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Letter to the Editor

Comment on "Dynamical model of mesoscales in z-coordinates" and "The effect of mesoscales on the tracer equation in z-coordinates OGCMs" by V.M. Canuto and M.S. Dubovikov

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Abstract

Canuto and Dubovikov [Canuto, V.M., Dubovikov, M.S., 2006. Dynamical model of mesoscales in z-coordinates. Ocean Modelling, 11, 123–166; Canuto, V.M., Dubovikov, M.S., 2007. The effect of mesoscales on the tracer equation in z-coordinates OGCMs. Ocean Modelling 16, 17–27] claim that present oceanographic models ignore a leading-order diapycnal term in the mean buoyancy and tracer equations, while previously published work shows this is not the case. It is important for the density and tracer equations of ocean models to be diabatic only to the extent of imposed diapycnal mixing. Present ocean modelling practice does achieve this property but the equations and parameterizations of Canuto and Dubovikov do not. Canuto and Dubovikov also argue that the same "missing" term in the mean buoyancy equation leads to extra diapycnal advection across mean isopycnals that should be included in the diapycnal advection–diffusion balance in the deep ocean. We show that this is incorrect: the relevant mean diapycnal advection of the Munk-Wunsch advective-diffusive balance is caused by small-scale mixing processes and processes due to the non-linear nature of the equation of state, not by the four so-called "mesoscale-adiabatic" terms identified by Canuto and Dubovikov.

1. The adiabatic nature of GM90 and TRM

Gent and McWilliams (1990) and Gent et al. (1995) perform averaging in isopycnal coordinates, and then transform their equations to z-coordinates retaining the same dependent variables. A great deal of the improvement in ocean modelling that occurs when the eddy-induced advection of Gent et al. (1995) is included in the tracer equations is because the fictitious diapycnal diffusion associated with the exactly horizontal

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diffusion (the so-called Veronis effect) can be avoided. Since 1995 the ocean modelling community has been able to control the amount of diapycnal diffusion and diapycnal advection that a model displays, and numerous papers have been written on this topic.

The Temporal-residual-mean (TRM) theory of McDougall and McIntosh (2001) provides an explanation for why the Gent and McWilliams (1990) (GM90) scheme is able to achieve adiabatic mean motion when the explicit diapycnal diffusion is zero. This is possible because under the TRM interpretation of model variables and of the quasi-Stokes streamfunction, the GM90 scheme mimics the thickness-weighted averaged equations of density coordinates. This has been shown very clearly in the papers of McDougall and McIntosh (1996, 2001) and the textbook of Griffies (2004), and will be outlined again below.

2. The diabatic nature of the Canuto and Dubovikov closure

Canuto and Dubovikov (2006) (hereinafter CD06) and Canuto and Dubovikov (2007) (hereinafter CD07) examine the temporal average, in z-coordinates, of the density conservation equation in the exact form (their Eq. (1c))

$$\bar{\gamma}_t + (\overline{\mathbf{V}} + \mathbf{V}^*) \cdot \nabla_H \bar{\gamma} + (\bar{w} + w^*)\bar{\gamma}_z = \bar{Q} - \Sigma_z, \tag{1}$$

where the eddy-induced velocity is non-divergent and is given by

$$\mathbf{V}^* = \left(-\frac{\overline{\mathbf{V}'\gamma'}}{\bar{\gamma}_z} \right)_z, \quad w^* = -\nabla_H \cdot \left(-\frac{\overline{\mathbf{V}'\gamma'}}{\bar{\gamma}_z} \right), \tag{2}$$

and the “residual flux” is defined by

$$\Sigma = \frac{\overline{\mathbf{V}'\gamma'} \cdot \nabla_H \bar{\gamma} + \overline{w'\gamma'}\bar{\gamma}_z}{\bar{\gamma}_z}. \tag{3}$$

Even though the Σ_z term in (1) looks like a diapycnal term, the correct interpretation is that when the flow is instantaneously adiabatic, the total 3-D flow $\overline{\mathbf{U}} + \mathbf{U}^+$ is adiabatic (the quasi-Stokes velocity $\mathbf{U}^+$ will be defined in (5) below). For purely adiabatic flow ($\bar{Q} = Q' = 0$), Reynolds-averaging results in both the mean ($\overline{\mathbf{U}}$) and the eddy-induced ($\mathbf{U}^+$) velocities having components through isopycnals, but these components exactly cancel. In other words, simply because the eddy flux of density is not directed along the tangent plane of the mean density field (so that Σ is nonzero) does not automatically imply that the flow is diabatic. CD06 argue that a diapycnal term is not required when the averaging is performed in density coordinates but is required when the same situation is represented in depth coordinates. We maintain that a correct coordinate transformation has the property that a flow that is adiabatic when averaged in one coordinate system must remain adiabatic when transformed into another coordinate system.

