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The Gent–McWilliams parameterization: 20/20 hindsight

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ABSTRACT

It has now been 20 years since the Gent and McWilliams paper on "Isopycnal Mixing in Ocean Circulation Models" was published in January 1990 issue of the Journal of Physical Oceanography. That paper was highlighted at the CLIVAR Working Group on Ocean Model Development "Workshop on Ocean Mesoscale Eddies" which was held at the UK Meteorological Office in April 2009, and this review paper is based on the talk given at that Workshop. It contains some hindsights on how the parameterization of the effect of mesoscale eddies on the mean flow came about; which is a question that I am asked quite often. A few important results from including the parameterization in a non-eddy-resolving ocean model are recalled. Including this parameterization, along with other improvements to all the components, in the first version of the Community Climate System Model resulted in the first non-drifting control simulation in a climate model that did not require flux corrections. Also included are brief comments on how the Gent and McWilliams eddy parameterization has been modified and improved since the original proposal in 1990.

1. Background

The first ocean general circulation model designed for use in a coupled climate model was created by Bryan (1969) and colleagues at the Geophysical Fluid Dynamics Laboratory (GFDL). The vertical coordinate used was depth, or *z*-coordinate, which is still used in a large majority of ocean climate components. The closure terms in the equations for potential temperature and salinity were the easily implemented Laplacian diffusion in the horizontal and vertical directions. These terms can be interpreted either as necessary to control numerical noise, or as a parameterization for the effects of mesoscale eddies that are not resolved by the numerical grid. However, it had already been known for 30 years that mixing occurs much more strongly along isopycnal surfaces of constant potential density than across these surfaces, see Iselin (1939) and Montgomery (1940). If this is not the case, then the old style "water mass" analysis of the World's Oceans would not have been valid, because deeper water masses would have mixed together too quickly. Therefore, it was not long before George Veronis and Henry Stommel showed a disadvantage of horizontal tracer mixing at a National Academy of Sciences symposium held in October 1972. The Veronis (1975) paper clearly showed that horizontal mixing has to be balanced by a false mean vertical velocity. This so called "Veronis Effect" occurs in the subtropics, and its main effect is to short-circuit the meridional overturning circulation in the North Atlantic Ocean. This strongly reduces the large and important northward ocean heat transport across 23°N, where it is estimated from observations to be 1.2 ± 0.3 Petawatts, see Hall and Bryden (1982). The point that large horizontal diffusion of $O(10^3\ \mathrm{m^2\ s^{-1}})$ implies much stronger cross-isopycnal mixing than the observed value below the mixed layer of $O(10^{-4}\ \mathrm{m^2\ s^{-1}})$ see Ledwell et al. (1993), even when the isopycnal slope is $O(10^{-4})$ or smaller, was hammered home in a later paper by McDougall and Church (1986).

Thus, it was agreed that tracer diffusion in *z*-coordinate models needed to be oriented along and perpendicular to isopycnals. More precisely, it should be along and perpendicular to "neutral surfaces", McDougall (1987), but I will ignore this subtlety here. The rotation to implement Laplacian diffusion in this manner without any approximation was derived by Redi (1982). However, the implementation using the small slope approximation into the GFDL model by Cox (1987) did not go smoothly, and the model was not able to run stably without the addition of horizontal diffusion, albeit with a much reduced coefficient. It was diagnosed much later that the Cox implementation caused a numerical instability when the equation of state is nonlinear, see Griffies et al. (1998). However, the results from this model obtained by Mike Cox were an improvement over the results from the original model using only horizontal Laplacian diffusion with a large coefficient.

By early 1989, I had finished building a reduced-gravity model with Mark Cane that was designed for the upper equatorial Pacific Ocean, with coupled El Nino studies in mind, see Gent and Cane (1989). Salinity was kept constant in the model, because the

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density depends primarily on temperature in this region. The temperature equation was closed by a high-order Shapiro filter, which was oriented in the horizontal. An NCAR visitor, possibly Claes Rooth, suggested that I use the accepted wisdom that temperature diffusion should be along isopycnals. I decided to try it, but fortunately before implementing it, I realized that it would have absolutely zero effect in the Gent–Cane model, because the density was just a function of temperature. This implied that mesoscale eddies would have no effect at all on the large-scale flow in this simplified, but never-the-less reasonably realistic, set up. Jim McWilliams and I thought this could not possibly be true, and set out to construct a parameterization for the effect of eddies that would operate in adiabatic flow, and was not merely diffusion of tracers along isopycnal surfaces.

2. The Gent and McWilliams 1990 paper

We decided to work in isopycnal coordinates (IC) because, as stated above, in the quiescent region of the ocean, mixing is fundamentally along and across surfaces of constant potential density. It is very important that the averaging is done in IC and not in depth coordinates, see the later discussion in Section 5.1. We stated three important properties of the adiabatic primitive equations, with the premise that the equations with our parameterization would still preserve these properties. These properties are that the volume between any two isopycnals is conserved, the domain-average of all passive tracers is conserved between any two isopycnals with insulating boundary conditions, and that the equation for a passive tracer is satisfied identically by the density. The operator mentioned was a time average, although to be slightly more correct, it is a low-pass filter operator in time and space, because the non-eddy-resolving model does have temporal variability.

