Multi-scale Theories and Models for the Madden–Julian Oscillation

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Workshop on Modelling Monsoon Intraseasonal Variability

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Outline

1. The skeleton of tropical intraseasonal oscillations
   Majda and Stechmann (2009) *Proc. Natl. Acad. Sci.*

   - Minimal dynamical model for the MJO's "skeleton"
   - Recovers robustly the MJO's fundamental features on intraseasonal/planetary scales

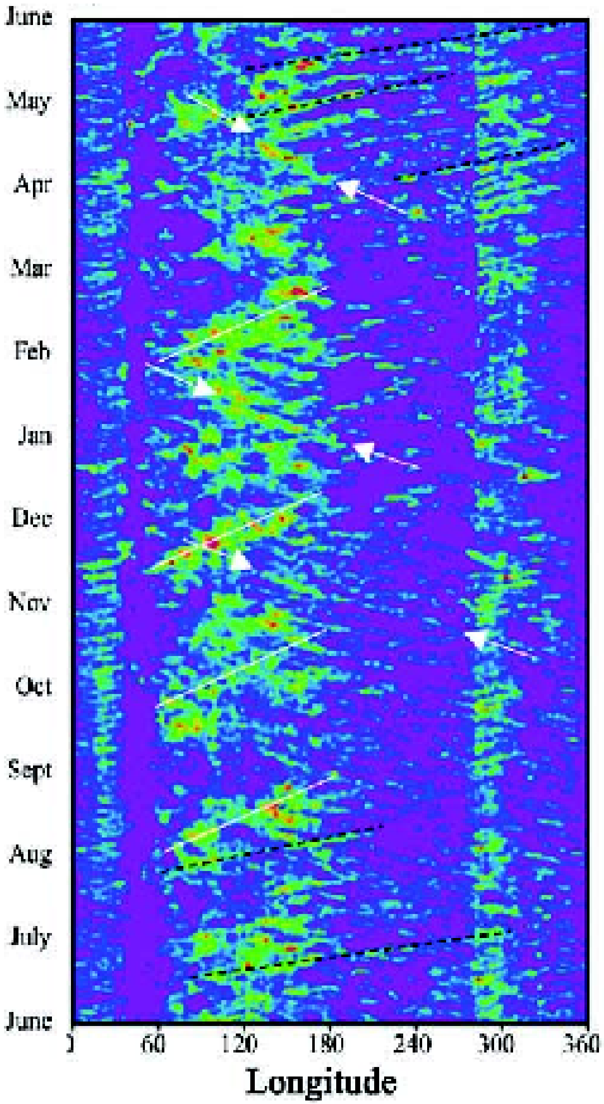
2. A simple dynamical model with features of convective momentum transport
   Majda and Stechmann (2009) *J. Atmos. Sci*

   - One aspect of the MJO's "muscle"
   - Convectively coupled wave–mean flow interactions

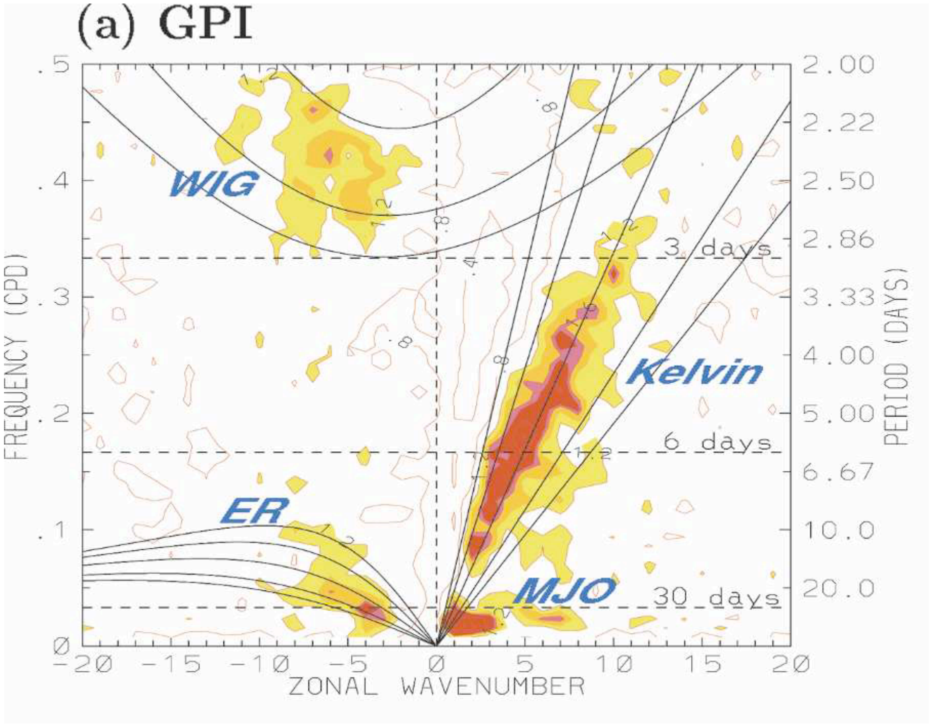
Observed features of the MJO

Precipitation

2000–2001 (from Zhang 2005)



Spectral Power



from Lin et al. 2006

MJO: slow eastward propagation ≈ 5 m/s

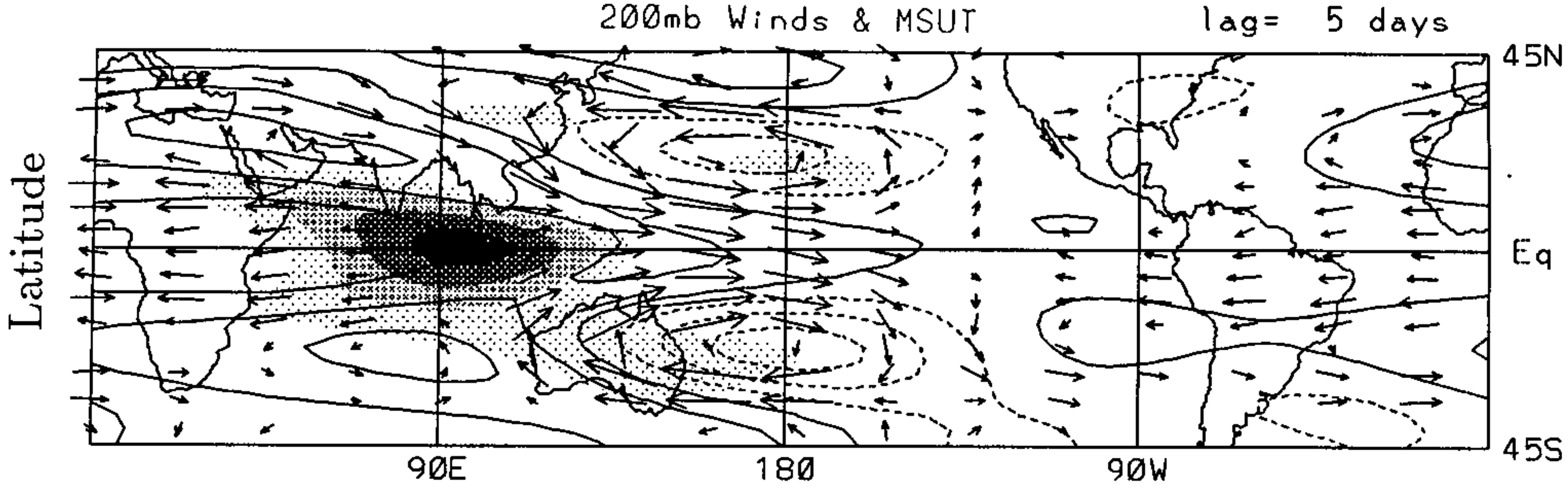
MJO: peculiar dispersion relation $\frac{d\omega}{dk} \approx 0$