In contrast to the adiabatic GM90 and TRM schemes, the eddy parameterization scheme of CD06 and CD07 (see Eqs. (7a) and (7b) of CD07) for the mean density equation is a parameterization for both the eddy-induced velocity (2) and for the residual flux (3), so that the mean density Eq. (1) has a diabatic source term $-\Sigma_z$. Their total flow $\overline{\mathbf{U}} + \mathbf{U}^*$ then has the parameterized component $-\Sigma_z/\bar{\gamma}_z$ flowing through the model's density surfaces. This apparent diapycnal component is not caused by diapycnal (i.e., small-scale diabatic) mixing processes but rather is caused by the adiabatic stirring of mesoscale eddies.

Because of the presence of the Σ_z term, (1) will not preserve the adiabatic property that the volume of water between any two isopycnals remains constant except for the imposed diapycnal mixing. The diabatic nature of the CD06,07 approach for modelling mesoscale motions lacks a physical justification and if adopted, would rob ocean modelling practice of its new-found ability (since the early 1990s) of being diabatic only to the extent imposed by the explicit diapycnal diffusion. In this comment, we refute the claims made by Canuto and Dubovikov about the need for an extra term in the mean density equation, while in McDougall et al. (submitted for publication) we comment on their similar claims that three extra terms are required in the mean tracer equation.

It is important to note that we are not claiming that in reality mesoscale eddies should mix in an exactly adiabatic fashion. It would not be surprising to learn in the future that energetic mesoscale eddies induce internal waves to break preferentially or that the strength of double-diffusive convection is mediated by the presence of the water-mass contrasts contained in mesoscale and sub-mesoscale eddies, even though to date observations of microscale dissipation do not strongly suggest such connections. If such a link between microscale mixing intensity and the presence of mesoscale eddies were found, then this would be accommodated by specifying the explicit diapycnal diffusivity according to our best understanding of the processes at work. Such a deliberate inclusion of diapycnal diffusion or near-surface diabatic forcing is not the issue at stake in the present dispute. Rather, we refute the claim of CD06 and CD07 that depth-coordinate models must be diabatic even though the same flow when analyzed in density coordinates is adiabatic.

3. The TRM approach accounts for the residual density flux Σ

CD06 and CD07 assert that the present ocean modelling practice of using an extra eddy-induced velocity in the tracer equations (Gent and McWilliams (1990), Gent et al. (1995)) implies that the residual density flux Σ is being ignored in the mean density equation. We agree that this Σ term is too large to ignore and that many papers, particularly prior to 1996 (including the paper of Gent and McWilliams (1996)), have ignored this term. Prior to the work of McDougall and McIntosh (1996) and especially McDougall and McIntosh (2001) (which we hereinafter refer to as MM01) the idea that ocean models are ignoring this Σ term may have been a reasonable view, but the field has moved on and now adopts a different type of averaging than is used in (1) in order to interpret the variables and parameterizations involved in ocean models. McDougall and McIntosh (1996) and MM01 developed the temporal residual mean (TRM) form of averaging and showed that the component of the modified eddy density flux that is directed through the mean density surface (similar to the residual flux Σ) can be interpreted as a non-divergent part of the modified eddy density flux (see Figure 2 and Eqs. (18) and (48) of MM01). Because this component of the modified density flux is non-divergent it does not contribute to the conservation equation for modified density. The TRM form of averaging does not ignore the Σ term, but neither does this term need to be parameterized.