We proposed that eddy effects be parameterized in IC by the term $\nabla\cdot\mathbf{F}$ in the equation for thickness h_ρ, where h and ρ are depth and potential density. The equation becomes

$$\frac{\partial}{\partial t}h_\rho + \nabla\cdot(\mathbf{u}h_\rho) + \nabla\cdot\mathbf{F} = 0. \tag{1}$$

In hindsight, the clearest way to describe this is that it represents an extra, eddy-induced advection by a horizontal velocity $\mathbf{F}/h_\rho$. I have kept some written comments from Trevor McDougall on the first draft of the paper that suggest this interpretation. This extra advection also occurs in Eq. (10) of the GM 1990 paper governing a passive tracer, and the parameterization was written in advective flux form in z-coordinates in Eq. (19). So, why did not we describe GM as an extra advection right at the start? I think the first reason is that we did not write down the density equation in GM 1990 as an extra advection, even though by the third important property mentioned above, density does satisfy the passive tracer equation. We wrote that, "Our non-eddy-resolving model is not adiabatic"; in hindsight, I think it would have caused less confusion if we had described it as adiabatic. The second reason is that our particular choice for $\mathbf{F}$ results in a thickness equation of the form

$$\frac{\partial}{\partial t}h_\rho + \nabla\cdot(\mathbf{u}h_\rho) = \nabla\cdot(\kappa\nabla h)_\rho. \tag{2}$$

As we wrote in GM 1990, "κ can be spatially varying, but if it is a constant, Eq. (2) has the familiar form of Laplacian mixing acting upon the equation variable". For this reason, we called κ the thickness diffusivity, a term which is still widely used today.

In hindsight, the right-hand-side of Eq. (2) is not Laplacian diffusion of thickness. In IC, the equation for a passive tracer, τ, is given by Eq. (2) of GM 1990; namely

$$\frac{\partial}{\partial t}\tau + \mathbf{u}\cdot\nabla\tau = \nabla\cdot(\mu h_\rho\nabla\tau)/h_\rho, \tag{3}$$

where the right-hand-side is Laplacian diffusion using the small slope approximation. This form is required to ensure global conservation of the tracer density τh_ρ, and a sink of tracer variance. It is also clear that what is an extra advection in z-coordinates must be an extra advection in IC; a coordinate transformation cannot change an advection into a diffusion. A consequence of this discussion is that I think κ should not be called the "thickness diffusivity", because this gives the impression that GM is a diffusive process, whereas it is an adiabatic process. κ is the coefficient in the GM extra advection of tracers; the coefficient, μ, in the tracer diffusion along isopycnals, or Redi (1982), term shown in Eq. (3) is the diffusivity.

The realization that GM is best described as an extra advection had been made by early 1993. However, the Gent et al. (1995) paper, where it was fully explained in this way, was not published until April 1995. The review process was delayed for over a year by a reviewer insisting that GM was a form of diffusion, even though the main point of the paper was to show it was an extra advection! In addition, Eq. (19) of the 1995 paper showed very simply that the GM choice created a positive definite sink of the global potential energy. This had only been shown approximately in the geostrophic limit in the Appendix of GM 1990, whereas it could have been demonstrated in IC as follows. The potential energy equation is formed by multiplying the thickness Eq. (2) by the Montgomery potential, ϕ, which is defined as

$$\phi = (p - g\rho h)/\rho_0, \quad \phi_\rho = -gh/\rho_0, \tag{4}$$

where p is the pressure, g gravity and ρ_0 a reference density. The additional term in the potential energy equation due to GM on the right-hand-side of Eq. (2) can be written as

$$\phi\nabla\cdot(\kappa\nabla h)_\rho = \nabla\cdot[\phi(\kappa\nabla h)_\rho] - [\kappa\nabla\phi\cdot\nabla h]_\rho - \kappa g\nabla h\cdot\nabla h/\rho_0. \tag{5}$$

When Eq. (5) is integrated over the global domain, the last term is a positive definite sink of the global potential energy. However, well before this second paper was published in 1995, GM had been implemented into a new global ocean model by Gokhan Danabasoglu. Exciting results and a large improvement over an older model version using horizontal tracer diffusion had been obtained by the summer of 1993.

3. Results from ocean models

The first results from implementing GM into an ocean model were published in May, 1994 by Danabasoglu et al. (1994). The global model had very coarse resolution; $4^\circ \times 3^\circ$, and 20 levels in the vertical, and was forced by observed winds and restoring of temperature and salinity to observations. Two equilibrium solutions were compared; one using horizontal tracer diffusion and the other GM, which was implemented as an extra advection. The first result was that the model with GM was stable over ten thousand years of integration, and no horizontal diffusion at all was needed to keep it stable. This was the first equilibrium solution of a z-coordinate ocean model without any horizontal diffusion. In retrospect, this was due to the original implementation of the Redi term in the GFDL model; now models with just the Redi term can be integrated to equilibrium.