MJO is envelope of smaller-scale convectively coupled waves

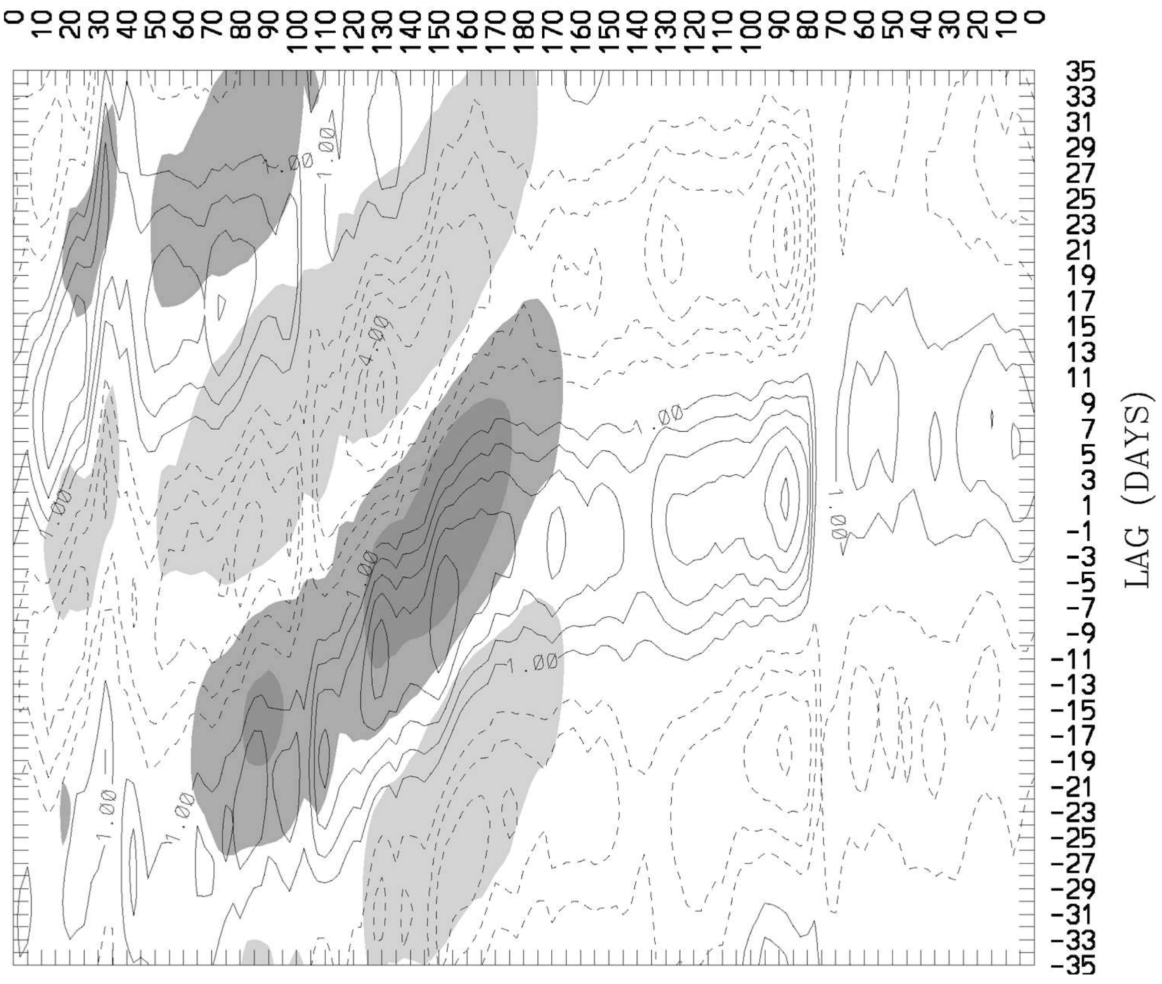
Horizontal structure of MJO

Quadrupole vortices:

Hendon and Salby (1994)



Moisture preconditioning in the MJO



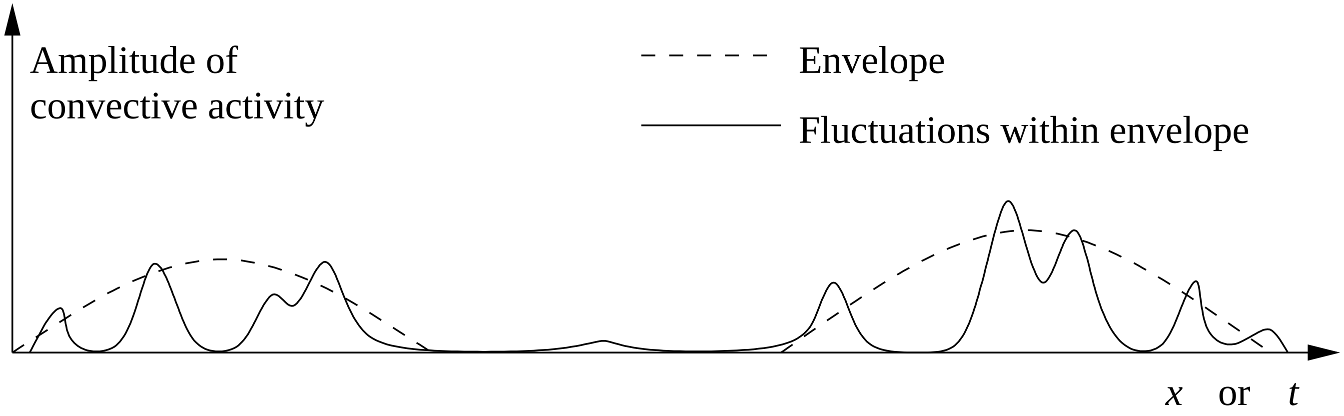
Kiladis et al (2005)

Lower tropospheric moisture (contours) leads enhanced convection (dark shading)

Fundamental mechanism proposed for MJO skeleton

Neutrally stable interactions between

1. planetary-scale, lower-tropospheric moisture
2. synoptic-scale, convectively-coupled-wave activity