The TRM approach is motivated by the clean separation between diapycnal and isopycnal processes that is natural in density coordinates. Mindful of the very weak diapycnal mixing processes in the ocean (compared with the atmosphere, for example), MM01 found that after the density equation was averaged in density coordinates, the resulting mean equation could be represented in z-coordinates in the following exact TRM form:

$$\tilde{\gamma}_t + (\overline{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla \tilde{\gamma} = \hat{Q}. \tag{4}$$

Here $\overline{\mathbf{U}}$ is the three-dimensional Eulerian-mean velocity, the eddy-induced three-dimensional velocity $\mathbf{U}^+$ is given in terms of the quasi-Stokes streamfunction $\boldsymbol{\Psi}$ by

$$\mathbf{U}^+ \equiv \nabla \times (\boldsymbol{\Psi} \times \mathbf{k}) = \boldsymbol{\Psi}_z - \mathbf{k}(\nabla_H \cdot \boldsymbol{\Psi}), \tag{5}$$

and $\hat{Q}$ is the thickness-weighted density flux divergence of small-scale ocean turbulence and air–sea surface forcing. The modified density $\tilde{\gamma}$ is not the same as the Eulerian-averaged density in (1). Rather $\tilde{\gamma}(z)$ is the density of the isopycnal whose average height is z and, from (11) of MM01, we have

$$\tilde{\gamma} \approx \bar{\gamma} - (\bar{\phi}/\bar{\gamma}_z)_z \tag{6}$$

where $\bar{\phi}$ is half the density variance measured at fixed height. The Σ term does not appear explicitly in (4) but it influences both the eddy-induced velocity and the mean density; both being different from those in (1).

The quasi-Stokes streamfunction is the contribution of mesoscale perturbations to the horizontal volume flux of water that is denser than $\tilde{\gamma}$. This streamfunction is defined by MM01 as exactly

$$\boldsymbol{\Psi} \equiv \overline{\int_z^{z+z'} \mathbf{V}\,\mathrm{d}z''}, \tag{7}$$

or approximately (when distant from the ocean boundaries, see Killworth (2001)) by the Taylor series expression

$$\boldsymbol{\Psi} \approx -\frac{\overline{\mathbf{V}'\gamma'}}{\overline{\gamma}_z} + \frac{\bar{\mathbf{V}}_z}{\overline{\gamma}_z}\left(\frac{\bar{\phi}}{\overline{\gamma}_z}\right). \tag{8}$$

Jacobson and Aiki (2006) have recently confirmed that (4) is exact when the quasi-Stokes streamfunction is defined by (7).

4. The Σ term is as often positive as negative

The density variance equation is (see for example Eq. (14) of MM01, and the last term here indicates that cubic and higher order terms in perturbation quantities have been ingnored)

$$\bar{\phi}_t + \overline{\mathbf{U}} \cdot \nabla\bar{\phi} = \overline{Q'\gamma'} - \overline{\mathbf{U}'\gamma'} \cdot \nabla\overline{\gamma} + O(\alpha^3) = \overline{Q'\gamma'} - \overline{\gamma}_z\Sigma + O(\alpha^3). \tag{9}$$

For a statistically steady situation, when the mean flow is moving into a region of high eddy activity the left-hand side is positive, while at locations where the mean flow is moving towards lower eddy activity the left-hand side is negative. Concentrating on the mesoscale effects (rather than the microscale diabatic term $\overline{Q'\gamma'}$) on the right-hand side of (9), this implies that Σ takes both signs in the ocean, being approximately as often positive as negative. This is illustrated in Figure 2(a) of MM01. However CD06 and CD07 consider Σ to always be positive. It is hard to determine how much damage this one assumption of theirs does to their parameterization, but it seems a very fundamental error.

5. Interpretation of GM90 as TRM

From the pioneering paper of Gent and McWilliams (1990) through Gent et al. (1995), Gent and McWilliams (1996), McDougall and McIntosh (1996) to MM01 and Jacobson and Aiki (2006), an evolution occurred in the interpretation of the mean and eddy-induced velocities and the meaning of the mean tracer and density variables. Section 10 of MM01 discusses this evolution of ideas, although not exhaustively. MM01 argued that if the Gent et al. (1995) scheme is regarded as a parameterization for the quasi-Stokes streamfunction (7) of the TRM theory, then the temperature and salinity carried by an ocean model can be interpreted as the thickness-weighted values; that is, the values obtained by thickness-weighting the instantaneous values between density surfaces. The density of the ocean model is then the modified density $\tilde{\gamma}$. This interpretation seems to have been adopted by most ocean modelers, so avoiding the criticism leveled by CD06 and CD07 that ocean modelers are neglecting the residual density flux term Σ in (1).