Having zero horizontal tracer diffusion eliminated the false large diapycnal diffusion in the model, which resulted in a much colder ocean below 1 km that agreed much better with observations, as shown in Fig. 1 of Danabasoglu et al. (1994). It also eliminated the Veronis effect, which resulted in the North Atlantic maximum northward heat transport increasing by more than 50%, as shown in their Fig. 3(b). Their Fig. 3 also showed that GM strongly reduced the poleward heat transport in the southern hemisphere, and greatly reduced the surface heat flux loss to the

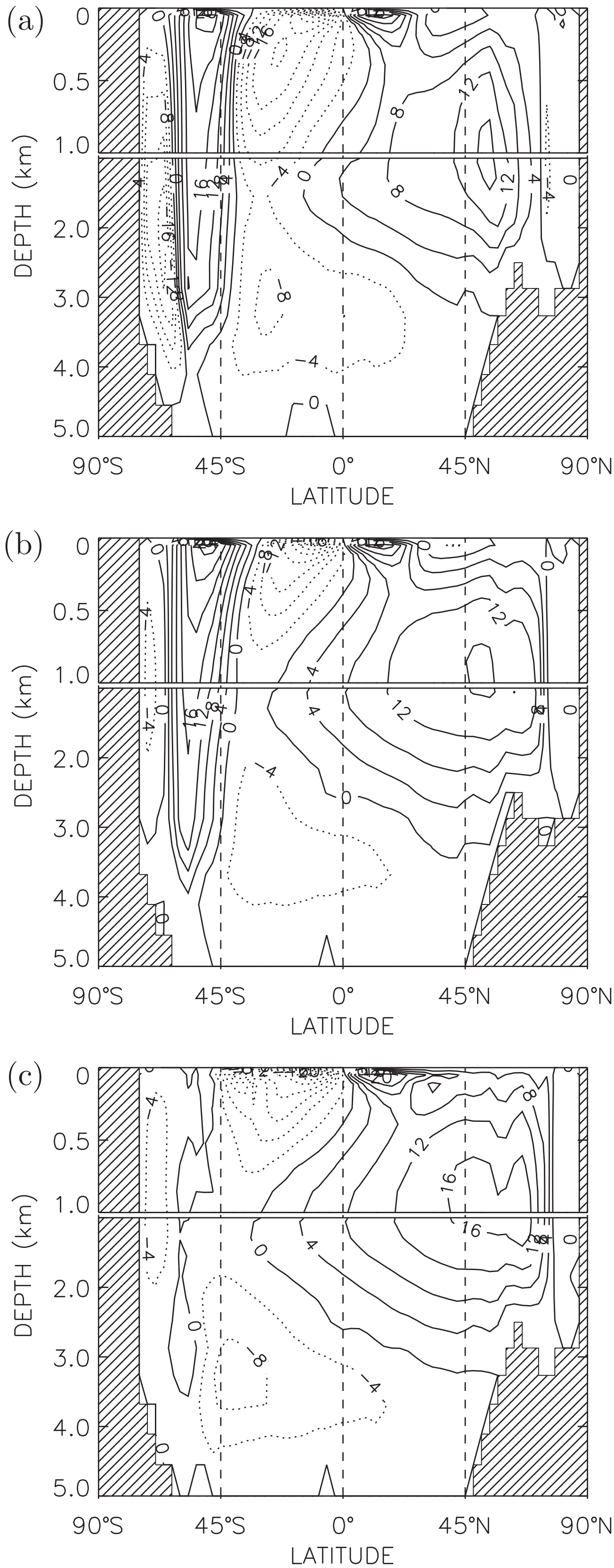

Fig. 1. Global meridional overturning streamfunction from solutions using (a) mean velocity using horizontal tracer diffusion, (b) mean velocity using GM, and (c) total transport velocity using GM. Taken from Danabasoglu et al. (1994).

atmosphere south of 50°S. These positive aspects of the results were due to a large change in the meridional overturning circulation (MOC) in the region of the Antarctic Circumpolar Current (ACC) when using GM.

Fig. 1(a) shows the mean velocity MOC from the horizontal diffusion solution, which has three main features below 1 km. The first is the strong overturning in the North Atlantic, the second is the strong, so called Deacon, cell centered at 50°S, and the third is the quite strong overturning near Antarctica. The MOC of the mean and total transport velocities in the GM solution are shown in Fig. 1(b) and (c). Fig. 1(b) shows the North Atlantic cell strengthened in the subtropics with the elimination of the Veronis effect, the Deacon cell remained the same, and the overturning near Antarctica weakened. The only change in Fig. 1(c) from Fig. 1(b) is the overturning near the ACC reduced to only a few Sverdrups, which implies that the MOC due to the eddy-induced velocity almost cancels out the Deacon Cell due to the mean flow. This cancellation was close to complete in this model setup; in subsequent setups the cancellation has not been nearly this exact.

