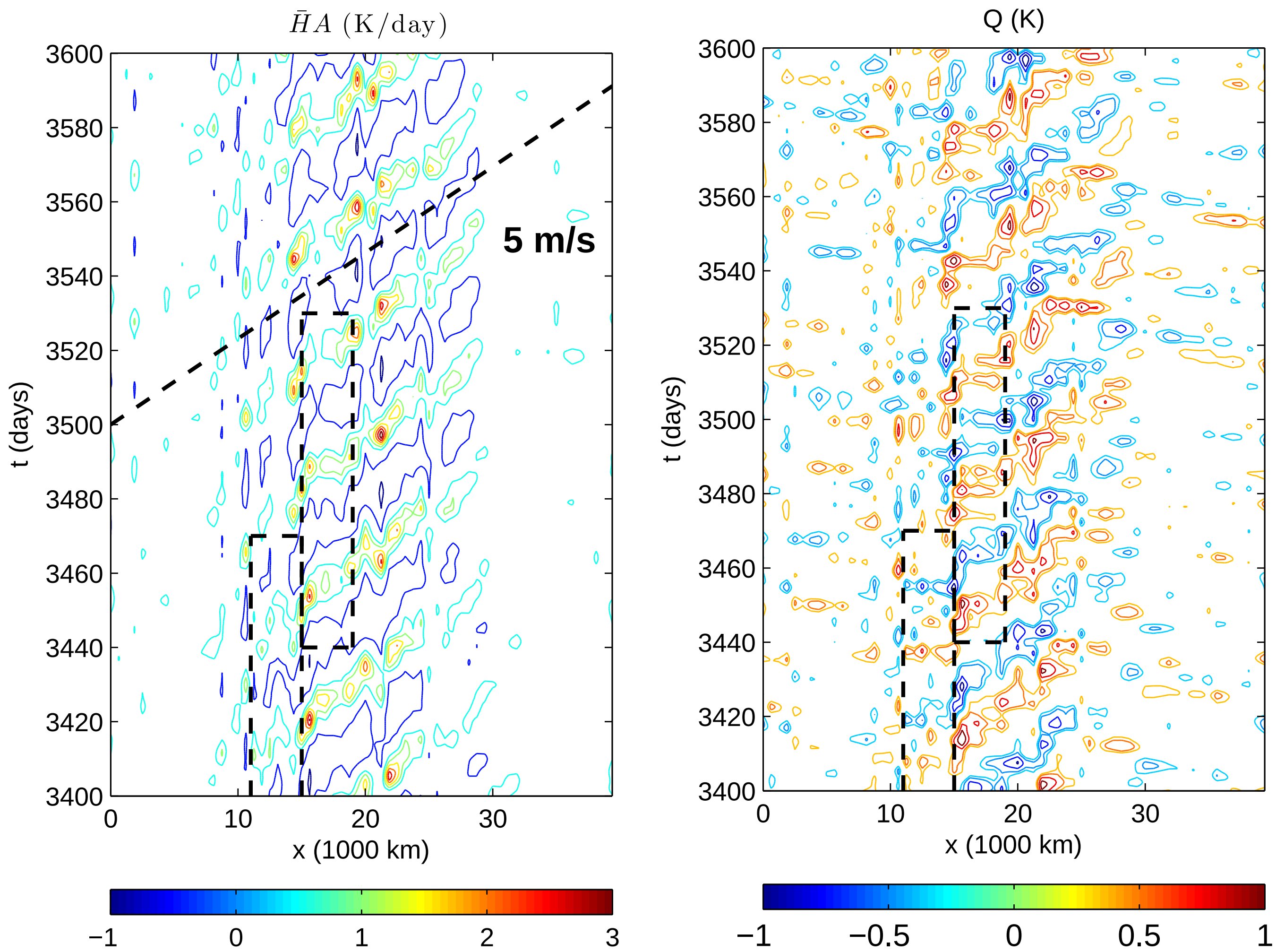


convective heating anomaly (x,z) (K/day) contours at equator at time t=150 days
z (km)
x (1000 km)
$\bar{H}a(x,y)$ (K/day) anomaly contours at time t=150 days
y (1000 km)
x (1000 km)
$q(x,y)$ (K) contours at time t=150 days
y (1000 km)
x (1000 km)