The question arises as to whether the simple parameterization of the quasi-Stokes streamfunction (7) suggested by Gent and McWilliams (1990), is accurate. Using MM01's interpretation of the GM90 streamfunction as the quasi-Stokes streamfunction, we see that the GM90 streamfunction involves the eddy diffusivity κ and the slope of the resolved scale density surfaces according to

$$\boldsymbol{\Psi} \equiv \overline{\int_z^{z+z'} \mathbf{V}\,\mathrm{d}z''} \approx -\frac{\overline{\mathbf{V}'\gamma'}}{\overline{\gamma}_z} + \frac{\bar{\mathbf{V}}_z}{\overline{\gamma}_z}\left(\frac{\bar{\phi}}{\overline{\gamma}_z}\right) \approx \kappa\frac{\nabla_H\tilde{\gamma}}{\tilde{\gamma}_z}. \tag{10}$$

We believe that the science of parameterizing $\boldsymbol{\Psi}$ is in its infancy and the GM90 form for $\boldsymbol{\Psi}$ (the right-hand most part of (10)) should be regarded as simply a first attempt at its parameterization; perhaps the analysis techniques of CD06, when applied to the two separate parts of (8), might lead to a superior parameterization of the quasi-Stokes streamfunction. We wish to make two important points about this. First, we emphasize that in the ocean interior the eddy-induced velocity should be based on the quasi-Stokes streamfunction (7) and not simply on the first term in (8). Present modelling practice interprets the Gent et al. (1995) streamfunction as an attempt to parameterize (7), or approximately, to parameterize both terms in (8), not simply the first term $-\overline{\mathbf{V}'\gamma'}/\overline{\gamma}_z$ of the streamfunction. Second, it is incorrect to say that ocean models have been ignoring the residual density flux (3). Rather, the simple re-interpretation of variables as recommended by MM01 eliminates the need to separately parameterize Σ. The key feature that will be retained by any reasonable parameterization of the quasi-Stokes streamfunction (7) in the TRM formalism is that the total flow will have a component through the model's isopycnals only when there is real small-scale diapycnal mixing specified in the model's parameters. A decade of implementing the GM90 scheme has taught us the importance of attaining this adiabatic property. Unfortunately, the averaging method and the parameterization of CD06 and CD07 do not possess this adiabatic property.

Essentially, (1) is based on the straightforward Reynolds averaging in z-coordinates whereas (4) results from the more sophisticated TRM form of averaging in z-coordinates, which is identical to averaging the equations in density coordinates. With different forms of averaging, the physical interpretation of all the variables will change. For example, the Generalized Lagrangian Mean averaging not only introduces the GLM velocity, but the tracers that emerge are the GLM tracers not the Eulerian-mean tracers. Since diapycnal mixing is so small in the ocean, it is natural to average in density coordinates, as this will achieve the cleanest separation between mean diapycnal advection and diffusion. Averaging along density surfaces motivated the GM90 and Gent et al. (1995) papers, and the TRM approach achieves the clean separation between epineutral and dianeutral processes.

6. Tapering of the quasi-Stokes streamfunction

CD06 and CD07 state that in the GM90 scheme the eddy-induced horizontal velocity is

$$\kappa(\nabla_H\tilde{\gamma}/\tilde{\gamma}_z)_z \tag{11}$$

(see (4a) of CD06 and (6c) of CD07), whereas in fact the GM90 scheme is based on the streamfunction $\kappa\nabla_H\tilde{\gamma}/\tilde{\gamma}_z$, so that the eddy-induced horizontal velocity $\mathbf{V}^+$ is

$$(\kappa\nabla_H\tilde{\gamma}/\tilde{\gamma}_z)_z. \tag{12}$$

It was difficult at first to justify putting the diffusivity κ inside the vertical derivative, although Gent et al. (1995) justified this choice (in their Section 3) because it assures a domain-averaged sink of potential energy. When the idea emerged in the 1996 papers that the GM90 scheme was some type of residual mean, the inclusion of the diffusivity inside the vertical derivative gained additional justification.