We already knew that the eddy-induced overturning would oppose the Deacon Cell because we had plotted it using Levitus (1982) observations in early 1993; these plots were eventually published as Figs. 6 and 7 in Gent et al. (1995). However, the almost exact cancellation in the region of the ACC was a surprise to us. In hindsight, it should not have been because of the non-acceleration theorem of Andrews and McIntyre (1978). Their work was applied to the stratosphere, and the mean circulation is defined as the zonal average. Under some simplifying assumptions, they proved that the eddy advection due to zonal perturbations exactly balanced advection by the mean flow, so that the solution was steady and did not accelerate. These conditions do not hold for ocean eddies, but we should have anticipated that the eddy-induced overturning would strongly oppose, but not exactly cancel, the mean flow Deacon Cell in the ACC region. In order to be valid, the non-acceleration theorem requires the elimination of the pressure gradient term from the averaged zonal momentum equation. Therefore, it cannot apply in midlatitude basins where there is a pressure difference from one side of the basin to the other. In these basins, the zonally-averaged eddy-induced overturning is rather small, even though it can be much larger locally where there is strong baroclinicity, such as near the Gulf Stream and Kuroshio, or the Agulhas retroflection region. However, by far the largest zonally-averaged eddy-induced overturning is in the region of the ACC, where it opposes the Deacon Cell due to the mean flow.

It is clear from Fig. 1(a) that the Deacon Cell transported cold water from south to north across the ACC in the upper ocean. This often produced an unstable density profile in this region, which was stabilized in the model by applying convective adjustment. The result is in Fig. 2(a), which shows the percentage of all times and model levels where convective adjustment occurred in the horizontal diffusion case. Convective adjustment occurs throughout the southern hemisphere in the region of the ACC and in the high latitude North Atlantic. With GM, Fig. 1(c) shows that this transport in the upper ocean was greatly reduced, and so was the percentage of time convective adjustment occurred, which is shown in Fig. 2(b). With GM, convective adjustment was reduced to just the Weddell, Ross, Labrador, and Greenland–Iceland–Norwegian Seas, which are precisely the locations where deep water formation is known to occur in the real ocean. This was a complete surprise to us, and this figure is probably the favorite of my career. What it showed was that, even in a coarse resolution $4^\circ \times 3^\circ$ model, deep water formation was confined to the correct, small locations. This convinced us that using GM in the ocean component would make a real improvement to the results from coupled climate models.

First, however, an interesting aside. Eric Chassignet had spent time during his post-doctoral fellowship at NCAR comparing North Atlantic circulation solutions from *z*-coordinate and IC models. The