Warm-Pool SST




$\bar{H}A$ (K/day)
t (days)
3200
3100
3000
2900
2800
2700
2600
2500
2400
0
10
20
30
x (1000 km)
−1
0
1
2
3



$\bar{H}A$ (K/day)
Q (K)
t (days)
x (1000 km)
5 m/s
3600
3580
3560
3540
3520
3500
3480
3460
3440
3420
3400
0
10
20
30
−1
0
1
2
3
−0.5
0.5

Outline

1. A model for the MJO "skeleton"

    - A minimal, nonlinear oscillator model

    - Based on multiscale ideas

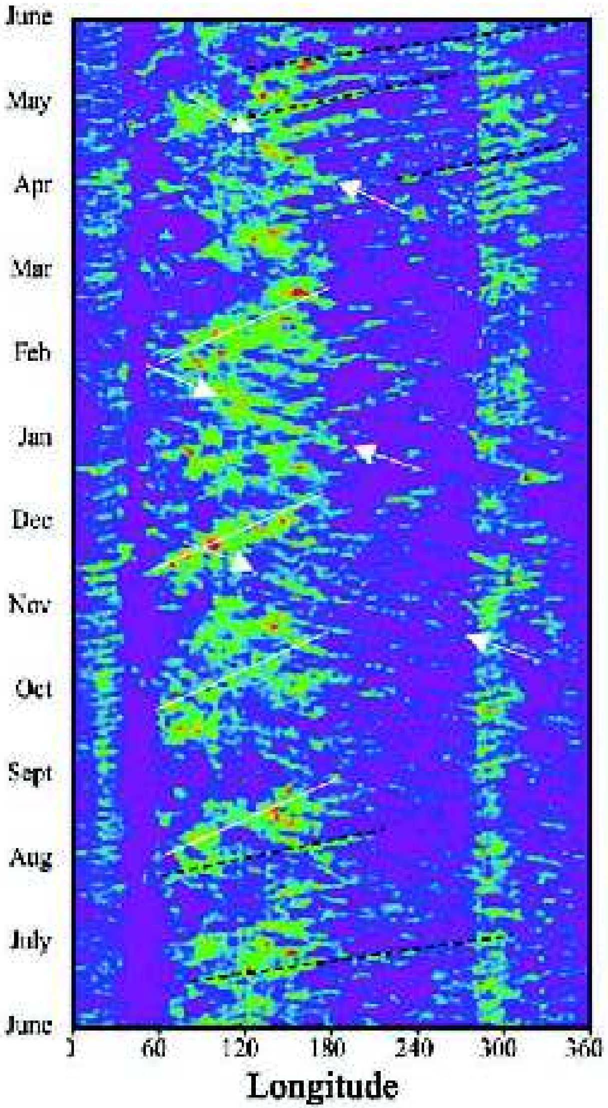
2. A model for part of the MJO's "muscle"

    - Multiscale model for convectively coupled wave–mean flow interactions

    - Acceleration of lower tropospheric jet during westerly wind burst (due to convective momentum transports)

The Madden–Julian Oscillation (MJO) is an envelope of smaller-scale convection/waves

Precipitation



2000–2001 (from Zhang 2005)

How does the MJO envelope interact with the CCW within it?

- MJO $\longrightarrow$ CCW?
- MJO $\longleftarrow$ CCW?
- MJO $\longleftrightarrow$ CCW?