CD06 states that previous implementations of eddy-induced advection did not automatically achieve zero horizontal volume flux on each water column (see the discussion around Eq. (1f) of CD06) and that tapering was therefore adopted near the sea surface and the ocean floor. This is incorrect: zero horizontal barotropic volume flux has always been an attribute of the GM90 scheme. This is guaranteed by the boundary conditions on the quasi-Stokes streamfunction at the top and bottom of the ocean. Moreover, the isopycnal interpretation of the averaging scheme provides the physical justification for the tapering of the quasi-Stokes streamfunction as ocean boundaries are approached (see Section 8 of MM01 and see also Killworth (2001) and Treguier et al. (1997)). The zero barotropic transport of the Gent et al. (1995) scheme is an attribute that is quite independent of the way the diffusivity varies in the vertical, but rather is due to the imposed boundary conditions on the quasi-Stokes streamfunction. These surprising comments by CD06 and CD07 appear to result from a misconception of the streamfunction adopted by Gent et al. (1995); the eddy-induced horizontal velocity was never (11) (which is Eq. (4a) of CD06 and (6c) of CD07) but was always (12) above.

The adiabatic interpretation that TRM affords means that the Σ term does not need to be parameterized in the ocean interior, and the temperature and salinity there are best interpreted as thickness-weighted variables. Near the sea surface mesoscale eddies provide mainly horizontal motion rather than motion along isopycnals. This horizontal motion, coupled with mixed-layer dynamics and air–sea interaction results in substantial diapycnal mixing near the sea surface. We support the suggestion of Treguier et al. (1997) that it is sensible to increase the purely horizontal diffusion of properties as the quasi-Stokes streamfunction is reduced near the sea surface. We add that it seems sensible to interpret the model's temperature and salinity as the thickness-weighted values in the ocean interior, but as the Eulerian-mean values at the sea surface, as the skew diffusion is reduced and replaced by horizontal diffusion. This has the added advantage that it is more natural to cast air–sea exchanges (for example, bulk formulae) in terms of Eulerian-mean quantities (such as Eulerian-mean SST) rather than that of the least density isopycnal over an averaging interval of several eddy lifetimes (Killworth (2001)). It is clear however that we are a long way from achieving satisfactory parameterizations of near-surface mixing processes.

7. The meaning of the mean velocity

The mean horizontal velocity defined in GM90 is that averaged on isopycnal surfaces. An important improvement introduced by the TRM theory is that the mean horizontal velocity that appears in the salinity and temperature equations is the Eulerian-mean horizontal velocity, and this velocity comes from the Eulerian-mean horizontal momentum equation. Section 10 of MM01 explains that the GM90 scheme used in the tracer equations is a parameterization of not only the bolus velocity but also the difference between the horizontal velocity averaged in isopycnal and z-coordinates, $\tilde{\mathbf{V}}$–$\bar{\mathbf{V}}$. The Reynolds stresses in the horizontal momentum equations are often parameterized with a simple Laplacian operator. Gent and McWilliams (1996) and others have suggested that advection in the horizontal momentum equation be by the total velocity (i.e., the mean plus the eddy-induced velocity), but this has not been widely adopted. The scale analysis in Appendix B of CD07 (see especially the last term in (8d)) suggests that it would be advisable to adopt this suggestion of Gent and McWilliams (1996), and thereby gain more confidence that the velocity from the solution of the horizontal momentum equation can be interpreted as the Eulerian mean horizontal velocity. We concur with this suggestion.

McDougall and McIntosh (1996) and MM01 pointed out that the mean horizontal pressure gradient $\nabla_H \bar{p}$ cannot be accurately evaluated in a coarse-resolution ocean model for several reasons. The two main reasons are (i) the lack of knowledge of temperature variance and its influence on the mean horizontal density gradient through the cabbeling nonlinearity of the equation of state (see Eq. (B3) of Appendix B of McDougall and McIntosh (1996)); and (ii) that the Eulerian-mean horizontal gradient of pressure is more naturally expressed in terms of the Eulerian-mean salinity and potential temperature rather than in terms of the thickness-weighted salinity and potential temperature (see Appendix B of MM01). These effects were estimated to cause uncertainty in the horizontal pressure gradient of order 1–3% but the errors spatially average to zero over regions of high eddy activity. In other words there are local errors in the calculation of the horizontal velocity components of 1–3% due to these issues. Such errors do not seem large in comparison with the uncertainty in parameterizing the Reynolds stresses as there are good reasons for doubting that the Reynolds stresses should be parameterized in a down-gradient manner.