- Tacit assumption: primary instabilities/damping occur on synoptic scales
- MJO "muscle" from other potential upscale transport effects from synoptic scales
  - convective momentum transports from synoptic-scale waves
  - variations in surface heat fluxes

Minimal dynamical model

$$\begin{aligned} u_t - yv &= -p_x \\ yu &= -p_y \\ 0 &= -p_z + \theta \end{aligned}$$

$$u_x + v_y + w_z = 0$$

$$\begin{aligned} \theta_t + w &= \bar{H}a \\ q_t - \tilde{Q}w &= -\bar{H}a \\ a_t &= \Gamma q(\bar{a} + a) \end{aligned}$$

Momentum equations:

- Equatorial long-wave scaling
- Coriolis term: equatorial β-plane approx.
- Hydrostatic balance

Thermodynamic equations:

- q: lower tropospheric moisture
- a: amplitude of convective activity envelope

Key mechanism: positive q creates a tendency to enhance convective activity a

Minimal number of parameters: $\tilde{Q}, \Gamma, \bar{a}$

Minimal dynamical model

(vertical truncation)

$$\boxed{\begin{aligned} u_t - yv - \theta_x &= 0 \\ yu - \theta_y &= 0 \\ \theta_t - u_x - v_y &= \bar{H}a \\ q_t + \tilde{Q}(u_x + v_y) &= -\bar{H}a \\ a_t &= \Gamma \bar{a} q \end{aligned}}$$

- Truncate at first vertical baroclinic mode
- Matsuno–Gill-like model
  without dissipative mechanisms
  but with
  - lower tropospheric moisture, q
  - envelope of synoptic-scale wave activity, a, provides dynamic planetary-scale heating

Minimal dynamical model

(vertical and meridional truncation)

$$
\begin{aligned}
K_t + K_x &= -\frac{1}{\sqrt{2}}\bar{H}A \\
R_t - \frac{1}{3}R_x &= -\frac{2\sqrt{2}}{3}\bar{H}A \\
Q_t + \frac{1}{\sqrt{2}}\tilde{Q}K_x - \frac{1}{6\sqrt{2}}\tilde{Q}R_x &= \left(-1 + \frac{1}{6}\tilde{Q}\right)\bar{H}A \\
A_t &= \Gamma\bar{a}Q
\end{aligned}
$$

Meridional structures:

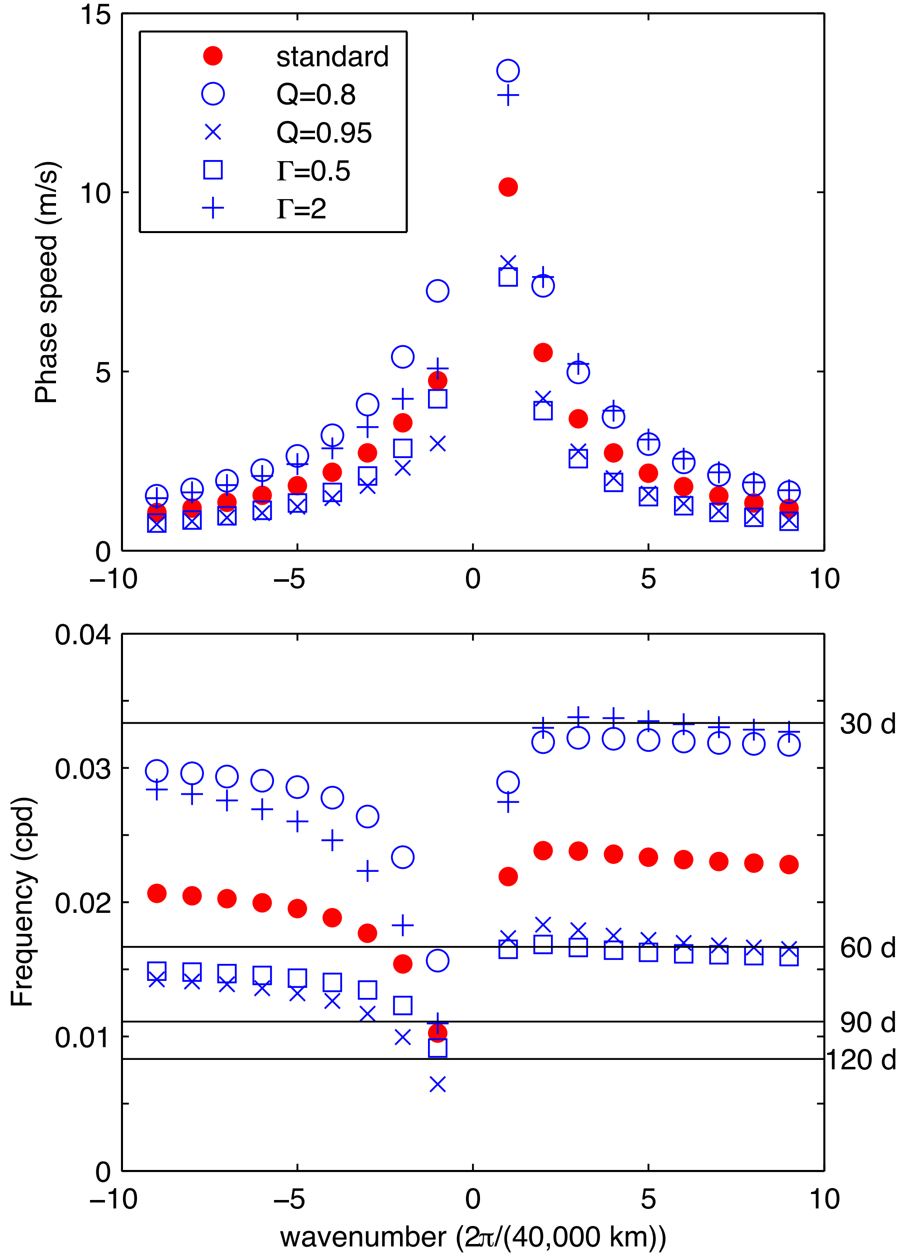
K: Kelvin wave

R: first symmetric equatorial Rossby wave

Q: $\exp(-y^2/2)$

A: $\exp(-y^2/2)$

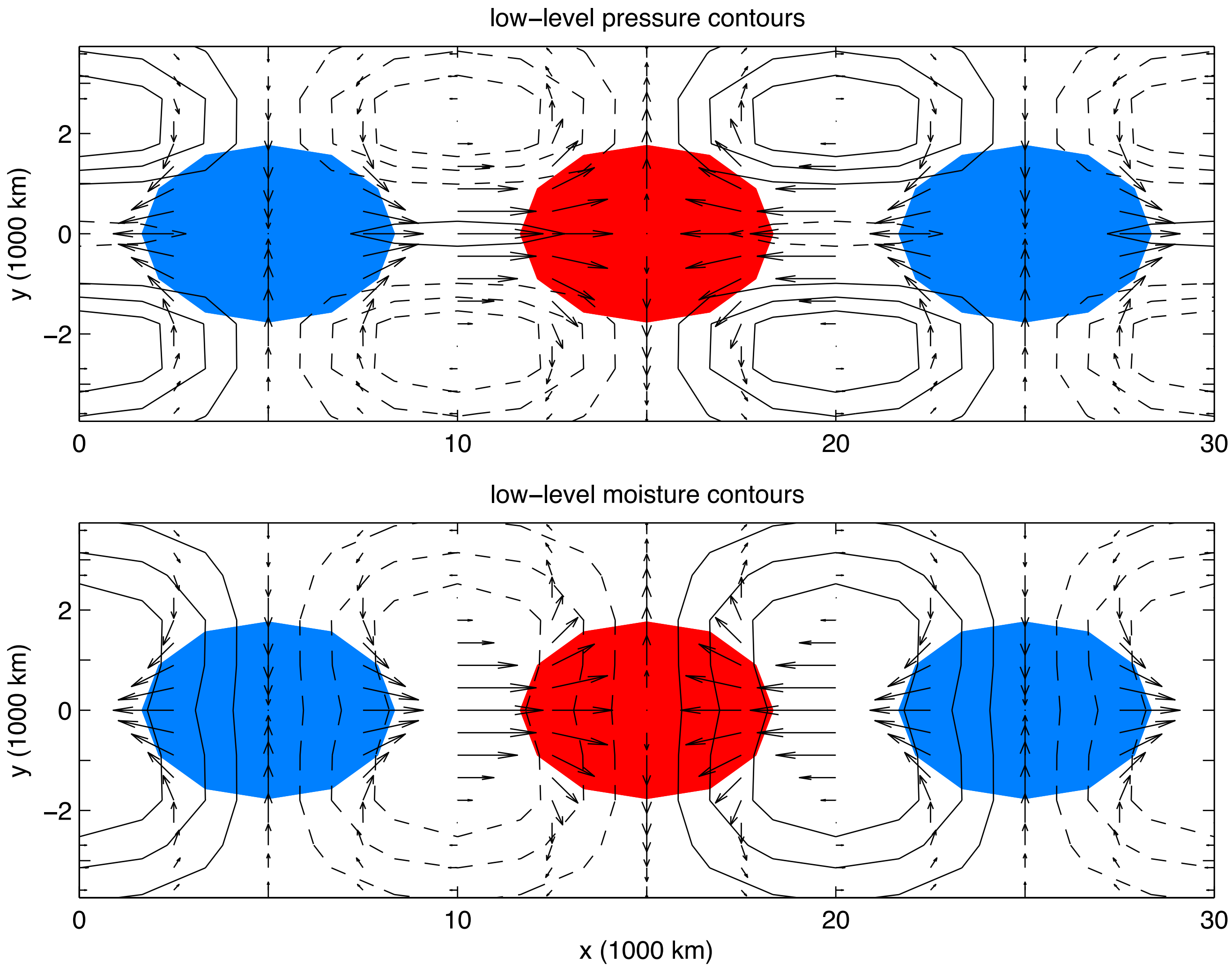
Phase speed and oscillation frequency



- Phase speeds of ≈ 5 m/s
- Results robust over parameter space

- Eastward MJO branch: $\frac{d\omega}{dk} \approx 0$ on *intraseasonal* time scales
- Westward branch: *seasonal* time scales for wavenumbers 1 and 2

Physical structure of MJO skeleton



suppressed convection ($A < 0$)

enhanced convection ($A > 0$)

- horizontal quadrupole vortices
- moisture leads convection

- Kelvin wave structure on equator
- off-equatorial quadrupole Rossby gyres

Formula for MJO frequency

Simplified case: 1D dynamics above the equator (ignore y variations and v)

- No rotation ⇒ Perfect east–west symmetry ⇒

Exact solution: $$2\omega^2 = \Gamma\bar{R} + k^2 \pm \sqrt{(\Gamma\bar{R} + k^2)^2 - 4\Gamma\bar{R}k^2(1 - \tilde{Q})}$$

Approx. solution: $$\omega \approx \sqrt{\Gamma\bar{R}(1 - \tilde{Q})}$$

- *Model recovers peculiar dispersion relation* $d\omega/dk \approx 0$
- *Simple formula for MJO frequency in terms of model parameters*

Summary of Part 1

- New minimal dynamical model for the MJO
- Robustly recovers the MJO's fundamental features (i.e., its "skeleton") on intraseasonal/planetary scales:
  - slow phase speed of ≈ 5 m/s
  - peculiar dispersion relation of $d\omega/dk \approx 0$
  - horizontal quadrupole vortex structure
- Simple formula for MJO oscillation frequency: $\omega \approx \sqrt{\Gamma \bar{R}(1-\tilde{Q})}$
- Explanation of preferred eastward propagation of intraseasonal variability
- Neutrally stable model on planetary/intraseasonal scales
  - Tacit assumption: primary instabilities on synoptic scales
- "Muscle" of MJO provided by other upscale transports from synoptic scales

Majda and Stechmann (2009) *Proc. Natl. Acad. Sci.* 106, 8417–8422

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   - Convectively coupled wave–mean flow interactions

2 important multi-scale effects

$$\frac{\partial u}{\partial t} + u\partial_x u + w\partial_z u = \cdots$$

$$u = \bar{u} + u'$$

1. Eddy momentum flux

   "Convective momentum transport" (CMT)

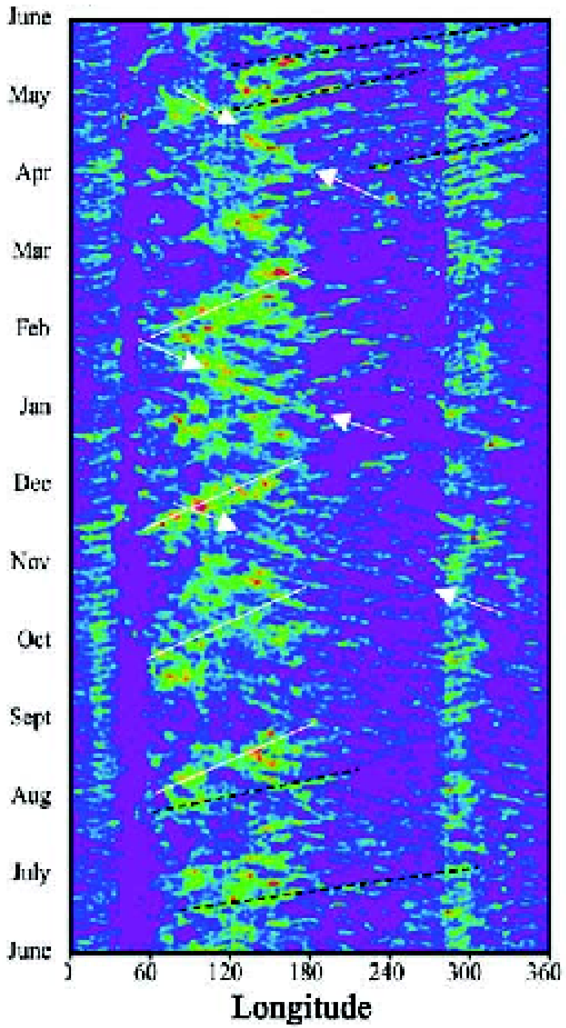
$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