Here: focus on momentum/wind shear rather than thermodynamics

2 important multi-scale effects

$$\frac{\partial u}{\partial t} + u\partial_x u + w\partial_z u = \cdots$$

$$u = \bar{u} + u'$$

1. Eddy momentum flux

   "Convective momentum transport" (CMT)

$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

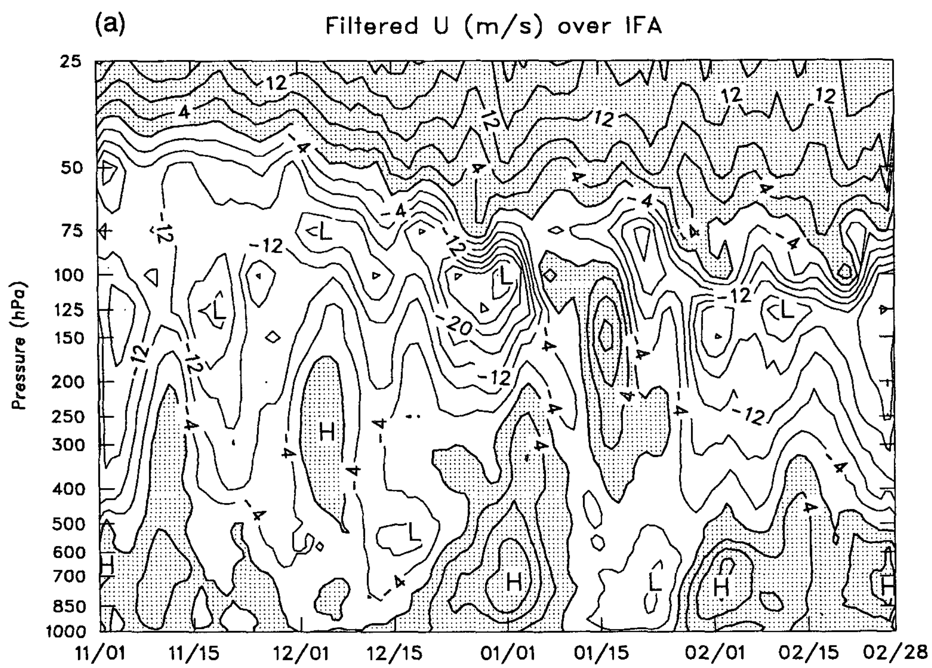
2. Background wind shear

$$\frac{\partial u'}{\partial t} + \bar{u}\partial_x u' + w'\partial_z \bar{u} = \cdots$$

CMT from CCW can drive the westerly wind burst aloft

Majda and Biello (2004),
Biello and Majda (2005):