For these reasons it is clear that the prognostic equations for the two horizontal velocity components are less than ideal in present ocean models. The key feature of the Gent et al. (1995) implementation of the TRM theory is that the tracer equations and therefore the density equations can be made adiabatic with respect to mesoscale parameterizations. That is, ocean models that adopt the TRM scheme will only exhibit diapycnal flow through their resolved-scale isopycnals to the extent that small-scale mixing is imposed in the model. This "adiabatic" attribute of the TRM approach is true whether the order 3% errors in the horizontal momentum equations are present or not.

8. The dianeutral advection-diffusion balance

The Eulerian-mean density Eq. (1) is analyzed in Sections 5.23 and 5.24 of CD06 to determine the component of the Eulerian-mean flow through Eulerian-mean density surfaces. In their Eq. (9e) Canuto and Dubovikov show there are six contributions to this particular version of diapycnal velocity, of which only the last two terms are due to small-scale mixing. The first two of these terms arise because their eddy-induced velocity $\mathbf{U}^* = \mathbf{V}^* + \mathbf{k}\, w^*$ has a component through $\bar{\gamma}$ surfaces, while the middle two terms are caused by the contribution of the Σ_z term in (1) to their diapycnal velocity, namely $-\Sigma_z/\bar{\gamma}_z$. While their expression (9e) is valid for the vertical component of the Eulerian-mean velocity through $\bar{\gamma}$ surfaces, this type of diapycnal velocity is not the one that is relevant to the diapycnal advective-diffusive balance first discussed by Munk (1966) and which has the connection to the dissipation of mechanical energy (Munk and Wunsch, 1998).

Under zonal averaging, it is well known that the Eulerian-mean transport across Eulerian-mean density surfaces (for example, the so-called Deacon Cell in the Southern Ocean (Döös and Webb (1994) and McIntosh and McDougall (1996))) has no physical interpretation in terms of mixing processes. The same lack of physical meaning applies to the "diapycnal" velocity of Sections 5.23 and 5.24 of CD06. The first four terms of CD06's Eq. (9e) are irrelevant to a discussion of the upwelling-diffusive balance and the consequent implications for the dissipation of mechanical energy discussed by Munk and Wunsch (1998). Similarly, the Eulerian-mean velocity $\overline{\mathbf{U}}$ calculated from the output of an eddy-resolving model often has a large "diapycnal component" even though small-scale diapycnal mixing makes only a small contribution (as found, for example, by Radko and Marshall (2004)). Such a "diapycnal component" of the Eulerian-mean velocity through modified density surfaces is due to the sum of (i) real small-scale diabatic mixing processes and (ii) the lateral divergence along the density surface of the quasi-Stokes streamfunction $\mathbf{\Psi}$ (see Eq. (57) of MM01). This second contribution represents adiabatic advection and should be described as such.

Rather than considering the vertical component of the Eulerian-mean velocity $\overline{\mathbf{U}}$ that flows through the Eulerian-mean density surfaces $\bar{\gamma}$, in order to understand the upwelling-diffusion balance one needs to switch reference frames to follow the instantaneous isopycnals and to perform the averaging in this reference frame. This shift in reference frame is achieved by the TRM approach in z-coordinates. Section 5 of MM01 shows that the temporal average of the instantaneous diapycnal velocity across a $\tilde{\gamma}$ surface is given by

$$\tilde{e} = \hat{Q}/\tilde{\gamma}_z, \tag{13}$$

and so depends only on mixing processes, not on the first four so-called "mesoscale-adiabatic" terms of CD06's Eq. (9e).