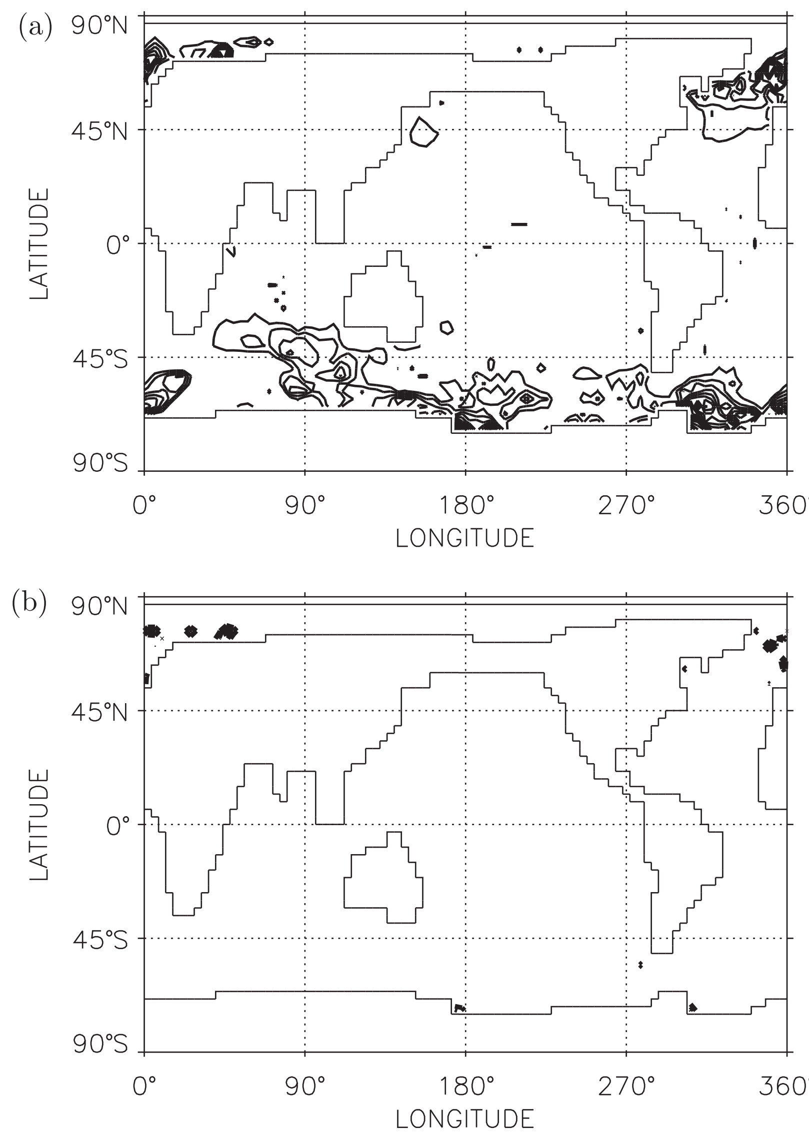

Fig. 2. Percentage of all times and model levels where convective adjustment occurred using (a) horizontal tracer diffusion, and (b) GM. Contour interval is 5%. Taken from Danabasoglu et al. (1994).

results were so different that he did not write them up. During 1994, Rainer Bleck, Trevor McDougall and I realized that the GM term on the right-hand-side of Eq. (2) had already been added to the thickness equation in the IC ocean model of Bleck and Boudra (1981). This had been done to suppress numerical noise and keep solutions smooth and stable, because it looked like a thickness diffusion term. Bleck and Chassignet then added the corresponding GM extra advection term into their tracer equation. Finally, for the first time ever, z-coordinate models with GM and Bleck's IC model were solving exactly the same density and tracer equations. Chassignet quickly reran his simulations of the North Atlantic in these two models and, lo and behold, they now produced comparable solutions, which were written up in Chassignet et al. (1996).

4. Results from climate models

In 1995 and early 1996, members of the Climate and Global Dynamics division at NCAR were assembling a new climate model. It was based on updated versions of the atmosphere, land and sea ice components that had been developed over the previous decade. However, the ocean component was revolutionary, rather than evolutionary. For the first time, the ocean component contained not only the GM parameterization, but also the K-profile parameterization of Large et al. (1994). This was the first ocean vertical mixing scheme that had been designed and tested for use in all regions of the global oceans. In the summer of 1996, the first experiment using this Community Climate System Model, version 1 (CCSM1) was run. It went for only 10 years. The model had no river runoff scheme to route the runoff calculated in the land component back into the ocean. Thus, the ocean was rapidly becoming saltier, because the total evaporation was somewhat larger than the precipitation over the global ocean. We decided to correct this in the simplest and quickest way possible; every day the ocean precipitation field was multiplied by the ratio of globally-averaged evaporation to precipitation over the ocean. This ensured exact fresh water conservation in the ocean component, but river runoff was obviously entering the ocean in completely the wrong locations.

We used this very crude "river runoff scheme" because we anticipated that this first CCSM1 coupled run would not go for very long before it drifted away from reality. The reason was that we had decided not to use flux corrections of heat and fresh water that up to that time had always been needed in climate models to prevent control runs drifting away from the realistic initial conditions from which they were started. A quite elaborate spin-up procedure for the various CCSM1 components, which is described in Boville and Gent (1998), had been used to try to prevent an initial shock to the coupled system. Despite this, the coupled solution did have an initial shock, and the surface temperatures changed considerably during the first 10 years. However, after that the solution, including the upper ocean, settled down and hardly drifted at all. In the end, the run continued for 300 years, which took three months to complete, but was then stopped because of the drift in the deep ocean salinity field caused by the crude river runoff scheme.

Surface temperatures from the CCSM1 present day control run are shown in Fig. 3, which is taken from Boville and Gent (1998). It shows the change over the first 10 years, but the trend in the combined surface temperature after that is 0.03 °C per century, which is much smaller than the standard deviation of the variability. This trend was about 30 times smaller than trends in previous runs of climate models without flux corrections. This run of the CCSM1 was the first ever to maintain the present day climate in a control run without flux corrections. This was headline news in the climate modeling community. Fairly quickly we wrote up this result in a short paper, and submitted it to Science. It was rejected. One reviewer said that the surface restoring of salinity to observations in the ocean component, that had been used in the spin-up procedure, was still applied during the fully coupled run. This was incorrect, but the editor would not change his decision. The result was eventually documented in Boville and Gent (1998).

Obviously, other climate centers were extremely interested in this result. The GM parameterization was implemented in the ocean component of two of their models quite quickly. It was put into the Australian climate model, see Hirst and McDougall (1996), with the result that the flux corrections needed in the model were very much smaller than before, especially in the high latitudes of the southern hemisphere. It was also included in the Hadley Centre model, which rather quickly was also running without flux corrections, and maintaining the present climate in a control run, see Gordon et al. (2000). These results are supporting evidence indicating that GM, which eliminated the need for any horizontal diffusion in the ocean component, was the major factor in eliminating the need for flux corrections in climate models. More evidence is given in Gent et al. (2002), which tested various eddy parameterizations and vertical mixing schemes in an ocean climate model. Fig. 1 of that paper shows that using GM instead of horizontal diffusion, is clearly the most important change that enables the ocean component to maintain the observed temperature and salinity initial conditions in a long integration. Presently, GM is used in virtually every climate model, and a large majority of them are now run successfully without flux corrections.