Kinematic multi-scale model including CMT due to CCW (one-way interaction)



Lin and Johnson (1996)

Majda and Stechmann (2009) JAS:

also include effect of mean flow $\bar{U}$ on CCW to give *dynamic* multi-scale model with two-way interactions between CCW and mean flow

Dynamic multi-scale model for convectively coupled wave–mean flow interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables: $\bar{\Theta}$, θ', $\bar{Q}$, q', etc.)

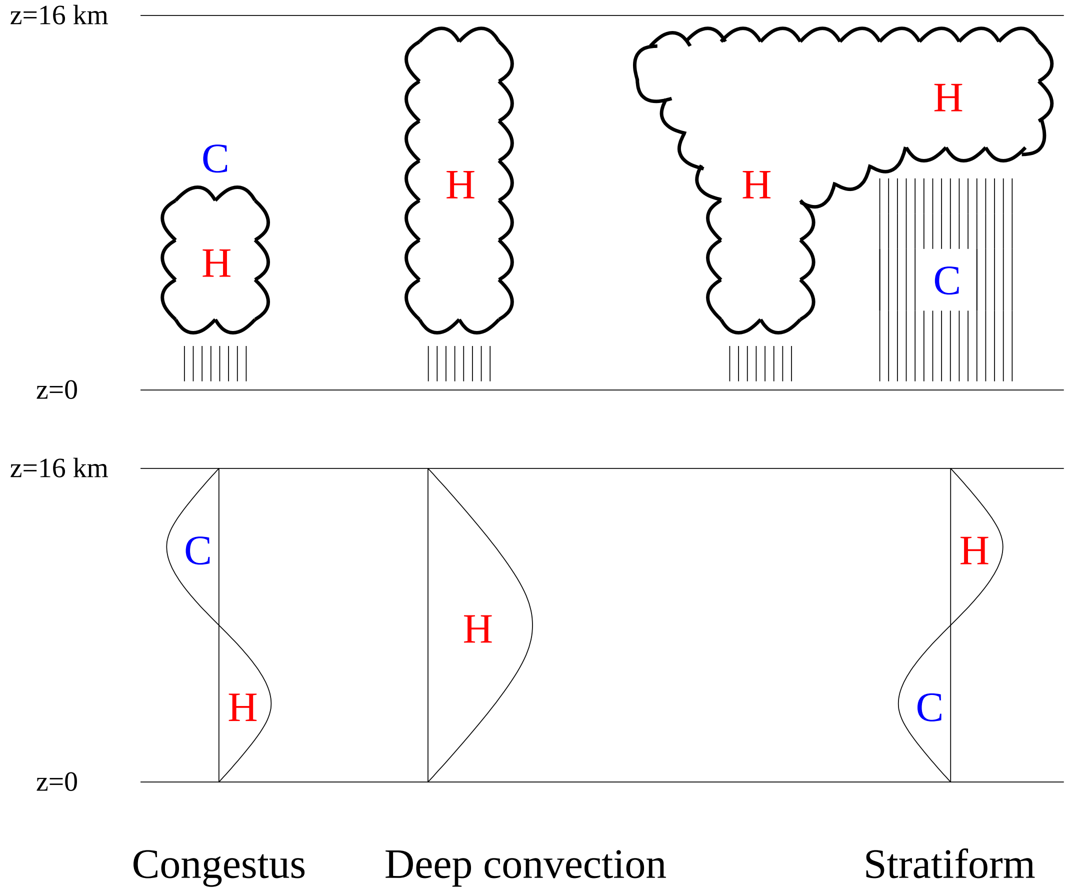
Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z\langle\overline{w'u'}\rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