Expressing this dianeutral velocity in terms of small-scale turbulent diffusion and epineutral diffusion, McDougall (1984) showed that, in addition to the last two terms in CD06's Eq. (9e), the equation for $\tilde{e}$ has terms called cabbeling and thermobaricity (McDougall (1987a)) caused by the nonlinear nature of the equation of state of seawater. These types of dianeutral advection occur even in the limit as the dissipation of kinetic energy tends to zero (McDougall (2003)). There are additional processes that also contribute to mean dianeutral advection, such as double-diffusive convection, the dianeutral motion of submesoscale coherent vortices (McDougall (1987b)) and the helical nature of neutral trajectories caused by the non-zero neutral helicity in the ocean (McDougall and Jackett (1988, in press)). We do not expound on these processes here, except to say that each displays a different relationship between its contribution to mean dianeutral advection and the dissipation of mechanical energy, and so serves to complicate the energy arguments presented by Munk and Wunsch (1998). Furthermore, McDougall (1988) (see Section 5.3 and particularly Figure 1 of that paper) has pointed out that the nonlinearity in the equation of state means that the vertical diffusive terms cannot be represented as Munk and Wunsch (1998) do. That is, McDougall (1988) showed that

$$\tilde{e} = gN^{-2}[\alpha(D\Theta_z)_z - \beta(DS_z)_z] \neq N^{-2}(DN^2)_z = 0.2N^{-2}\varepsilon_z, \tag{14}$$

where D is the vertical diffusivity of small-scale turbulence and ε is the rate of dissipation of mechanical energy. However, exploring the implications of this is beyond the scope of this note. We simply wish to point out that the link between the rate of dissipation of kinetic energy and dianeutral upwelling is far from simple.

9. Conclusions

We have made the following main points in relation to CD06 and CD07:

- The assertion by CD06,07 that present ocean models omit a term due to the residual flux Σ ignores the advances that have been made since 1996. In the more natural TRM or density-coordinate forms of averaging, this residual flux does not arise, and the tracer and density variables need to be reinterpreted in the ocean interior.
- CD06 and CD07 claim that "all ocean codes have been run with" $\Sigma = 0$, whereas in fact the whole TRM effort of McDougall and McIntosh (1996) and MM01 was undertaken precisely because of the presence of the Σ term in the mean density Eq. (1). If it were not for the Σ term then the eddy-induced velocity (2) would be sufficient, whereas, by specifically taking the non-zero nature of Σ into account, the eddy-induced velocity (5) is obtained by the TRM theory. The oft-repeated assertion in CD06 and CD07 that present ocean models have ignored the Σ term is simply false.
- CD06 and CD07 take Σ of (3) to be positive everywhere, whereas it is almost as often positive as negative; this is apparent from a glance at the conservation equation of density variance (9) and is illustrated in Figure 2 of MM01.

- Present ocean modelling practice employs the quasi-Stokes streamfunction of the TRM theory and (a) does not ignore the effects of Σ, and (b) manages to achieve the "adiabatic" property that there is no transport through resolved-scale density surfaces if the explicit diapycnal diffusivity is zero. This "adiabatic" attribute of the TRM approach applies despite remaining uncertainties in the exact representation and parameterization of the effects of mesoscale eddies in the horizontal momentum equations.
- By contrast, the parameterization of the Σ term advocated by CD06, CD07, and Dubovikov and Canuto (2005, 2006) ensures that their mean density Eq. (1) has diapycnal transport even when there is no diapycnal mixing. In this way their scheme encounters fictitious diapycnal diffusion of the same type as that caused by purely horizontal diffusion (the so-called Veronis effect).
- The near-surface tapering of the quasi-Stokes streamfunction is not a device to ensure that the eddy-induced velocity has no depth-integrated transport, but rather is justified physically in terms of the horizontal transport of water in each density class as the surface is approached (Treguier et al. (1997), Section 8 of MM01 and Killworth (2001)). The zero depth-integrated transport of the Gent et al. (1995) scheme is guaranteed by the boundary conditions on the quasi-Stokes streamfunction. CD06 are simply incorrect when they state that previous models of eddy-induced advection have not achieved zero barotropic transport (their Eq. (1f)).
- The balance between dianeutral upwelling and diffusion does not need to be reconsidered in the manner recommended in Sections 5.23 and 5.24 of CD06. The extra four terms identified by CD06 (see their Eq. (9a)) do not appear in the equation for $\tilde{e}$ which is the relevant diapycnal velocity for considering the balance between upwelling and diffusion.

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