2. Background wind shear

$$\frac{\partial u'}{\partial t} + \bar{u}\partial_x u' + w'\partial_z \bar{u} = \cdots$$

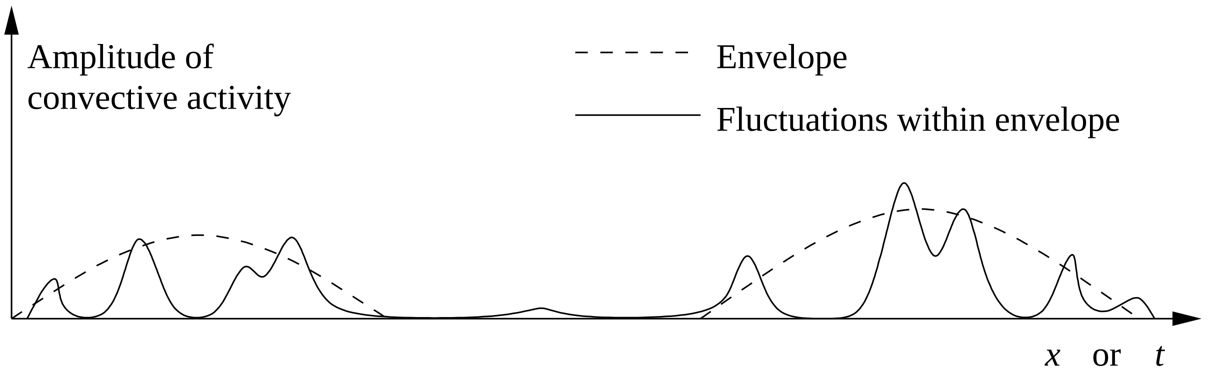
The Madden–Julian Oscillation (MJO) is an envelope of synoptic-scale convectively coupled waves (CCW)

Precipitation



2000–2001 (from Zhang 2005)

Schematic diagram



How does the MJO envelope interact with the waves embedded within it?

- MJO $\longrightarrow$ CCW?
- MJO $\longleftarrow$ CCW?
- MJO $\longleftrightarrow$ CCW?

Multi-scale organized convection

Key questions:

1. How does the MJO envelope interact with the waves embedded within it?
    - MJO $\longrightarrow$ CCW?
    - MJO $\longleftarrow$ CCW?
    - MJO $\longleftrightarrow$ CCW?
2. What is the missing physics of the MJO in GCMs?

    Proper representation of

    - CCW?
    - *interactions* between CCW and larger-scale environment (e.g., MJO)?

Here: focus on momentum/wind shear rather than thermodynamics

One effect of CCW on MJO envelope:

Convective Momentum Transport (CMT)

$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

Mesoscales and smaller:

CMT from squall lines and other mesoscale convection

- Moncrieff (1981), LeMone (1983), Moncrieff (1992), Wu and Yanai (1994), Tung and Yanai (2002), Moncrieff (2004)

Synoptic scales:

CMT from convectively coupled waves (CCW) (CCW-MT?)

- Can change velocity on the planetary scales (and MJO)
- Majda and Biello (2004), Biello and Majda (2005)

Kinematic multi-scale model

including CMT due to CCW

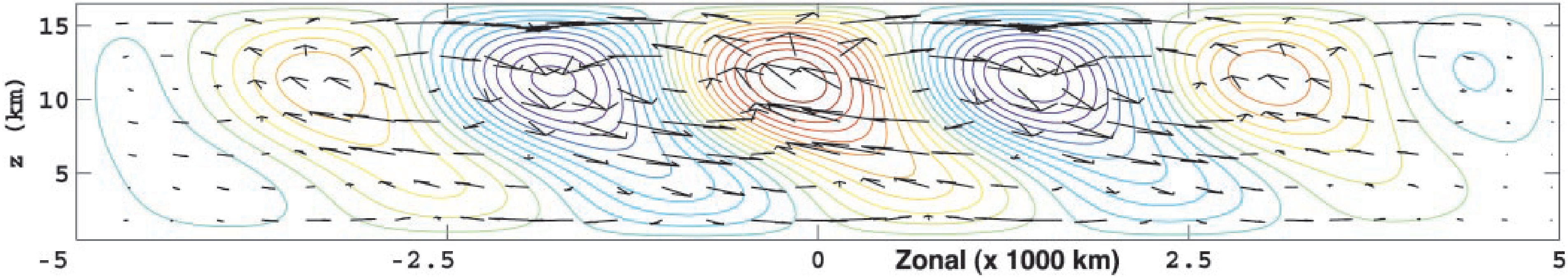
Majda and Biello (2004), Biello and Majda (2005)

- Kinematic model for CCW:

$$w' = S'_\theta(X, x, z, t), \text{ etc.}$$

- CMT from CCW drives mean flow:

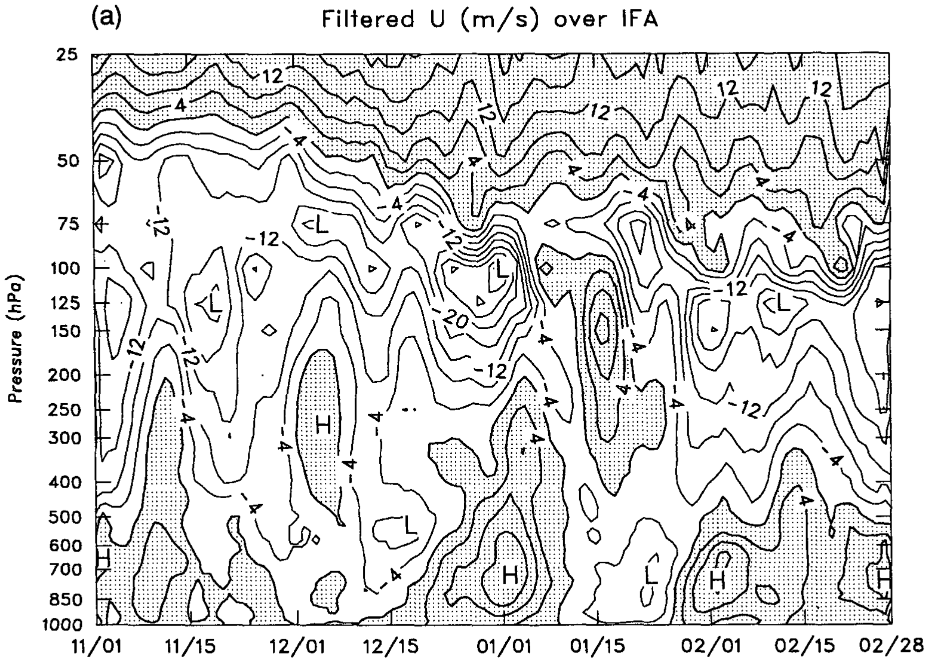
$$\partial_t \bar{U} = -\partial_z(\overline{w'u'}) + \cdots, \text{ etc.}$$