Need convectively coupled waves with *tilts* to have nonzero $\partial_z\langle\overline{w'u'}\rangle$

The Multicloud Model (Khouider and Majda 2006)
(a model for CCW)



Two vertical baroclinic modes $\Rightarrow$ waves with vertical tilts

Multi-scale effects: add nonlinear advection and a 3rd baroclinic mode

Dynamic multi-scale model for convectively coupled wave–mean flow interaction

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$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

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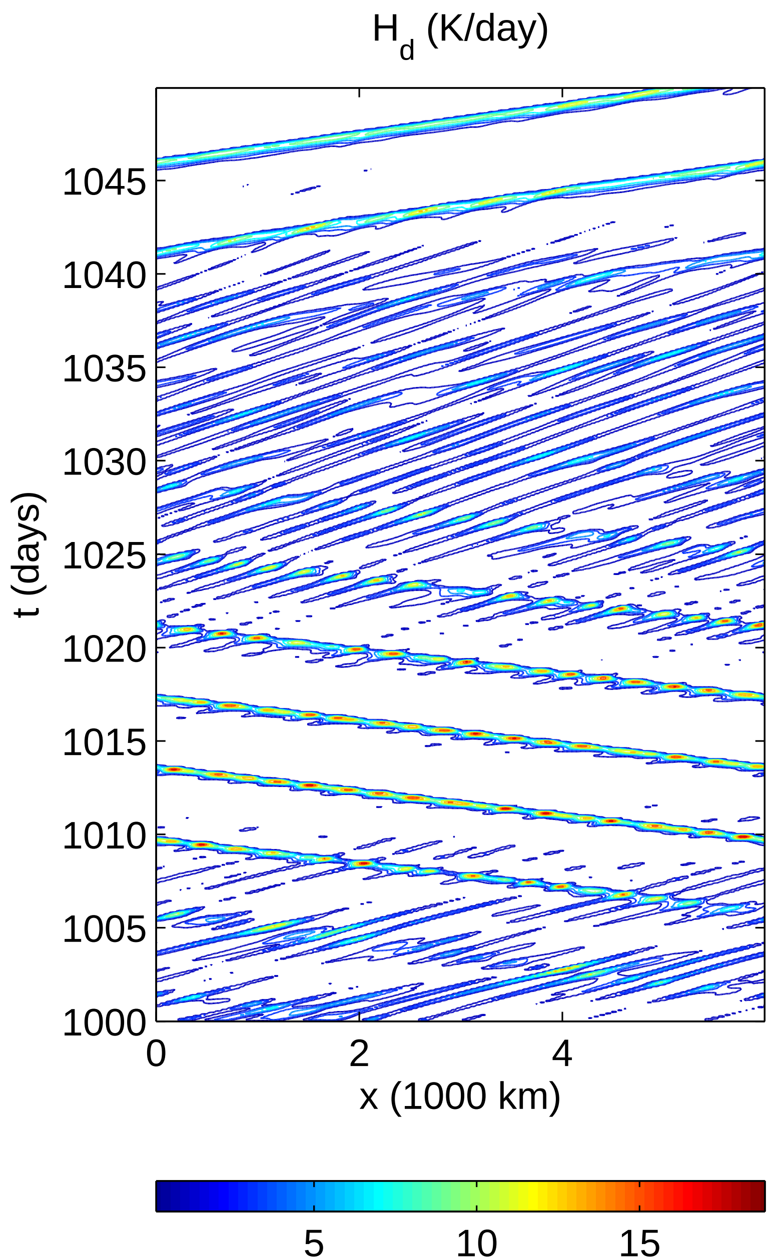
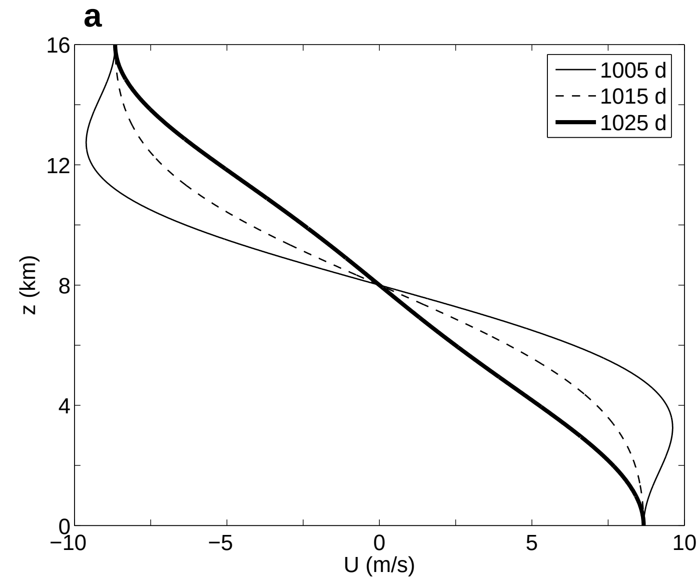
CCW–mean flow interactions on intraseasonal time scale