5. Modifications and improvements to GM

5.1. Temporal residual-mean interpretation

The GM parameterization was originally formulated in IC, so that averages are taken at constant density, not constant depth. McDougall and McIntosh (1996, 2001) produced an interpretation of GM that applies for appropriate averaging in a z-coordinate ocean model. They called it the Temporal Residual Mean (TRM), because it is closely related to the residual-mean theory developed by Andrews and McIntyre (1976). The TRM uses a time averaging operator, whereas the residual mean theory uses a zonal averaging operator. The TRM is adiabatic because its velocity does not have a component through appropriately defined density surfaces. These surfaces are defined by the density variable whose surface is, on average, at the depth of the averaging. TRM transport is defined as the sum of the Eulerian streamfunction and a quasi-Stokes streamfunction. The horizontal component of the TRM transport of fluid between two resolved-scale density surfaces is the same as occurs between the same two density surfaces when the averaging is done in IC. Thus, the three dimensional TRM velocity is the same as obtained by averaging with respect to instantaneous density surfaces. In addition, TRM defines a tracer in non-eddy-resolving models as the thickness-weighted tracer that results from doing the averaging in IC. Finally, GM is a parameterization for the quasi-Stokes streamfunction.

It is very important to use the TRM theory when interpreting results from z-coordinate models, because it retains the assumption that GM is an adiabatic parameterization. If the traditional averaging at a constant depth is applied to the z-coordinate density equation, then it can appear that there should be a diapycnal diffusion term, see Section 9.3.3 of Griffies (2004), but this is an incorrect inference about GM. It is a legitimate question to ask whether the effects of eddies should be purely adiabatic, or if they cause some diapycnal mixing as well? However, it is absolutely clear that any diabatic effect, which should be added to the model's vertical

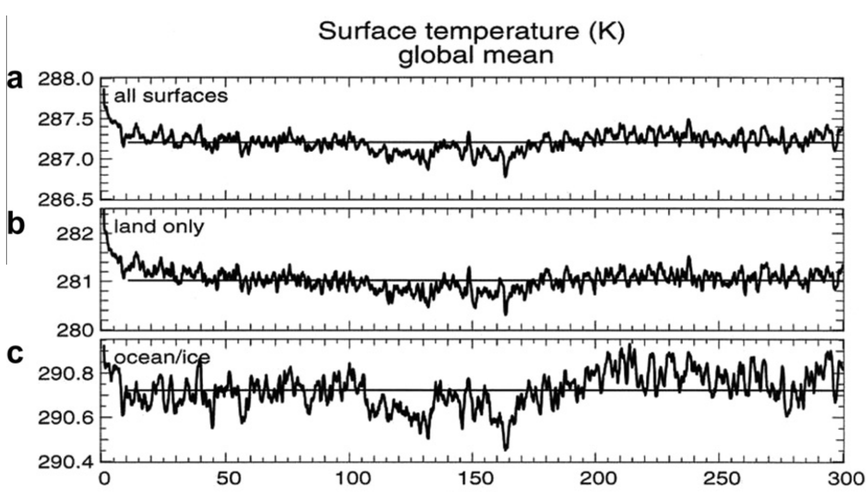

Fig. 3. Surface temperatures against time from the 300 year present day control run of the CCSM1; (a) all surfaces, (b) land only, and (c) ocean and ice. The horizontal lines are the mean of each series over years 11 to 300. Taken from Boville and Gent (1998).

mixing scheme, is very much smaller than the false diapycnal mixing implied by using horizontal tracer diffusion.

5.2. Implementing GM near the ocean surface

The boundary conditions on the vertical eddy-induced velocity are that it is zero at the ocean surface and bottom. This means that the GM coefficient has to reduce to zero at the ocean surface. How to do this is tied to the question of whether GM, which is designed for the nearly adiabatic interior, should be active in the upper mixed layer? If so, should the diffusion along isopycnals also be active, even when the isopycnals can be steep? These questions were addressed in early implementations of GM, but it became clear that global solutions depend rather strongly on how this is done. A clear and thorough discussion of work on this topic through 2003 is given in Chapter 15 of Griffies (2004).

Slope clipping had first been used by Cox (1987) when implementing the Redi diffusion term, which involved setting a maximum slope for the isopycnals. Hirst and McDougall (1996) used a similar technique for GM, and restricted the slope to 1/500 near the surface, increasing to 1/50 below 1 km. These slopes were chosen to restrict the eddy-induced velocity to a "maximum plausible level", which they chose to be 16 cm/s^{-1}. However, it became clear that this technique, which did not mix along and across steeper isopycnals, implied an unacceptable amount of diapycnal diffusion. This should not be confused with the small-slope approximation, which only implies a 1% error, even when the slope is 1/10. So instead, tapering of the coefficients when the slope is greater than a maximum value was introduced, with the diffusion still oriented along the isopycnals even when they are steep. Danabasoglu and McWilliams (1995) used a *tanh* function profile that reduces the coefficients very quickly for slopes greater than the maximum. Treguier et al. (1997) suggested that diffusion in the mixed layer should be in the horizontal direction because that is parallel to the ocean surface, and this is the direction of diapycnal mixing in the limit of an uniform density mixed layer.