Kinematic multi-scale model
including CMT due to CCW

Majda and Biello (2004),
Biello and Majda (2005):

CMT from CCW drives
the westerly wind burst aloft



Lin and Johnson (1996)

Majda and Stechmann (2009):

also include effect of mean flow $\bar{U}$ on CCW to give *dynamic* multi-scale model with two-way interactions between CCW and mean flow

Dynamic multi-scale model for convectively coupled wave–mean flow interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle \overline{w'u'} \rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables)

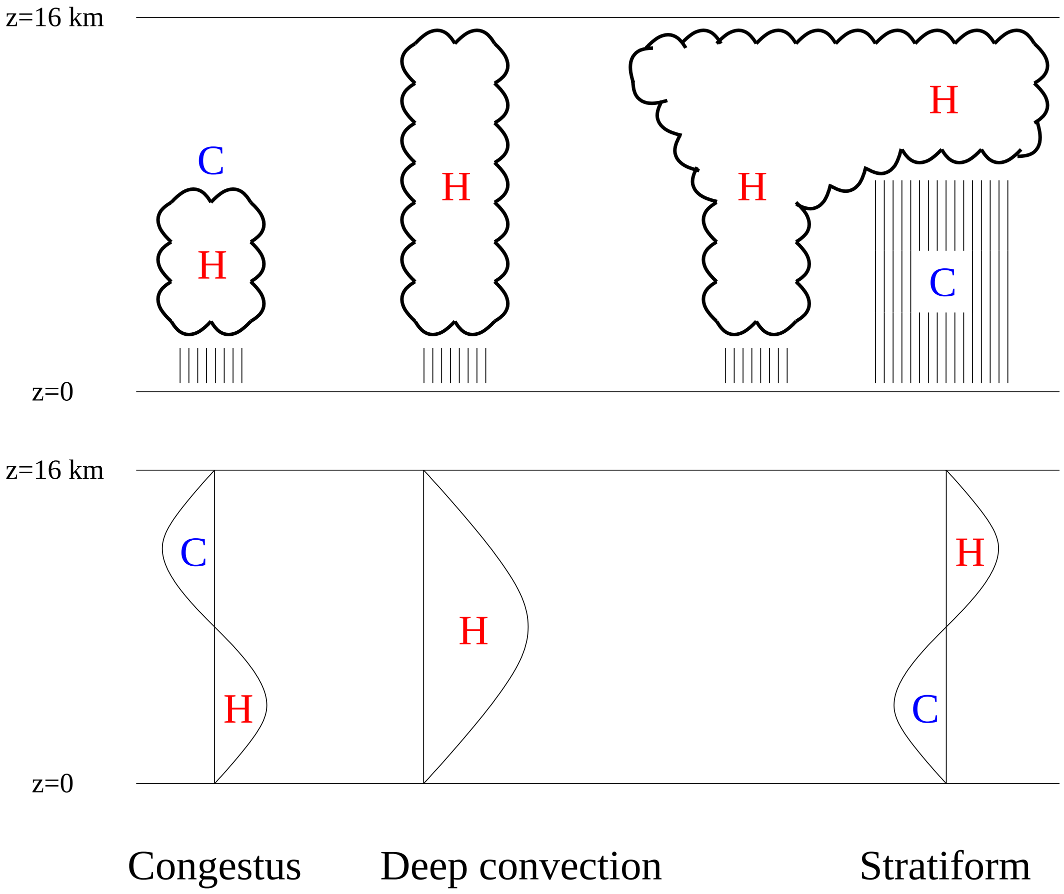
Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z \langle \overline{w'u'} \rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

Need convectively coupled waves with *tilts* to have nonzero $\partial_z \langle \overline{w'u'} \rangle$

The Multicloud Model (Khouider and Majda 2006)
(a model for CCW)



Two vertical baroclinic modes ⇒ waves with vertical tilts

Multi-scale effects: add nonlinear advection and a 3rd baroclinic mode

Dynamic multi-scale model for convectively coupled wave–mean flow interaction

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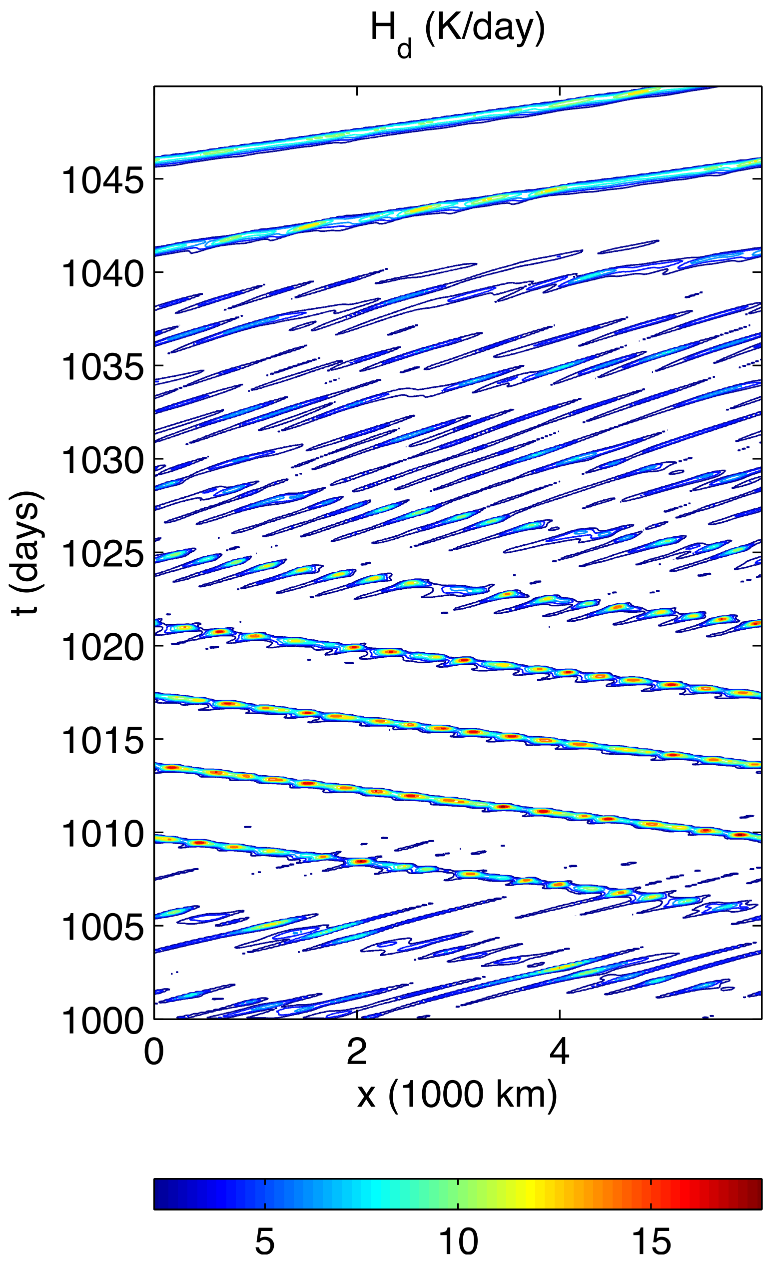
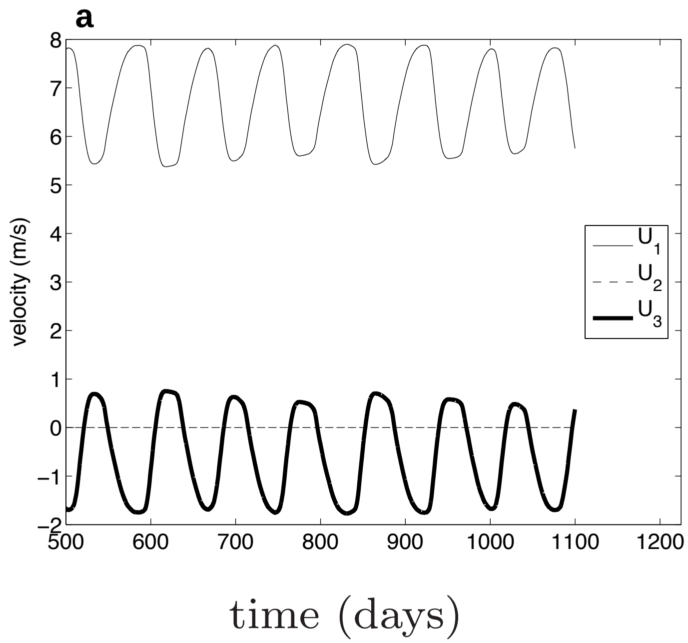
CCW–mean flow interactions on intraseasonal time scale