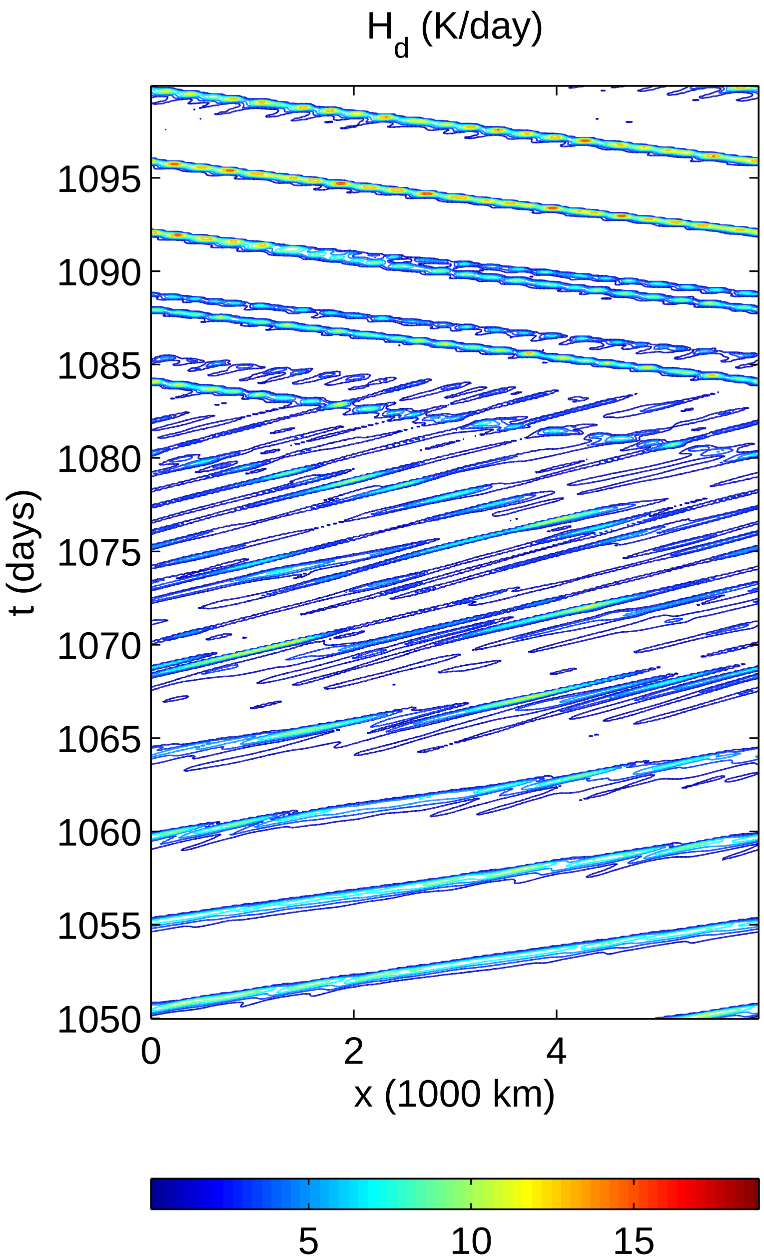


Mean wind $\bar{U}(z)$:

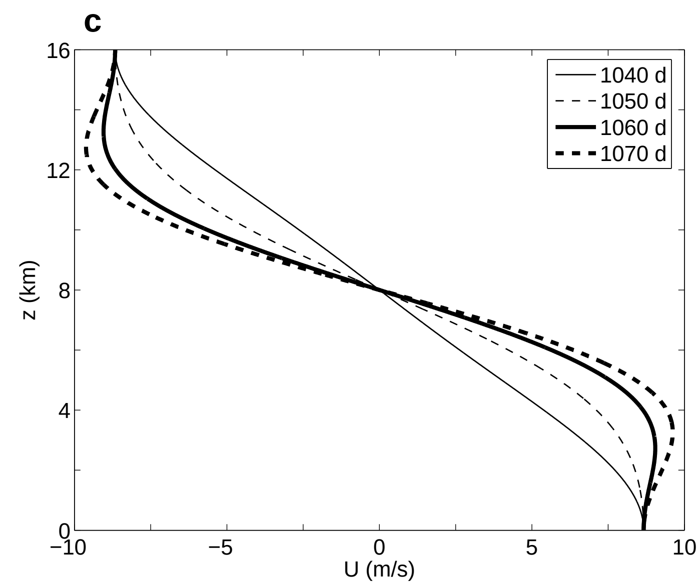


- Momentum transports from CCW drive changes in mean wind
- Advection by mean wind changes wave propagation direction
- Squall line-like fluctuations within wave envelope

Westerly Wind Burst Intensification



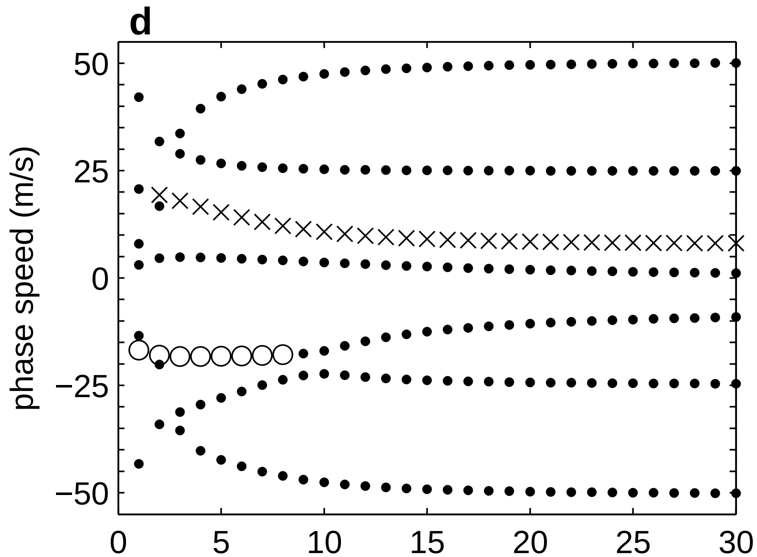
Mean wind $\bar{U}(z)$:

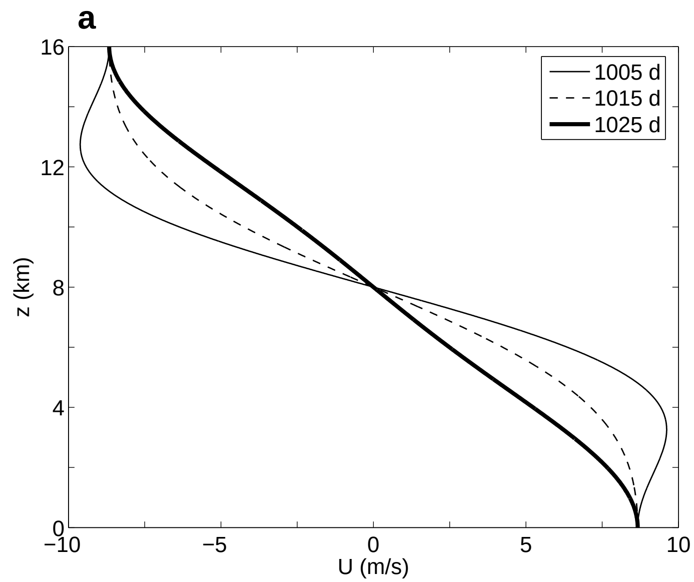


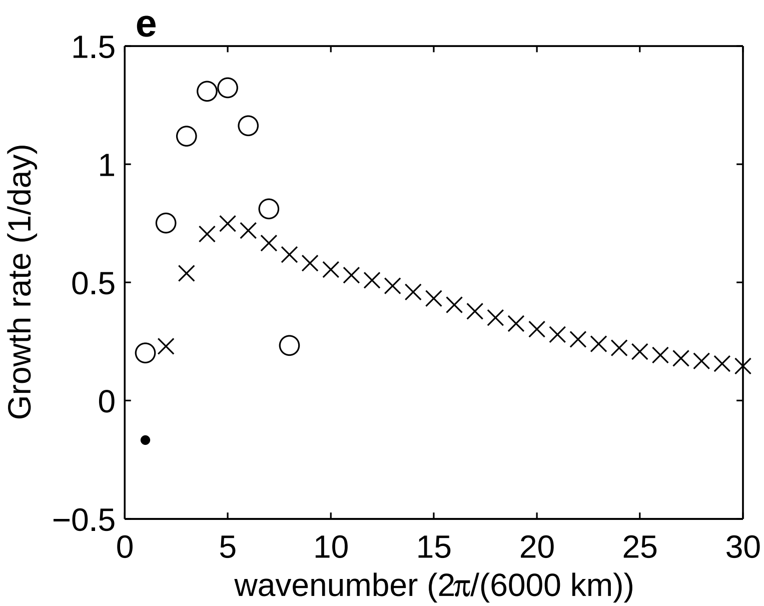
- Upscale CMT from eastward-moving CCW accelerates WWB aloft

Linear Stability Theory

$t = 1005$ days







Propagating envelopes of smaller-scale convection:

- Westward-propagating CCW favored at larger scales
- Eastward-propagating convection (squall line-like) favored at smaller scales

Summary

- Minimal *nonlinear oscillator* model for the MJO skeleton
  - slow eastward phase speed of ≈ 5 m/s
  - peculiar dispersion relation with $d\omega/dk \approx 0$
  - horizontal quadrupole vortex structure
  - asymmetries: stronger, narrower enhanced convection region
  - both standing oscillations and eastward propagation
- Multiscale model for part of the MJO's "muscle"
  - convectively coupled wave–mean flow interactions
  - convective momentum transport $\longleftrightarrow$ wind shear
  - acceleration of lower-tropospheric jet during westerly wind burst