Recently, there has been a more physically based proposal by Ferrari et al. (2008). They propose that in the mixed layer the eddy-induced velocity is horizontal and has no vertical shear. In addition, there is diffusion both along isopycnals and in the horizontal direction. Between this mixed layer and the adiabatic GM interior, there is a transition layer across which the mixed layer and interior forms are matched. A slightly simplified form of this proposal has been implemented into the CCSM ocean component, and improved solutions are documented in Danabasoglu et al. (2008). Fortunately, the solutions do not depend strongly on the details assumed about the transition layer, but they do still strongly depend on the magnitudes chosen for the GM and Redi coefficients. The best way to implement GM near the ocean surface continues to be an active area of research.

5.3. Making κ a function of space

Gent et al. (1995) says, "The best choice for κ as a function of space is a research question beyond the scope of this manuscript." It was not very long before proposals were being made, with the first one by Visbeck et al. (1997). They suggested that the coefficient be evaluated as the square of a length scale divided by a time scale. The length scale is the width of the baroclinic zone, and the inverse time scale is the product of the Coriolis parameter and a vertical integral of a function of the local Richardson number. There have been a number of other proposals, with a recent one by Eden and Greatbatch (2008), which consists of an additional prognostic equation for eddy kinetic energy, and an eddy length scale which is the minimum of the Rossby radius and Rhines scale.

Most of the early proposals for κ were to make it a function of horizontal position, but not depth. However, it is well known from observations that eddy energy levels decrease in the deeper ocean, so that the effect of eddies should also decrease with depth. Ferreira et al. (2005) used an inverse model technique to suggest that making κ proportional to the square of the buoyancy frequency would give the best comparison of model solutions with observations. This is not a theoretical result, but just a convenient decay scale to use for the vertical variation of κ. This parameterization is now used in the ocean component of the CCSM, and results are documented in Danabasoglu and Marshall (2007). This is just one of a number of possible choices for κ, and evaluating these choices is an important area of active research that will continue into the future.

5.4. Using GM in eddy-resolving models

Should GM be used in ocean models with eddy-permitting and even eddy-resolving resolution? The first paper to address this was Roberts and Marshall (1998), who answered in the affirmative. One reason is that the false diapycnal diffusion implied by horizontal diffusion is not just proportional to the coefficient, which gets smaller with better resolution, but is also related to the tracer gradients, which get larger with better resolution. This is true even with eddy-resolving resolution of 0.1° or finer, where biharmonic diffusion is most often used. Using the standard isotropic form of GM at these finer resolutions produced a low level of mean kinetic energy, with slower mean western boundary currents, and much broader fronts than obtained when using biharmonic diffusion.

Smith and Gent (2004) overcame these deficiencies by implementing an anisotropic form of GM into a model of the North Atlantic using 0.2° and 0.1° resolution. This allows GM to act mostly along the local flow direction, with a zero, or very small, GM coefficient in the cross-flow direction. The improvements using anisotropic GM compared to biharmonic diffusion were the same as in a non-eddy-resolving model, but with a reduced amplitude. In the 0.2° runs, the elimination of the Veronis effect increased the maximum North Atlantic northward heat transport by 0.2 Petawatts. In the 0.1° runs, anisotropic GM gave a stronger vertical density gradient in the Labrador Sea, which resulted in shallower winter mixed layer depths that were in much better agreement with observations.

5.5. Parameterization of mixed layer eddies

Recently, Fox-Kemper et al. (2008) and Fox-Kemper and Ferrari (2008) have proposed a parameterization for the effects of submesoscale eddies in the ocean mixed layer. These eddies have the effect of restratifying the mixed layer, countering the effects of strong vertical mixing due to the wind forcing. The form of this parameterization is exactly like GM with a variable coefficient, κ, which results in an additional advection in the density and tracer equations. In fact, the additional velocity due to submesoscale eddies can just be directly added to the GM velocity in the mixed layer. This parameterization has very recently been implemented in the CCSM ocean component, with the result that the diagnosed mixed layer depths now agree much better with observational estimates.

6. Discussion and conclusions

As discussed in Section 2, hindsight has shown that the Gent and McWilliams (1990) paper would have been much clearer if it had described GM as a purely adiabatic extra advection. However, the proposed parameterization form has stood the test of time, and is still used in a large majority of ocean circulation models and virtually all climate models with non-eddy-resolving resolution. There must be close to 100 published papers documenting the improvements when GM and Redi diffusion replace horizontal diffusion; I reviewed a very large number of them! I know several young scientists who started their careers by implementing GM into a particular ocean model, and wrote a paper documenting the new results. There have been suggestions that eddy effects should be parameterized differently, and that there should be additional terms to the GM form, but I am not aware of any published results from global circulation models using either of these proposals.