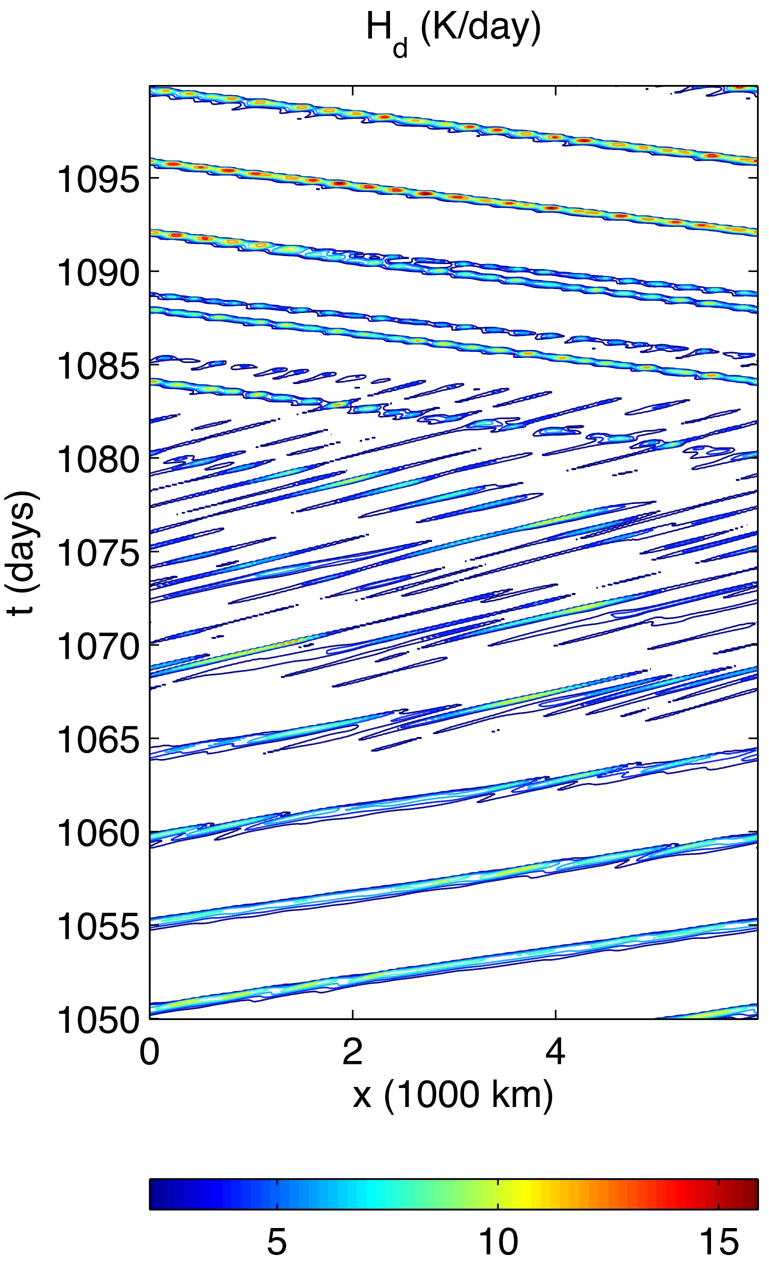


Mean wind $\bar{U}_j(t)$:

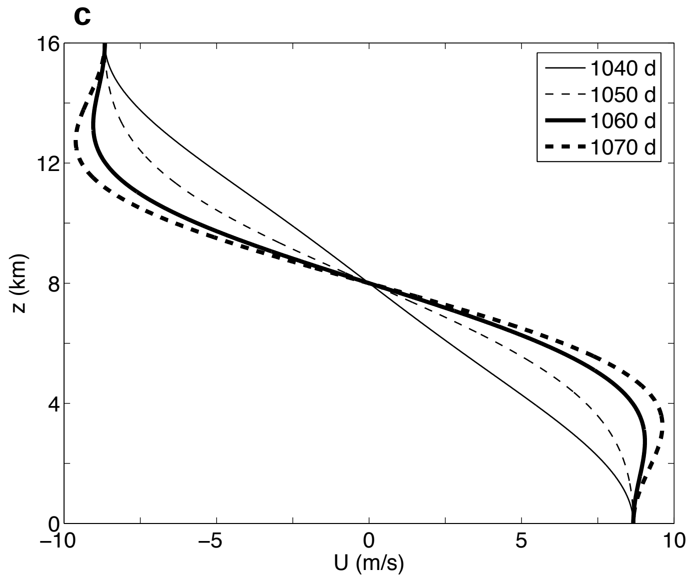


- Momentum transports from CCW drive changes in mean wind
- Advection by mean wind changes wave propagation direction