Why did GM work so well and was the major reason that climate models were able to run without flux corrections? As discussed in Section 3, one reason is that it allowed *z*-coordinate ocean models to run with no horizontal diffusion at all, which eliminated the Veronis effect and made the deep ocean much colder and more realistic. It later turned out that this could have been achieved just by implementing the Redi term. However, global solutions with just the Redi term would not benefit from the proactive results of GM, which are especially large near the ACC. Fig. 1 shows how the eddy-induced meridional overturning strongly opposes the mean flow near the ACC, and Fig. 2 shows that this results in much reduced southern hemisphere convection, so that deep water formation occurs in realistic small regions. It is these features that made the largest difference in climate models, and allowed stable control runs without flux corrections.

There are several active areas of research about how GM should be implemented. Should it be as an extra advection, as originally in Danabasoglu et al. (1994), or as a skew diffusion, as proposed by Griffies (1998)? He showed how the small-slope Redi and GM terms could be combined into a single tensor where two of the off-diagonal terms are zero if the coefficients are equal. What is the most physically realistic way to turn GM off in the mixed layer, where horizontal diffusion should be used? What is the best way to parameterize the GM and Redi coefficients as functions of space or mean flow variables? An important question is should an eddy parameterization be purely adiabatic, or have a diapycnal mixing term that is added to the vertical mixing scheme? Guidance on this question is likely to come from analyzing results from eddy-resolving simulations. This is not straightforward, however, especially if diapycnal mixing results from the *z*-coordinate model's advection scheme and the very often used horizontal biharmonic tracer diffusion.

There have been some proposals about the form of the momentum equation. Gent and McWilliams (1996) keep the mean velocity as the dependent variable, but propose that it is advected by the total velocity, which is consistent with the residual-mean and TRM theories. As far as I am aware, this has never been implemented in an ocean climate component, probably because it would make little difference at non-eddy-resolving resolution. However, it would make more of a difference if the model resolution is finer.

Greatbatch and Lamb (1990) showed very shortly after GM 1990 was published, that, if the Coriolis force in the momentum equation is written in terms of the total velocity, then the GM parameterization can be written as a vertical viscosity term. Using the geostrophic approximation, the viscosity coefficient becomes $\kappa f^2/N^2$, where f is the Coriolis parameter and N the buoyancy frequency. This idea was subsequently developed further, and two recent modeling studies by Ferreira and Marshall (2006) and Zhao and Vallis (2008) change all terms in the momentum equation to involve just the total velocity. This approximation has the advantage that all the model equations then only involve a single velocity variable. However, the vertical viscosity coefficient goes to zero at the equator, so that this GM implementation differs from the standard tracer equation implementation, which has a non-zero effect at the equator. Thus, solutions using momentum and tracer implementations of GM will differ near the equator and, therefore, differ globally. The momentum implementation of GM still must be blended with horizontal diffusion in the mixed layer, and the solutions are again quite sensitive to how this is done. In addition, it could be argued that the model diapycnal mixing scheme and surface flux calculations should use the mean velocity and not the total velocity. If this is done, then the momentum formulation does not eliminate the need to carry two velocities. Consequently, I do not see compelling reasons to change to this much less familiar GM formulation for ocean models.

Finally, I will briefly discuss GM in relation to alternative eddy parameterizations based on potential vorticity. Killworth (1997), Treguier et al. (1997) and Marshall et al. (1999) all propose that the additional advection velocity due to eddies should be based on a down-gradient assumption on potential vorticity, rather than the GM form based on thickness. Most frequently the potential vorticity is approximated by just its planetary vorticity component, f/h_ρ, but even this simple form gives an additional "beta" term proportional to the meridional gradient of f. The main reason I do not like this form for the eddy-induced velocity is because it breaks the important property of GM discussed in Section 2 that it is a positive definite sink of the global potential energy. In fact, Adcock and Marshall (2000) show that the advective form based on potential vorticity can produce a spurious source of potential energy when the flow interacts with large variations in bottom topography.

In addition, an idea originally proposed by Welander (1973) that predates GM is that mesoscale eddy effects should be parameterized as advection and diffusion along isopycnals of the full potential vorticity, because it satisfies the same inviscid equation as a passive tracer. However, Ringler and Gent (2010) show that this assumption leads to horizontal viscous terms in the momentum equation that do not satisfy two of the usual properties assumed for the viscous terms. In fact, none of the usual forms assumed for horizontal viscosity, even in models formulated in isopycnal coordinates, lead to potential vorticity being diffused along isopycnals, which is the assumption made for a passive tracer, see Gent and McWilliams (1996). After 20 years of hindsight, Jim McWilliams and I are very comfortable with this because we think that isopycnal flattening due to baroclinic instability, which GM parameterizes, is a much more generic and widespread property of ocean dynamics than is mixing of potential vorticity.

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