Westerly Wind Burst Intensification



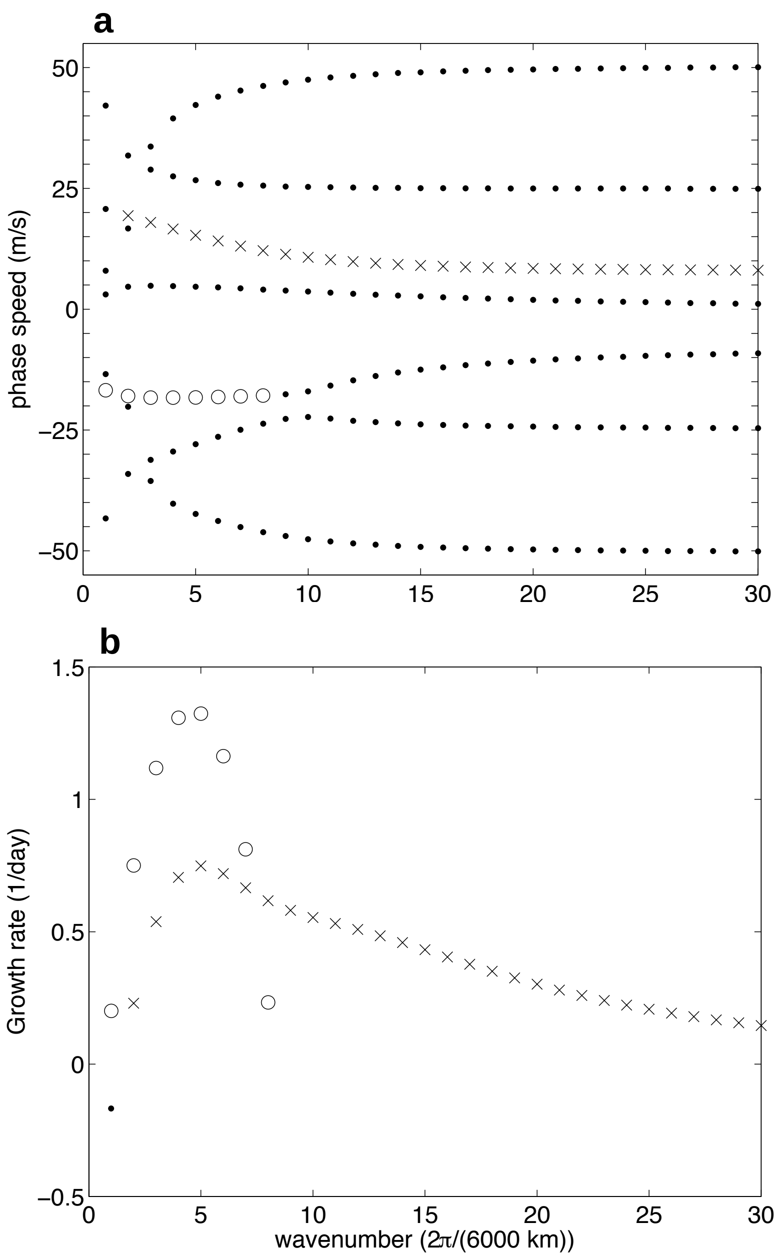
Mean wind $\bar{U}(z)$:

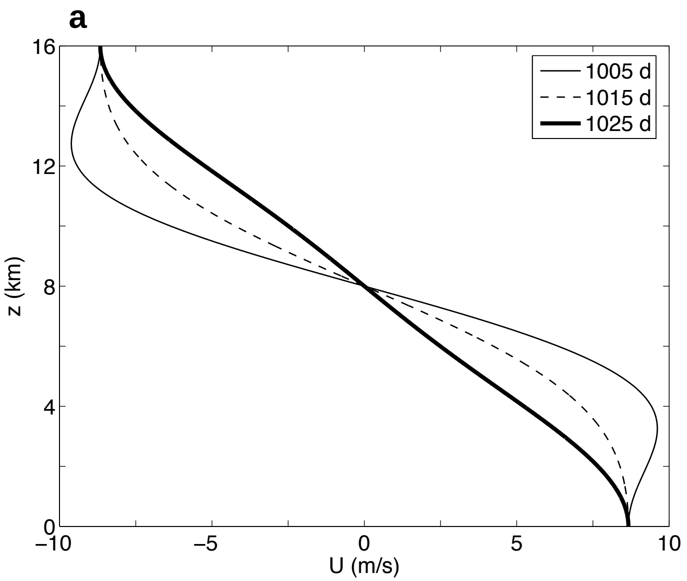


- Either coherent or multi-scale wave depending on mean wind
- Upscale CMT from eastward-moving CCW accelerates WWB aloft

Linear Stability Theory

$t = 1005$ days





Propagating envelopes of smaller-scale convection:

- Westward-propagating CCW favored at larger scales
- Eastward-propagating convection favored at smaller scales

Advertisement: Preferred propagation direction of CCW in shear

Previous slide:

- Theory based on synoptic-scale linear instability

Poster:

- Theory from smaller-scale perspective (wave trains of mesoscale convection)

Stechmann and Majda (2009) *J. Atmos. Sci.*
“Gravity waves in shear and implications for organized convection”

Convection–envelope interactions in comprehensive models

Are these CCW-mean flow effects seen in GCMs?

- Difficult to say because GCMs do not adequately resolve CCW and MJO

Analogous situation on smaller scales:

Cloud-resolving model (CRM) simulations of CCW

- CCW is envelope of mesoscale convective systems (MCS)
- How does CCW envelope interact with mesoscale convection?

  CCW $\rightarrow$ MCS? CCW $\leftarrow$ MCS? CCW $\leftrightarrow$ MCS?
- Does CMT from mesoscale convection affect the CCW envelope?

Cloud-Resolving Model (CRM) simulations of CCW:

What is the role of *resolved* CMT from mesoscale convection?

Results vary depending on strength of *parameterized* momentum damping:

$$\frac{\partial u}{\partial t} = -\frac{1}{\tau}u + \cdots$$

- Held et al. (1993): No momentum damping: Long-time oscillation develops
  - Is this due to CMT interactions or stratospheric interactions?
- Grabowski & Moncrieff (2001): Weak momentum damping: CCW develop with significant CMT
- Tulich et al. (2007): Stronger momentum damping: CCW develop with little or no CMT
- Held et al. (1993): Intense momentum damping: Convection shut down except at a few grid points

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   - Recovers the MJO's fundamental features on intraseasonal/planetary scales
   - Fundamental mechanism: neutrally stable interactions between low-level moisture and envelope of convective activity

2. A simple dynamical model with features of convective momentum transport
   Majda and Stechmann (2009) *J. Atmos. Sci*

   - Convectively coupled wave–mean flow interactions
   - Westerly wind burst intensification due to CCW-MT
   - Results suggest cooperative interactions between MJO envelope and CCW within it